

Name:

Result:

1.

(5 points)

The normal to the curve $y = a\sqrt{x} + \frac{b}{x}$ at $x = 1$ is given by the equation $2x - y + 3 = 0$.
Find the values of a and b .

2.*(5 points)*

Let $f(x) = \frac{1}{3}x^3 + ax^2 + bx - 2$. The graph of $f(x)$ has a local maximum at $x = -4$ and a local minimum at $x = 1$.

(a) Find the values of a and b . [3]

(b) Find the x -coordinate of the point of inflexion of the graph of $f(x)$. [2]

3.*(4 points)*

The following table shows value of $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ for various values of x :

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	2	1	2	5
1	3	4	1	2
2	3	1	-1	-4
3	1	0	-1	-1
4	0	2	-5	1

Let $h(x) = (f \circ g)(x)$. Find the equation of the normal to the graph of $h(x)$ at $x = 2$. Write your answer in the form $Ax + By + C = 0$, where $A, B, C \in \mathbb{Z}$.

4.

(5 points)

Consider the curve $y = \frac{1}{x}$ with $x > 0$. The tangent to this curve at the point where $x = a$ intersects the axes at points A and B . Find the area of the triangle OAB where O is the origin.

5.*(13 points)*

Consider the function $f(x) = \sqrt{x + \frac{1}{x-a}}$, where $x > a$ and $a > 0$.

- (a) Find $f'(x)$ in terms of a . [3]
- (b) The function is increasing for $x > 3$. Find a . [3]
- (c) State the coordinates of the minimum of the graph of $f(x)$. [1]
- (d) Using the graph of $f'(x)$, or otherwise, find the exact coordinates of the point of inflexion of the graph of $f(x)$. [4]
- (e) Sketch the graph of $f(x)$ indicating all the features you've found. [2]