Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 1

16 pages

1. height of triangle at roof $= 1.35 - 0.9 = 0.45$ ***(A1)***

Note: Award ***A1*** for 0.45 (height of triangle) seen on the diagram.

slant height $= \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ ***(M1)***

$= \sqrt{0.405}$ $(0.636396..., 0.45\sqrt{2})$ ***A1***

Note: If using $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ then ***(A1)*** for angle of 45°, ***(M1)*** for a correct trig statement.

area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756...)$ ***M1***

area painted $= (2 \times \sqrt{0.405} \times 0.9 = 2 \times 0.572756...)$

1.15 m^2 $(1.14551... \text{ m}^2, 0.81\sqrt{2} \text{ m}^2)$ ***A1***

[Total 5 marks]

2. (a) $\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01$ $(8.00812...)$ m ***A1***

[2 marks]

(b) $\text{F}\hat{\text{A}}\text{O} = \sin^{-1}\left(\dfrac{5.8}{8.00812...}\right)$ **OR** $\cos^{-1}\left(\dfrac{5.52177...}{8.00812...}\right)$ **OR** $\tan^{-1}\left(\dfrac{5.8}{5.52177...}\right)$ ***(M1)***

46.4° $(46.4077...^\circ)$ ***A1***

[2 marks]

[Total 4 marks]

3. (a) 1.2 metres ***A1***

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)$ 4.43 s $(4.431415...$ s$)$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \le t \le 4.43$ **OR** $[0, 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]

[Total 5 marks]

4. (a) midpoint $(1, 2.5)$ ***A1***

$m_{AB} = \frac{6-(-1)}{8-(-6)} = \frac{1}{2}$ ***(M1)A1***

Note: Accept equivalent gradient statements including using midpoint.

$m_{\perp} = -2$ ***M1***

Note: Award ***M1*** for finding the negative reciprocal of their gradient.

$y - 2.5 = -2(x-1)$ **OR** $y = -2x + \frac{9}{2}$ **OR** $4x + 2y - 9 = 0$ ***A1***

[5 marks]

(b) substituting $x = -6$ into their equation from part (a) ***(M1)***

$y = -2(-6) + \frac{9}{2}$

$y = 16.5$ ***A1***

Note: Award ***M1A0*** for $(-6, 16.5)$ as their final answer.

[2 marks]
[Total 7 marks]

5. (a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

Note: Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as x, y or z. Accept answers written in words: e.g. 308 adult tickets.

[2 marks]
[Total 5 marks]

6. (a) $\frac{1}{2}\left(0.6+0+2(1.2+1.2)\right)$ ***(A1)(M1)***

Note: Award ***A1*** for evidence of $h=1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

$2.7\ \text{m}^2$ ***A1***

[3 marks]

(b) $\int_{-1}^{2}\frac{-x^3-3x^2+4x+12}{10}\,\text{d}x$ **OR** $\int_{-1}^{2}f(x)\,\text{d}x$ ***(M1)***

Note: Award ***M1*** for using definite integration with correct limits.

$2.925\ \text{m}^2$ ***A1***

Note: Question requires exact answer, do not award final ***A1*** for 2.93.

[2 marks]

(c) $9-2.925$ ***(M1)***

Note: Award ***M1*** for 9 seen as part of a subtraction.

$=6.08\ \text{m}^2\ \ (6.075)$ ***A1***

[2 marks]
[Total 7 marks]

7. (a) H_0 : The die is fair **OR** $P(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1 : The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal ***A1***

[1 mark]

(b) 5 ***A1***

[1 mark]

(c) 10 ***A1***

[1 mark]

(d) $(p\text{-value} =)\ 0.287\ (0.28724163\ldots.)$ ***A2***

[2 marks]

(e) $0.287 > 0.05$ ***R1***

EITHER
Insufficient evidence to reject the null hypothesis ***A1***

OR
Insufficient evidence to reject that the die is fair ***A1***

Note: Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their p-value and their hypothesis.

[2 marks]
[Total 7 marks]

8. (a) 50% ***A1***

Note: Do not accept 0.5 or $\frac{1}{2}$.

[1 mark]

(b) $0.0478\ (0.0477903\ldots, 4.78\%)$ ***A2***

[2 marks]

(c) $P(X < k) = 0.98$ **OR** $P(X > k) = 0.02$ ***(M1)***

Note: Award ***(M1)*** for a sketch with correct region identified.

$506\ \text{g}\ (506.161\ldots)$ ***A2***

[3 marks]
[Total 6 marks]

9. (a) $f'(x) = -2x^{-2} + 6x$ **OR** $f'(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

Note: Award ***A1*** for $6x$ seen, and ***(M1)*** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from either x^{-2} or $\frac{2}{x^2}$ seen in their final answer), ***A1*** for $-\frac{2}{x^2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

[3 marks]

(b) finding gradient at $x = 1$

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_{\perp} = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

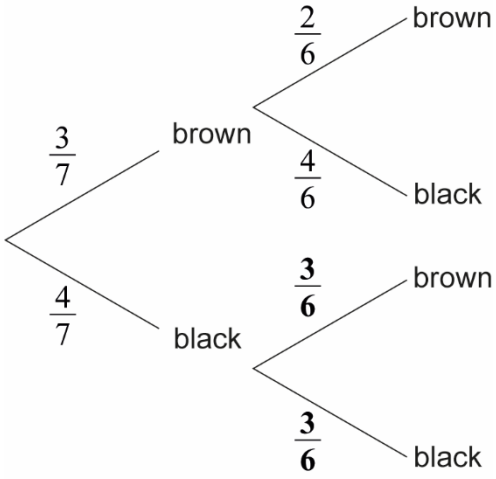
Note: Award ***M1*** for correctly substituting $x = 1$ and $y = 2$ and their $m_{\perp}$.

$x + 4y - 9 = 0$ ***A1***

Note: Do not award the final ***A1*** if the answer is not in the required form. Accept integer multiples of the equation.

[4 marks]
[Total 7 marks]

10. (a)



A1

Note: Award ***A1*** for both missing probabilities correct.

[1 mark]

(b) multiplying along branches and then adding outcomes ***(M1)***

$$\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{3}{6}$$

$$=\frac{18}{42}\left(=\frac{3}{7}\approx 0.429\ (42.9\%)\right)$$ ***A1***

[2 marks]

(c) use of conditional probability formula ***M1***

$$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$$ ***A1***

$$=\frac{6}{18}\left(=\frac{1}{3}\right)\ \left(\frac{252}{756},\ 0.333,\ 33.3\%\right)$$ ***A1***

[3 marks]
[Total 6 marks]

11. (a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ (= 10^5)$ ***A1***

[2 marks]

(c) $0.001 < N < 100000 \ (10^{-3} < N < 10^5)$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

[2 marks]

(d) $N = \frac{10^5}{10^{7.2}} \ (= 0.0063095...)$ ***(M1)***

length of time = $\frac{1}{0.0063095...} = 10^{2.2}$

= 158 years ***A1***

[2 marks]

[Total 8 marks]

12. (a) **METHOD 1**

(when $t = 2$)

$\frac{\mathrm{d}P}{\mathrm{d}t} = -4$ **OR** $\frac{\mathrm{d}P}{\mathrm{d}t} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore P is decreasing ***A1***

METHOD 2

sketch with $t = 2$ indicated in 4th quadrant **OR** t-intercepts identified ***M1***

therefore P is decreasing ***A1***

[2 marks]

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

Note: Award ***M1*** for substituting $(1, 4)$ into their equation with $+c$ seen.

$c = 7$

$P(t) = t^3 - 4t^2 + 7$ ***A1***

[4 marks]

[Total 6 marks]

13. (a) use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68$ m

$= 68$ cm ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68$ cm ***AG***

[2 marks]

continued…

Question 13 continued

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \dfrac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \dfrac{(1.8)(1 - 0.85^4)}{1 - 0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

Note: Answers may be given in cm.

[3 marks]
[Total 7 marks]