

Name:

Result:

1.*(9 points)*

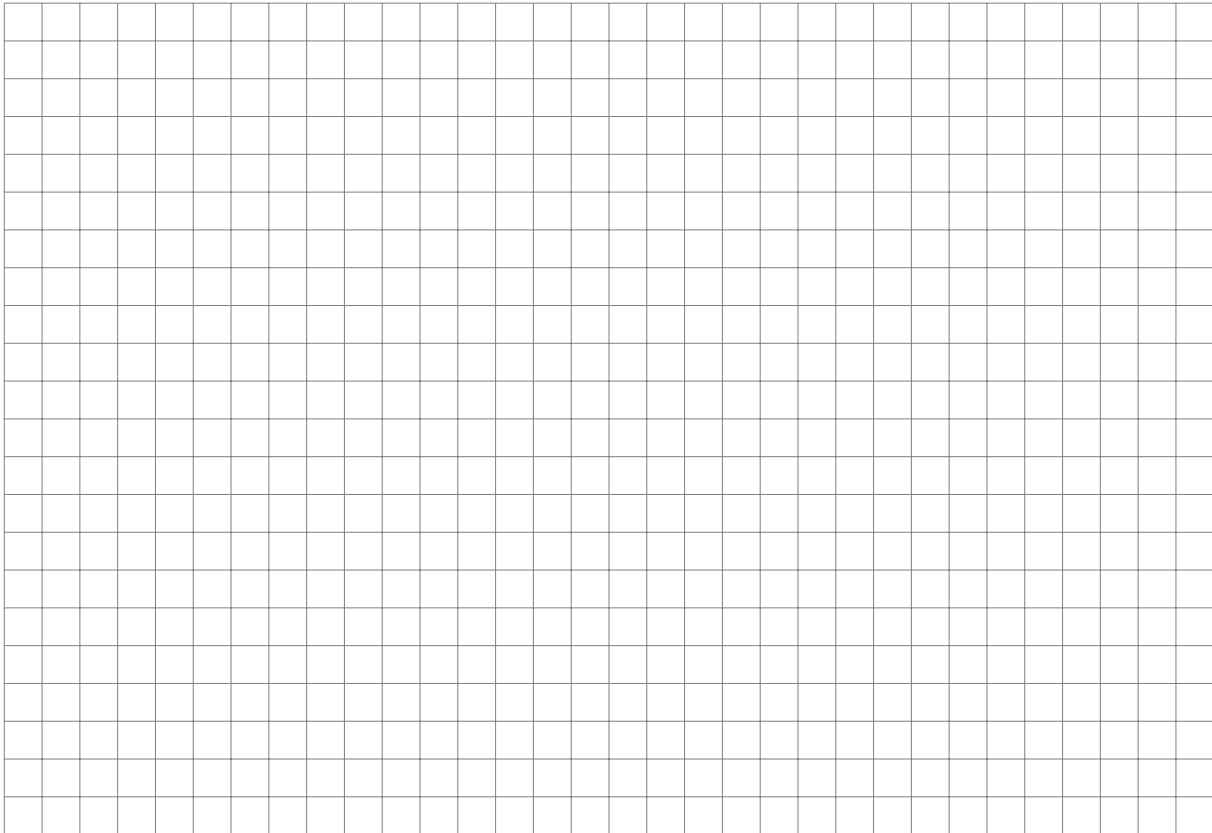
Consider a function $f(x) = \frac{2x + 4}{x + 3}$.

(a) The graph of $y = f(x)$ can be obtained from the graph of $y = \frac{1}{x}$ using a sequence of transformations. Describe these transformations.

(b) Sketch the graph of $y = f(x)$. Clearly indicate the equations of asymptotes and any axes intercepts.

(c) On the same diagram sketch the graph of $y = |2x + 1|$.

(d) Solve the inequality $\frac{2x + 4}{x + 3} \geq |2x + 1|$



2.*(7 points)*

Consider a function $f(x) = x^2 + 2x + 5$, where $x \in \mathbb{R}$.

(a) Let $g(x) = \sqrt{8 - x}$. Find the domain of $(g \circ f)(x)$.

(b) Let $h(x) = 2^{x-5}$. Find the range of $(h \circ f)(x)$.

(c) Find the maximum possible domain, in the form $x \leq a$, so the $f(x)$ has an inverse function and find this inverse function.

3.*(21 points)*

Solve the following equations and inequalities. In part (d) give your answer in the form $\frac{\ln p}{\ln q}$, where $p, q \in \mathbb{Q}$.

(a) $|3x - 2| \leq |x + 1|$

(b) $|x - 2| + |x + 1| = x + 2$

(c) $3^{2x+1} + 4 = 13 \cdot 3^x$

(d) $5^{x-2} = 3^{2x+1}$

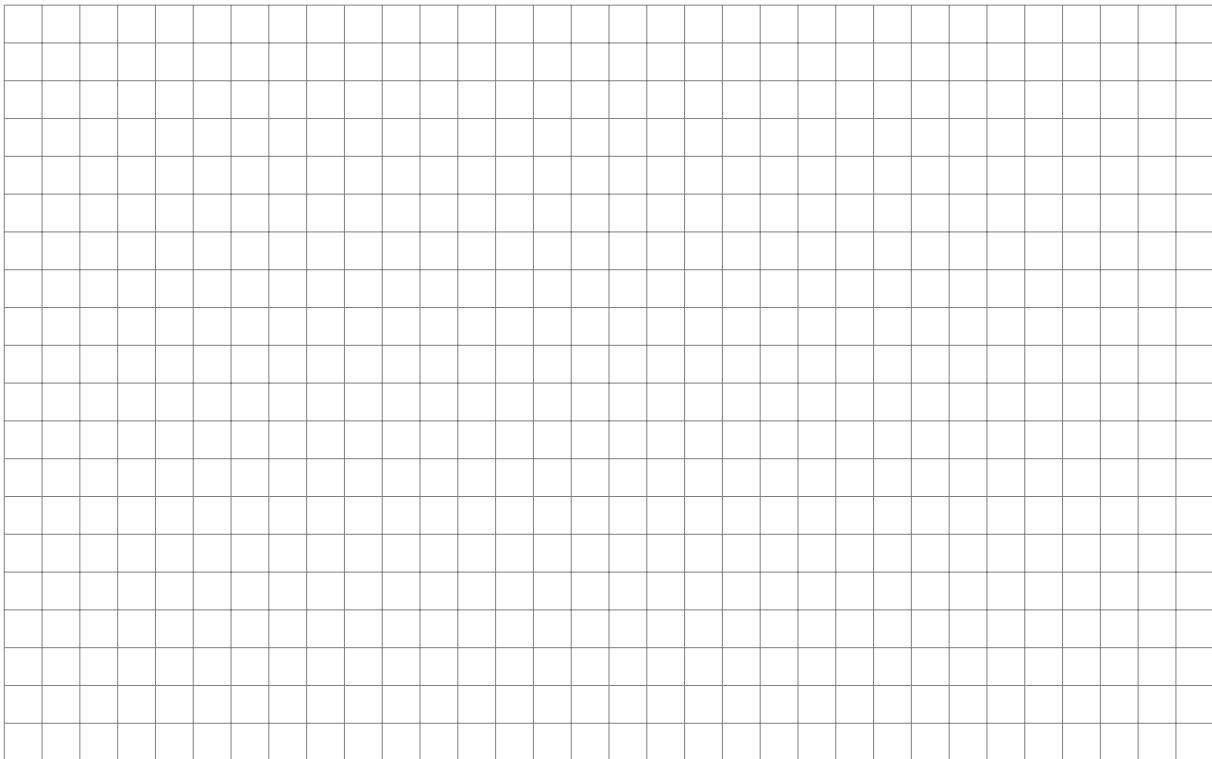
(e) $\left(\frac{1}{4}\right)^{x+2} < (\sqrt[3]{4})^{6x-9}$

(f) $\log_2(x-2) + \log_2(x-3) = 2 + \log_2 3$

(g) $2 \log_4 x = 2 \log_x 4 + 3$

4. (7 points)
 T denotes the temperature, measured in $^{\circ}\text{C}$, of a bowl of soup in a fridge after t minutes. It is given that the difference between T and 4°C decreases by 30% every 5 minutes. Initially the temperature of the soup was 50°C .

- (a) Find the formula for $T(t)$.
- (b) Sketch the graph of $T(t)$.
- (c) Interpret the 4°C given in the question.
- (d) Rearrange your answer to part (a) to find t as a function of T .



5.*(6 points)*

Consider the function $f(x) = |2^{2-|x|} - 1|$.

- (a) Sketch the graph of $y = f(x)$.
- (b) Explain why f does not have an inverse function.
- (c) The equation $f(x) = k$ has exactly two solutions. Find all possible values of k .

