Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 2

1. (a) **EITHER**
annual cycle for daylight length ***R1***

OR
there is a minimum length for daylight (cannot be negative) ***R1***

OR
a quadratic could not have a maximum and a minimum or equivalent ***R1***

Note: Do not accept "Paula's model is better".

[1 mark]

(b) (i) 4 ***A1***

(ii) 12 ***A1***

(iii) $y = 12$ ***A1A1***

Note: Award ***A1*** "$y =$ (a constant)" and ***A1*** for that constant being 12.

[4 marks]

(c) $f(t) = -4\cos(30t) + 12$ **OR** $f(t) = -4\cos(-30t) + 12$ ***A1A1A1***

Note: Award ***A1*** for $b = 30$ (or $b = -30$), ***A1*** for $a = -4$, and ***A1*** for $d = 12$. Award at most ***A1A1A0*** if extra terms are seen or form is incorrect. Award at most ***A1A1A0*** if x is used instead of t.

[3 marks]

(d) $10.5 = -4\cos(30t) + 12$ ***(M1)***

EITHER
$t_1 = 2.26585\ldots,\ t_2 = 9.73414\ldots$ ***(A1)(A1)***

OR
$t_1 = \frac{1}{30}\cos^{-1}\frac{3}{8}$ ***(A1)***
$t_2 = 12 - t_1$ ***(A1)***

THEN
$9.73414\ldots - 2.26585\ldots$
7.47 $(7.46828\ldots)$ months $(0.622356\ldots$years) ***A1***

Note: Award ***M1A1A1A0*** for an unsupported answer of 7.46. If there is only one intersection point, award ***M1A1A0A0***.

[4 marks]

continued…

Question 1 continued

(e) $\left|\dfrac{16-\left(16+\dfrac{14}{60}\right)}{16+\dfrac{14}{60}}\right|\times 100\%$ ***(M1)(M1)***

Note: Award ***M1*** for correct values and absolute value signs, ***M1*** for $\times 100$.

$=1.44\%$ $(1.43737...\%)$ ***A1***

[3 marks]
[Total 15 marks]

2. (a) (i) 30 ***A1***

(ii) 40 ***A1***

[2 marks]

(b) arithmetic formula chosen ***(M1)***

(i) $w_n = 20+(n-1)10$ $(=10+10n)$ ***A1***

(ii) $l_n = 30+(n-1)10$ $(=20+10n)$ ***A1***

[3 marks]

(c) (i) $740 = 30+(n-1)10$ **OR** $740 = 20+10n$ ***M1***
$n = 72$ ***A1***
144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

(ii) $w_{72} = 730$ ***A1***

[3 marks]

(d) $(10\times 20)\times 144$ ***(M1)***
$= 28800$ ***(A1)***
2.88×10^4 cm^2 ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

continued…

Question 2 continued

(e) **EITHER**

1 square metre $= 100\,\text{cm} \times 100\,\text{cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10\text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2\,\text{m} \times 0.1\,\text{m} \times 5 =)\ 0.1\ \text{m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$\$2.45$ per pack (of 5 tiles) ***A1***

[3 marks]

(f) $\frac{1.08 \times 144}{5}\ (= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

(g) $35 + (32 \times 2.45)$ ***(M1)***

$\$113\ \ (113.4)$ ***A1***

[2 marks]

[Total 19 marks]

3. (a) (i) $\frac{370+472}{2}$ ***(M1)***

> **Note:** This ***(M1)*** can also be awarded for either a correct Q_3 or a correct Q_1 in part (a)(ii).

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their Q_1 (clearly stated) ***(M1)***

IQR $= (421 - 318 =)\ 103$ ***A1***

[4 marks]

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

> **Note:** The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.

[3 marks]

(c) not appropriate ("no" is sufficient) ***A1***

as r is too close to zero / too weak a correlation ***R1***

[2 marks]

(d) (i) 6 ***A1***

(ii) 4.5 ***A1***

(iii) 4.5 ***A1***

[3 marks]

(e) (i) $r_s = 0.683\ (0.682646...)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

OR

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of 0.249). ***A1***

[3 marks]

(f) lowering the top score by 20 does not change its rank so r_s is unchanged ***R1***

> **Note:** Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."

[1 mark]

[Total 16 marks]

4. (a) (i) $\left(\frac{1}{2}\hat{AOB}=\right)\ \arccos\left(\frac{4}{4.5}\right)=27.266...$ ***(M1)(A1)***

$\hat{AOB}=54.532...\approx 54.5^{\circ}$ $(0.951764...\approx 0.952$ radians) ***A1***

Note: Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) finding area of triangle

EITHER

area of triangle = $\frac{1}{2}\times 4.5^2\times\sin(54.532...)$ ***(M1)***

Note: Award ***M1*** for correct substitution into formula.

$=8.24621...\approx 8.25\ \text{m}^2$ ***(A1)***

OR

$AB=2\times\sqrt{4.5^2-4^2}=4.1231\ldots$

area triangle $=\frac{4.1231\ldots\times 4}{2}$ ***(M1)***

$=8.24621...\approx 8.25\ \text{m}^2$ ***(A1)***

finding area of sector

EITHER

area of sector = $\frac{54.532...}{360}\times\pi\times 4.5^2$ ***(M1)***

$=9.63661...\approx 9.64\ \text{m}^2$ ***(A1)***

OR

area of sector = $\frac{1}{2}\times 0.9517641..\times 4.5^2$ ***(M1)***

$=9.63661...\approx 9.64\ \text{m}^2$ ***(A1)***

THEN

area of segment = $9.63661...-8.24621...$

$=1.39\ \text{m}^2$ $(1.39040...)$ ***A1***

[8 marks]

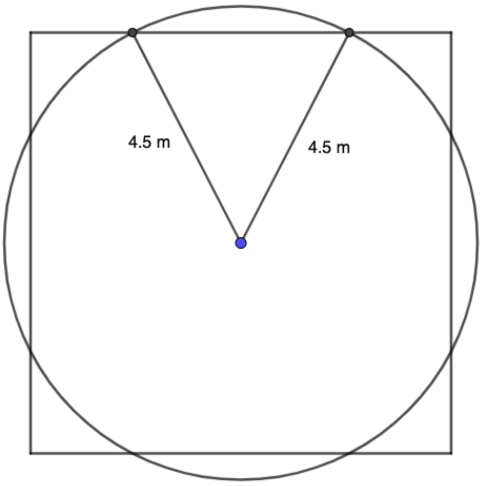
continued…

Question 4 continued

(b) (i) $\pi \times 4.5^2$ ***(M1)***

$63.6\ \text{m}^2\ \ (63.6172\ldots\ \text{m}^2)$ ***A1***

(ii) **METHOD 1**



$4 \times 1.39040\ldots\ \ (5.56160)$ ***(A1)***

subtraction of four segments from area of circle ***(M1)***

$= 58.1\ \text{m}^2\ \ (58.055\ldots)$ ***A1***

METHOD 2

$4(0.5 \times 4.5^2 \times \sin 54.532\ldots) + 4\left(\frac{35.4679}{360} \times \pi \times 4.5^2\right)$ ***(M1)***

$= \ 32.9845\ldots + 25.0707$ ***(A1)***

$= 58.1\ \text{m}^2\ \ (58.055\ldots)$ ***A1***

[5 marks]

(c) sketch of $\frac{\mathrm{d}V}{\mathrm{d}t}$ **OR** $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.110363\ldots$ **OR** attempt to find where $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$ ***(M1)***

$t = 1$ hour ***A1***

[2 marks]

[Total 15 marks]

5. (a) (let T be the number of passengers who arrive)

$\left(\mathrm{P}(T > 72) =\right)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ \ (0.00379124...)$ ***A1***

> **Note:** Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

> **Note:** Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

OR

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600\ \ (= 1.26070...)$ ***(M1)A1A1***

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ ***(M1)***

$= 11098.73..\ \ (= \$11099)$ ***A1***

THEN

income for 72 tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

continued…

Question 5 continued

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$P(T = 73) = 0.00338014\ldots,\ P(T = 74) = 0.000411098\ldots$ ***A1A1***

$E(C) = 0.003380\ldots \times 300 + 0.0004110\ldots \times 600\ \ (= 1.26070\ldots)$ ***(M1)A1A1***

extra expected revenue $= 300 - 1.01404\ldots - 0.246658\ldots\ \ (300 - 1.26070\ldots)$

(A1)(M1)

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$= \$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T \le 72$	$T = 73$	$T = 74$
Change in income	300	0	−300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$P(T \le 73) = 0.9962\ldots,\ P(T = 74) = 0.000411098\ldots$ ***A1A1***

$E(D) = 300 \times 0.9962\ldots + 0 \times 0.003380\ldots - 300 \times 0.0004110$ ***(M1)A1A1***

$= \$299$ ***A1***

[8 marks]

[Total 15 marks]