Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Billy is a keen walker who keeps a record of his performance. The following table shows the time, in minutes, it takes him to walk one kilometre up hills with different gradients. The gradient of each hill is constant.

Gradient G (%)	0	4	10	15	20
Time T (min.)	6.85	8.42	11.20	14.49	17.88

(a) (i) Find the equation of the regression line of T on G.

(ii) Describe the correlation between T and G with reference to the value of r, the Pearson's product-moment correlation coefficient. [4]

On Sunday, Billy intends to walk up a hill with a gradient of 13%.

(b) Estimate the time it will take Billy to walk one kilometre up the hill. [2]

This morning, Billy walked one kilometre up a hill, and it took 22 minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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3. [Maximum mark: 6]

Consider the graph of the following function:

$$f(x) = \frac{16}{x} + \frac{x^2}{8}, \text{ for } x \neq 0.$$

(a) Write down the equation of the vertical asymptote of $f(x)$. [1]

(b) Find $f'(x)$. [3]

(c) Write down the interval in which $f(x)$ is increasing. [2]

4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $\mathrm{A}(0, 3)$, $\mathrm{B}(8, 3)$ and $\mathrm{C}(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

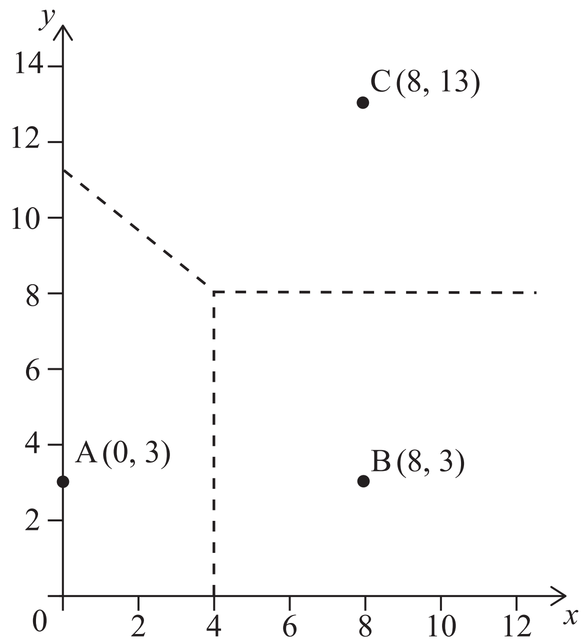
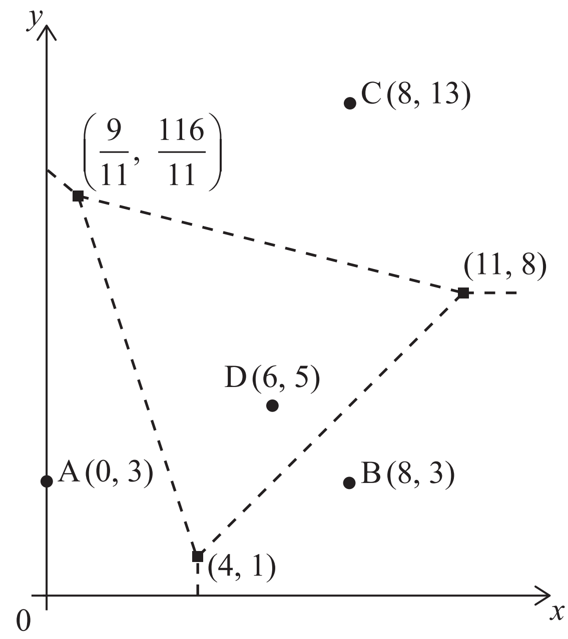


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[\mathrm{AC}]$. [3]

An additional farmhouse $\mathrm{D}(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

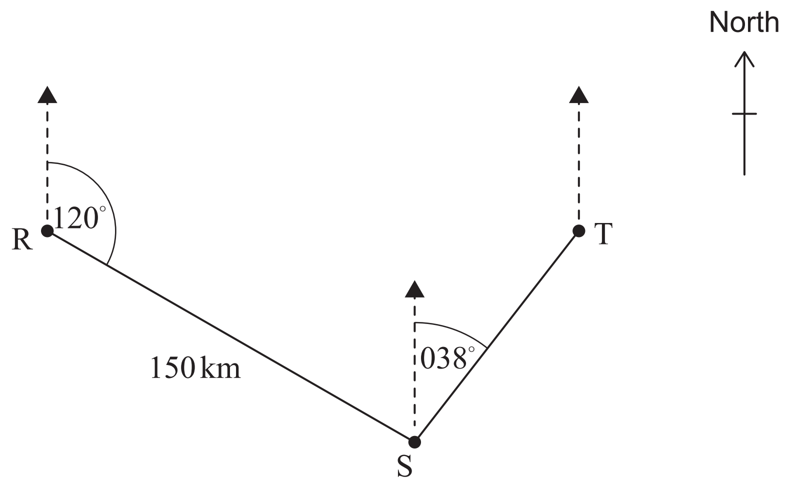
(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

Ron sails his boat from point R for a distance of $150\,\text{km}$, on a bearing of 120°, to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.

diagram not to scale



(a) Find $\text{R}\hat{\text{S}}\text{T}$. [2]

Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

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6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}\text{, for } x > 1\,.$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

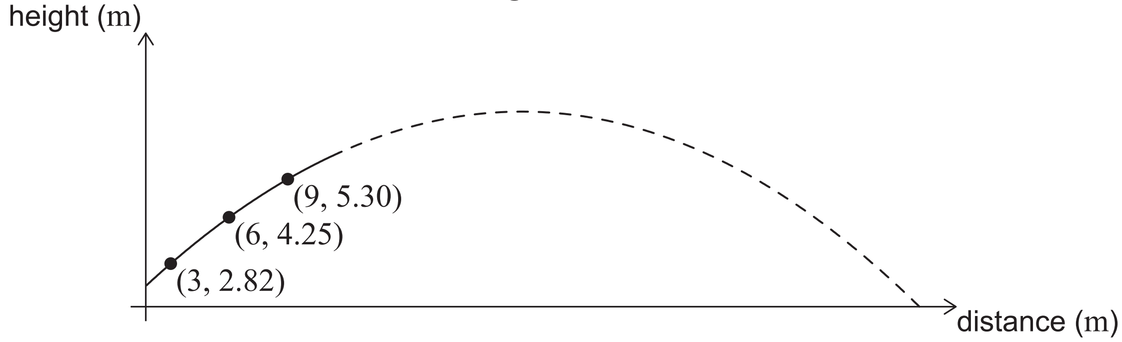
7. [Maximum mark: 9]

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points $(3, 2.82)$, $(6, 4.25)$ and $(9, 5.30)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



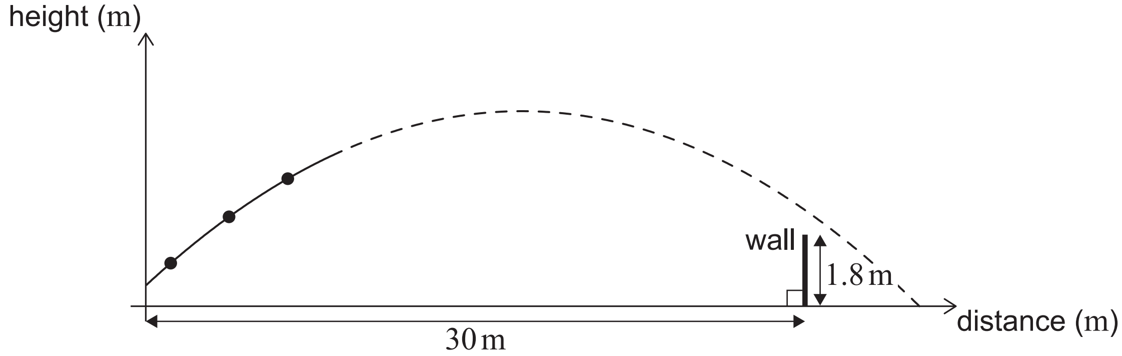
(a) Use the coordinates $(3, 2.82)$ to write down an equation in terms of a, b and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the discus will go over the wall. [3]

(d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 7 continued)

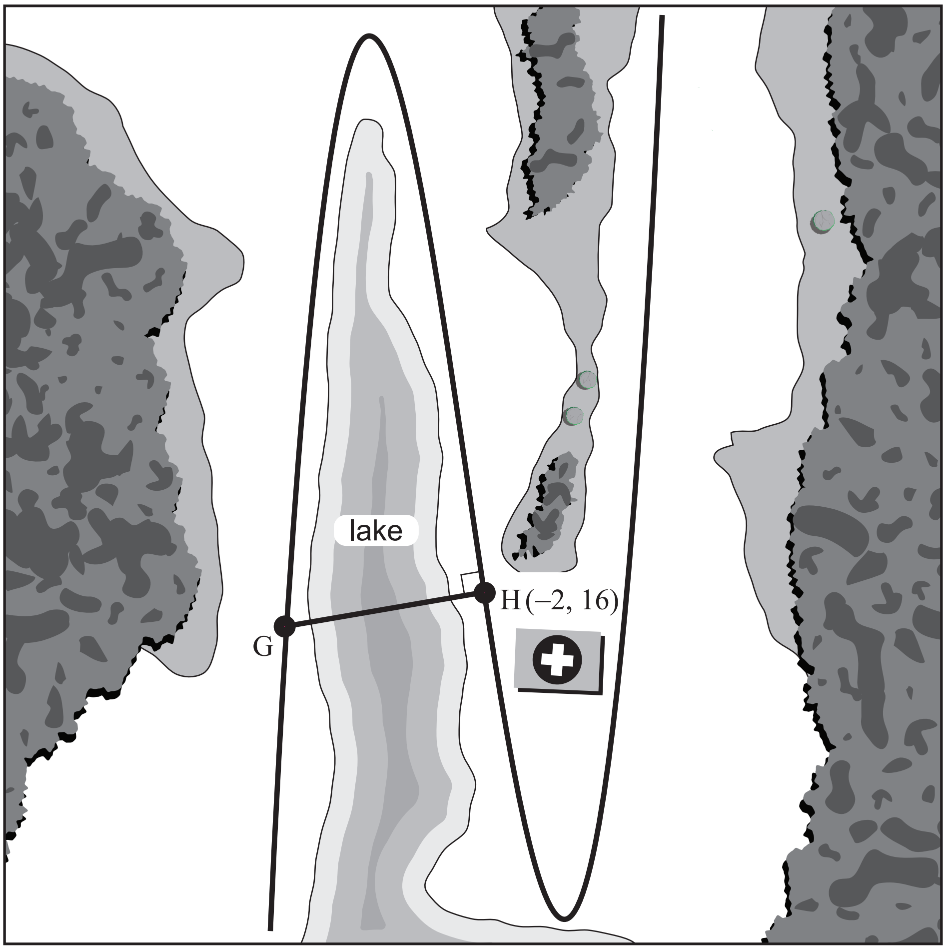
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+2)^3 + 3x^2 - 2x$. All distances in the map are in kilometres.

The local hospital is located at point H, which has coordinates $(-2, 16)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point H and will connect the hospital to point G.

diagram not to scale



(a) Using your graphic display calculator, find the value of $r'(-2)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point H, which can be used to model the new bridge. [2]

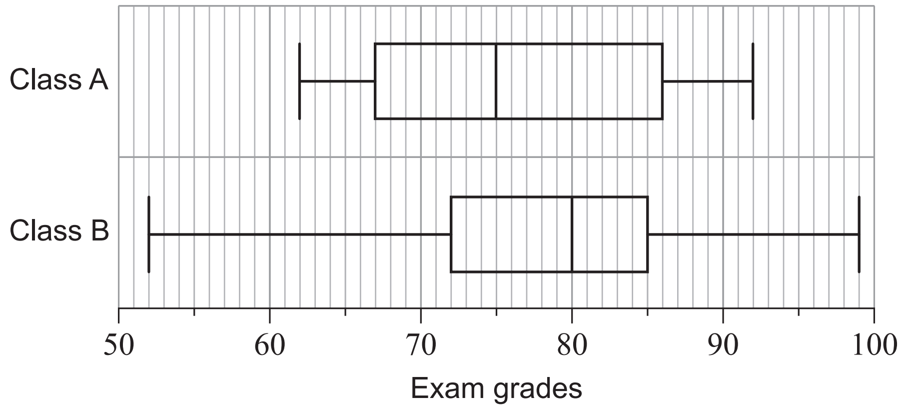
(c) Hence, determine the length of the new bridge. [3]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

Mrs Whitehouse is a chemistry teacher. After grading her final exams, she creates the following box and whisker diagram to compare the grades of her two classes.



(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

Statement	True (✓)
The data for Class A is normally distributed.	
A higher percentage of students in Class A received a grade less than 70 on the exam, than in Class B.	
More students in Class B received a grade greater than 90 on the exam than in Class A.	
The interquartile range for Class B is less than the interquartile range for Class A.	

At the end of the year, Mrs Whitehouse surveyed a random sample of students from each of her two large classes to determine how satisfied they were with her teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

Her collected data from the student surveys is shown.

Class A	7	5	3	4	3	8	6	5

Class B	6	9	8	10	1	9	10	9	8	3

(This question continues on the following page)

(Question 9 continued)

Mrs Whitehouse believes that there was no difference in the general satisfaction between the two classes. She assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a t-test at the 5% significance level.

(b) Write down the null and alternative hypotheses for her test. [2]

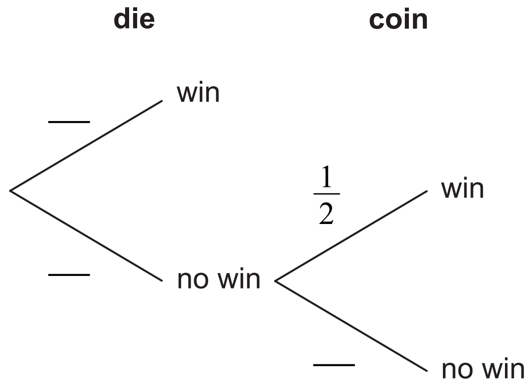
(c) Find the p-value for her test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

10. [Maximum mark: 7]

Rita is playing a game. In the game, she must roll a fair six-sided die. If she gets a five or six then she wins a prize. If not, then she has another chance but this time she must flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then Rita wins a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Rita does **not** win a prize. [2]

(c) Given that Rita won a prize, find the probability that she got a five or six when she rolled the die. [3]

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11. [Maximum mark: 6]

Nicole works at a local school 5 days each week. She drives an old car to work that has a 72% probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Nicole's car starts on exactly three mornings in a particular 5 day workweek. [2]

Nicole walks to work on mornings when her car does not start and it is **not** raining.
Nicole takes the bus to work on mornings when her car does not start and it is raining.

Where Nicole lives, there is a 42% probability of rain on any given morning, independent of any other morning. The probability of Nicole's car starting is independent of the weather.

(b) Find the probability that Nicole will **not** have to take the bus in a particular workweek. [4]

12. [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

Table 1

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Number of social media followers (in thousands)	11 500	12 400	1300	2300	674	49 500	315	94 400
Number of albums sold in first week (in thousands)	123	62.4	17.4	94.9	52.5	27	21.6	595.5

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

Table 2

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week								1

(This question continues on the following page)

(Question 12 continued)

(b) Calculate the value of r_s, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a 10% significance level with the following null hypothesis:

H_0: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of r_s for this test is 0.643.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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References:

2. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.