Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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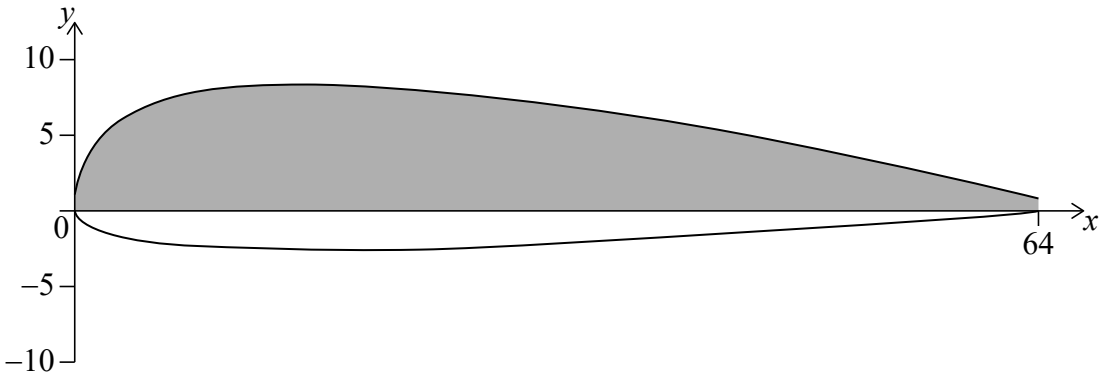
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{ for } 0 \le x \le 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table. [3]

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

(b) Find Jan's estimate of the shaded area of the cross-section. [3]

(c) (i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section.

(ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]

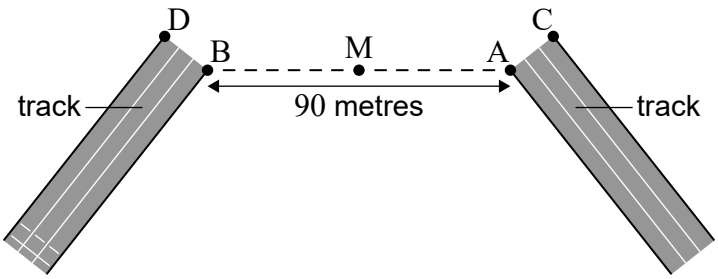
(d) Calculate the percentage error of Jan's estimate in part (b). [2]

2. [Maximum mark: 15]

Ansel is designing a racing track for a local bicycle club. The following diagram shows an incomplete portion of the track.

Ansel wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 90 metres.

diagram not to scale

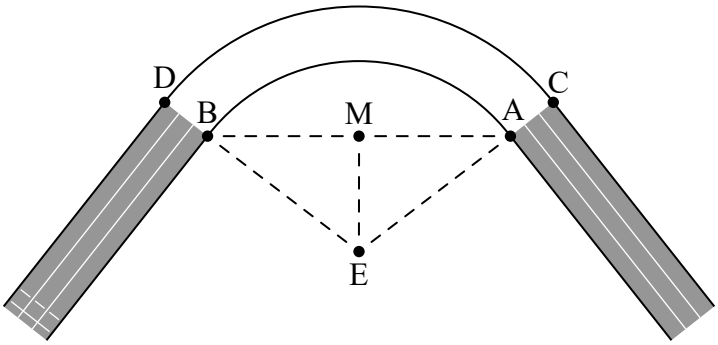


To create a smooth curve, Ansel first walks to M, the midpoint of [AB].

(a) Write down the length of [BM]. [1]

Ansel then walks 32 metres in a direction perpendicular to [AB] to get from point M to point E. Point E is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BE].

(ii) Find $\hat{BEM}$. [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre E, such that the track is 4 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 15 cm deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

3. [Maximum mark: 18]

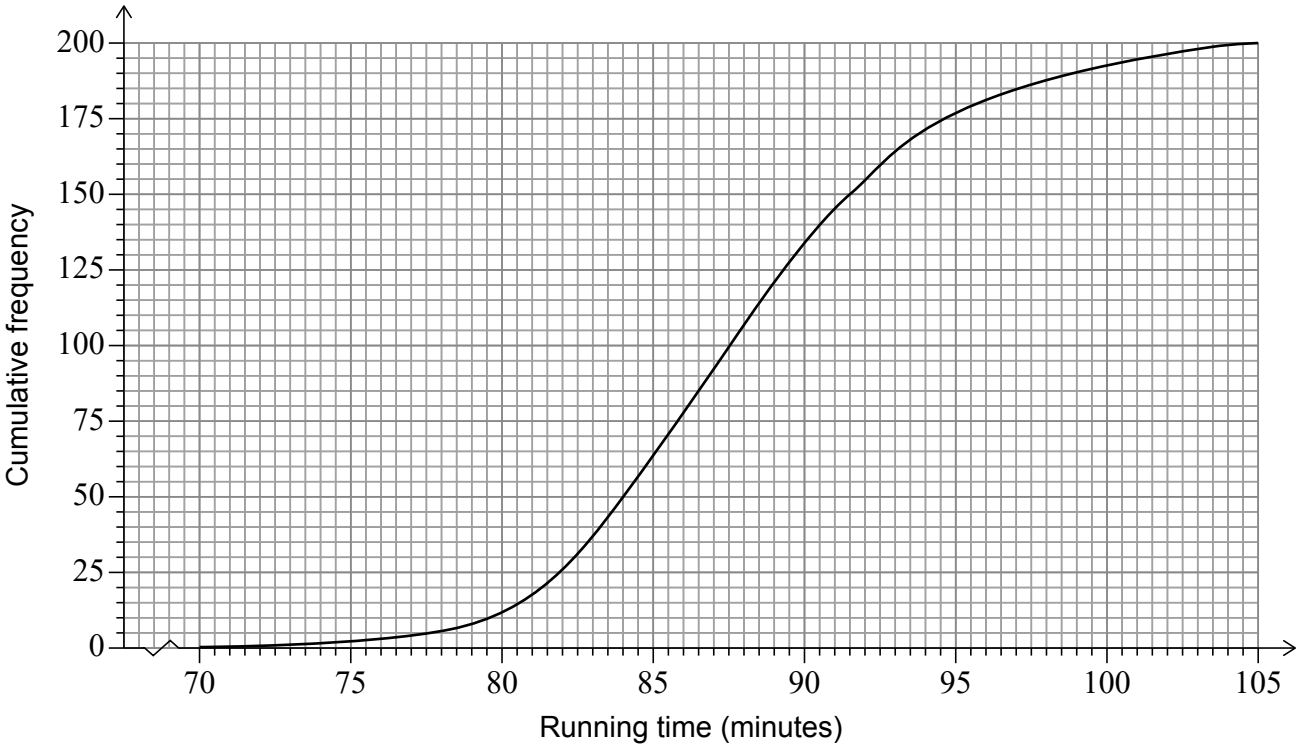
The running time, t (minutes), of 200 family movies are recorded in the following table.

Running time (minutes)	Frequency
$70 \le t < 80$	11
$80 \le t < 85$	51
$85 \le t < 90$	68
$90 \le t < 95$	47
$95 \le t < 105$	23

(a) (i) Write down the mid-interval value of $70 \le t < 80$.

(ii) Calculate an estimate of the mean running time of the 200 movies. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

"Star Feud" is a movie in the data set and its running time is 100 minutes.

(c) Use your answer to part (b) to estimate whether "Star Feud's" running time is an outlier for this data. Justify your answer. [3]

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(Question 3 continued)

It is believed that the running times of family movies follow a normal distribution with mean 88 minutes and standard deviation 6.75 minutes.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 movies could have plausibly been drawn from an underlying distribution $\mathrm{N}(88,\ 6.75^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Movie running time (minutes)	Observed frequency	Expected frequency
$t < 80$	11	23.6
$80 \leq t < 85$	51	42.1
$85 \leq t < 90$	68	a
$90 \leq t < 95$	47	46.7
$95 \leq t$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

Ruben wants to buy a car for a price of $285\,000$ South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to $25\,\%$ of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a $4.5\,\%$ nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays $4600\,\text{ZAR}$ each month.

(c) Find the total number of monthly payments he will need to make to pay off the loan. [2]

This strategy will result in Ruben's final payment being less than $4600\,\text{ZAR}$.

(d) Determine the amount of Ruben's final payment, to two decimal places. [4]

(e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

5. [Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for $16\,\text{EUR}$ each.

(a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]

(b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]

(c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x. [1]

Mosaic Industries has $10\,000\,\text{EUR}$ of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier $10\,\text{EUR}$ for each case.

(d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of $16\,\text{EUR}$. [3]

(e) Show that the average monthly profit for any selling price, x, can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]

(f) (i) Find $P'(x)$.

(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

(g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]

(h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]