Financial Mathematics

Introduction

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In almost all cases you will want to use the GDC for these problems. We will review this in detail, but we will also discuss a more algebraic approach.

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Interest rates

There are two types of interest rates: **simple interest** and **compound interest**. Both may appear on the exam, but vast majority of the questions are on the compound interest.

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Fixed amount is added each period.

Compound interest

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Simple interest

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If we invest 1000 PLN in a fund that pays 5% simple interest per year, it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) each year. So after 5 years we will have $1000 + 5 \cdot 50 = 1250$PLN.

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Simple interest - exercise

Tomasz invests 20000 PLN into savings account that pays 6% simple interest per year. Find the value of his investment after 7 years.

$$6\% \cdot 20000 = 120$$

So he gets 120 PLN each year.

After 7 years his investment will be worth:

$$20000 + 7 \cdot 120 = 28400$$

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Compound interest

In **compound interest** fixed percentage is added each period.

If we invest 1000 PLN in a fund that pays 5% compound interest (compounded annually), it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) after the first year, but we will get 52.5 PLN (because 5% of 1050 PLN is 52.5 PLN) in the second year and the amount we receive increases as we have more money in the account.

Because the amount we have increases by a fixed percentage, we can multiply it by 1.05 each year. Note that multiplying by 1.05 denotes the increase by 5%.

So after 5 years we will have:

$$1000 \cdot 1.05^5 = 1276.28$$

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Compound interest

Tomasz invests 50000 PLN into savings accound that pays 3% annual interest compounded yearly. Calculate the value of his investment after 8 years.

$$50000 \cdot 1.03^8 = 63338.50$$

We multiplied by 1.03 to denote the 3% increase. 1.03 is to the 8th power because we want to know the value of the investment after 8 years.

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Compound interest GDC

To solve compound interest problems on **Ti-84** you need to press APPS and find Finance.

Compound interest GDC

When using GDC you need to input the following data:

- n - number of periods/payments,
- I% - annual interest rate,
- PV - present value,
- PMT - payments,
- FV - future value,
- P/Y - payments (or periods) per year,
- C/Y - compounding per year.

Note that we use sign (+ or -) to indicate the direction of the money. For example -1000 means that we lose (or invest) the money, +1000 means that we get (or borrow, or withdraw) the money.

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Compound interest GDC

In our example we have

$$\begin{aligned}
\text{n} &= 8 \text{ (our period is a year and we want 8 years)}\\
\text{I\%} &= 3\\
\text{PV} &= -50000 \text{ (minus sign indicates that we invest this money)}\\
\text{PMT} &= 0 \text{ (there are no payments)}\\
\text{FV} &= ? \text{ (this doesn't matter, we need find it)}\\
\text{P/Y} &= 1 \text{ (there is one year in a year)}\\
\text{C/Y} &= 1 \text{ (it is compounded once a year)}
\end{aligned}$$

You need to highlight FV and press ALPHA-ENTER (Solve).

You should get FV= 63338.50

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You should get FV$=$ 63338.50

Compound interest - exercise 1

Let's consider a different example.

Tomasz invests 20000 PLN. The bank pays 4% annual interest rate and the interest is compounded every 6 months. Calculate the value of the investment after 7 years.

The annual interest rate is 4%, but it is compounded twice a year, so every 6 months we receive 2%. In 7 years there are 14 6-month periods, so we can calculate the value of the investment as follows:

$$20000 \cdot 1.02^{14} = 26389.58$$

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Compound interest - exercise 1

Doing the same calculation using TVM on GDC:

n = 14 (we have 14 6-month periods)
I% = 4 (this is the annual interest rate)
PV = −20000 (remember about the minus sign)
PMT = 0 (there are no payments)
FV = ? (this is what we're looking for)
P/Y = 2 (there are 2 6-month periods in a year)
C/Y = 2 (the interest is compounded twice a year)

We get FV= 26389.58

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Compound interest - exercise 2

Tomasz invests 40000 PLN. The bank pays $r\%$ annual interest rate and compounded monthly. Calculate r if after 5 years the value of the investment is 45320.04 PLN.

We can try solving this algebraically by solving the following equation:

$$45320.04 = 40000 \cdot \left(1 + \frac{r}{12 \cdot 100}\right)^{60}$$

We should get $r = 2.50$. Think about this equation. It helps if you understand what is happening here. The 12 in the denominator comes from the fact that we are looking for annual interest rate, but since it is compounded monthly than each month we get $\frac{r}{12}\%$. It is, of course, much easier to simply use TVM on GDC.

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Compound interest - exercise 2

We have:

$n = 60$ (we have 60 months in 5 years)
$I\% = ?$ (unknown)
$PV = -40000$ (remember about the minus sign)
$PMT = 0$ (there are no payments)
$FV = 45320.04$ (no minus here, we're getting this money)
$P/Y = 12$ (12 months in a year)
$C/Y = 12$ (interest is compounded 12 times a year)

We get that the interest rate is $r = 2.50$.

Compound interest - exercise 2

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\begin{aligned}
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We get that the interest rate is $r = 2.50$.

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Compound interest - exercise 3

Tomasz would like to have 100 000 PLN on his account in 10 years time. How much does he need to invest into savings account that pays an interest of 2% compounded quarterly?

Algebraically we would do it as follows:

$$100000 = x \cdot (1.005)^{40}$$

We multiply by 1.005 because we earn 0.5% per quarter. The power is 40, because we have 40 quarters in 10 years. The answer we get is $x = 81913.89$.

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Compound interest - exercise 3

Now let's use TVM:

n = 40 (we have 40 quarters in 10 years)
I% = 2 (annual interest rate)
PV = ? (this will be negative)
PMT = 0 (there are no payments)
FV = 100 000 (we want to have this much)
P/Y = 4 (4 quarters in a year)
C/Y = 4 (compounded 4 times a year)

We get that $PV = -81913.89$. So he should invest 81913.89 PLN. Do not use the negative sign in your final answer.

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Payments

Note that in all examples so far we've set $PMT = 0$ as there were no payments made during the investment.

PMT is any money that is paid in or withdrawn during the investment.

Note that in the IB examination payments will always be made at the end of the period. So make sure you have Payment:END.

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Note that in the IB examination payments will always be made **at the end** of the period. So make sure you have Payment:END.

Payments

Tomasz invests 100 000 PLN into savings account that pays 3% annual interest compounded monthly. At the end of each month he withdraws 1500 PLN from his account. Calculate how much money does he have on his account after 4 years.

Here we will go directly to TVM.

Payments

Tomasz invests 100 000 PLN into savings account that pays 3% annual interest compounded monthly. At the end of each month he withdraws 1500 PLN from his account. Calculate how much money does he have on his account after 4 years.

Here we will go directly to TVM.

Payments

We have:

n = 48 (we have 48 months in 4 years)
I% = 3 (annual interest rate)
PV = −100000 (must be negative)
PMT = 1500 (must be positive, he gets this money)
FV = ? (unknown)
P/Y = 12 (12 payments per year)
C/Y = 12 (compounded 12 times per year)

And we get $FV = 36335.99$.

Payments

We have:

$$
\begin{aligned}
\text{n} &= 48 \text{ (we have 48 months in 4 years)}\\
\text{I\%} &= 3 \text{ (annual interest rate)}\\
\text{PV} &= -100000 \text{ (must be negative)}\\
\text{PMT} &= 1500 \text{ (must be positive, he gets this money)}\\
\text{FV} &= ? \text{ (unknown)}\\
\text{P/Y} &= 12 \text{ (12 payments per year)}\\
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\end{aligned}$$

And we get $FV = 36335.99$.

Payments 2

Tomasz invests 50 000 PLN into savings account that pays 2.2% annual interest compounded quarterly. At the end of each quarter he puts another 5000 PLN into his account. Calculate how much money he has on his account after 5 years.

We will use TVM. Note that this time PMT will be negative, since he puts the money into the account.

Payments 2

Tomasz invests 50 000 PLN into savings account that pays 2.2% annual interest compounded quarterly. At the end of each quarter he puts another 5000 PLN into his account. Calculate how much money he has on his account after 5 years.

We will use TVM. Note that this time PMT will be negative, since he puts the money into the account.

Payments 2

We should input:

n = 20 (we have 20 quarters in 5 years)
I% = 2.2 (annual interest rate)
PV = −50000 (must be negative)
PMT = −5000 (negative, he invests the money)
FV = ? (unknown)
P/Y = 4 (4 payments per year)
C/Y = 4 (compounded 4 times per year)

We get $FV = 161198.61$.

Payments 2

We should input:

$$
\begin{aligned}
\text{n} &= 20 \text{ (we have 20 quarters in 5 years)} \\
\text{I\%} &= 2.2 \text{ (annual interest rate)} \\
\text{PV} &= -50000 \text{ (must be negative)} \\
\text{PMT} &= -5000 \text{ (negative, he invests the money)} \\
\text{FV} &= \text{? (unknown)} \\
\text{P/Y} &= 4 \text{ (4 payments per year)} \\
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\end{aligned}
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We get $FV = 161198.61$.

Payments 2

We should input:

$$\begin{aligned} n &= 20 \text{ (we have 20 quarters in 5 years)} \\ I\% &= 2.2 \text{ (annual interest rate)} \\ PV &= -50000 \text{ (must be negative)} \\ PMT &= -5000 \text{ (negative, he invests the money)} \\ FV &= ? \text{ (unknown)} \\ P/Y &= 4 \text{ (4 payments per year)} \\ C/Y &= 4 \text{ (compounded 4 times per year)} \end{aligned}$$

We get $FV = 161198.61$.

Payments 2

Let's do the second example algebraically too. The annual interest is 2.2%, so we get 0.55% each quarter. After the first quarter we should get:

$$FV = 50000 \cdot 1.0055 + 5000$$

After the second quarter:

$$FV = (50000 \cdot 1.0055 + 5000) \cdot 1.0055 + 5000$$

We could continue this way or note that the initial 50000 stays in the account for the full 20 quarters, the first payment of 5000 is in the account for 19 quarters, the second payment for 18 quarters etc. This gives us:

$$FV = 50000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{19} +$$
$$+ 5000 \cdot 1.0055^{18} + \ldots + 5000 \cdot 1.0055^{1} + 5000$$

Payments 2

Let's do the second example algebraically too. The annual interest is 2.2%, so we get 0.55% each quarter. After the first quarter we should get:

$$FV = 50000 \cdot 1.0055 + 5000$$

After the second quarter:

$$FV = (50000 \cdot 1.0055 + 5000) \cdot 1.0055 + 5000$$

We could continue this way or note that the initial 50000 stays in the account for the full 20 quarters, the first payment of 5000 is in the account for 19 quarters, the second payment for 18 quarters etc. This gives us:

$$FV = 50000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{19} +$$
$$+ 5000 \cdot 1.0055^{18} + \ldots + 5000 \cdot 1.0055^{1} + 5000$$

Payments 2

Let's do the second example algebraically too. The annual interest is 2.2%, so we get 0.55% each quarter. After the first quarter we should get:

$$FV = 50000 \cdot 1.0055 + 5000$$

After the second quarter:

$$FV = (50000 \cdot 1.0055 + 5000) \cdot 1.0055 + 5000$$

We could continue this way or note that the initial 50000 stays in the account for the full 20 quarters, the first payment of 5000 is in the account for 19 quarters, the second payment for 18 quarters etc. This gives us:

$$FV = 50000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{19} +$$
$$+ 5000 \cdot 1.0055^{18} + \ldots + 5000 \cdot 1.0055^{1} + 5000$$

Payments 2

Let's do the second example algebraically too. The annual interest is 2.2%, so we get 0.55% each quarter. After the first quarter we should get:

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After the second quarter:

$$FV = (50000 \cdot 1.0055 + 5000) \cdot 1.0055 + 5000$$

We could continue this way or note that the initial 50000 stays in the account for the full 20 quarters, the first payment of 5000 is in the account for 19 quarters, the second payment for 18 quarters etc. This gives us:

$$FV = 50000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{20} + 5000 \cdot 1.0055^{19} +$$
$$+ 5000 \cdot 1.0055^{18} + ... + 5000 \cdot 1.0055^{1} + 5000$$

Payments 2

We get $50000 \cdot 1.0055^{20}$ plus a geometric series with first term 5000, ratio 1.0055 and 20 terms, so:

$$FV = 50000 \cdot 1.0055^{20} + \frac{5000(1.0055^{20} - 1)}{1.0055 - 1} = 161198.61$$

Payments 2

Let's use yet another method. Recurrences!

Let's set up a recurrence. Here (unlike in most cases) it is good to start with a_0 rather than a_1 (because a_n will represent the value after n quarters, so a_0, not a_1, represents the initial investment). The recurrence is $a_n = a_{n-1} \cdot 1.0055 + 5000$ and $a_0 = 5000$. We are looking for a_{20}. You should get the same answer as we did with the previous methods.

Payments 2

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Let's set up a recurrence. Here (unlike in most cases) it is good to start with a_0 rather than a_1 (because a_n will represent the value after n quarters, so a_0, not a_1, represents the initial investment). The recurrence is $a_n = a_{n-1} \cdot 1.0055 + 5000$ and $a_0 = 5000$. We are looking for a_{20}. You should get the same answer as we did with the previous methods.

Loans

Tomasz takes a loan of 120 000PLN. The loan is to be repaid in 20 years with equal monthly payments. The annual interest on the loan is 4.5% compounded monthly. Calculate the monthly repayments.

Again we will use TVM.

Loans

Tomasz takes a loan of 120 000PLN. The loan is to be repaid in 20 years with equal monthly payments. The annual interest on the loan is 4.5% compounded monthly. Calculate the monthly repayments.

Again we will use TVM.

Loans

We have:

n = 240 (we have 240 months in 20 years)

I% = 4.5 (annual interest rate)

PV = 120 000 (this time it's positive as we get this money)

PMT = ? (we want to calculate the repayments, they will be negative)

FV = 0 (the loan needs to be repaid)

P/Y = 12 (12 payments per year)

C/Y = 12 (compounded 12 times per year)

And we get $PMT = -759.18$. So Tomasz needs to pay 759.18 PLN per month to repay his loan in 20 years.

Loans

We have:

$$\begin{aligned} n &= 240 \text{ (we have 240 months in 20 years)} \\ I\% &= 4.5 \text{ (annual interest rate)} \\ PV &= 120\,000 \text{ (this time it's positive as we get this money)} \\ PMT &= ? \text{ (we want to calculate the repayments, they will be negative)} \\ FV &= 0 \text{ (the loan needs to be repaid)} \\ P/Y &= 12 \text{ (12 payments per year)} \\ C/Y &= 12 \text{ (compounded 12 times per year)} \end{aligned}$$

And we get $PMT = -759.18$. So Tomasz needs to pay 759.18 PLN per month to repay his loan in 20 years.

Loans

We have:

$$\begin{aligned}
\text{n} &= 240 \text{ (we have 240 months in 20 years)}\\
\text{I\%} &= 4.5 \text{ (annual interest rate)}\\
\text{PV} &= 120\,000 \text{ (this time it's positive as we get this money)}\\
\text{PMT} &= ? \text{ (we want to calculate the repayments, they will be negative)}\\
\text{FV} &= 0 \text{ (the loan needs to be repaid)}\\
\text{P/Y} &= 12 \text{ (12 payments per year)}\\
\text{C/Y} &= 12 \text{ (compounded 12 times per year)}
\end{aligned}$$

And we get $PMT = -759.18$. So Tomasz needs to pay 759.18 PLN per month to repay his loan in 20 years.

Annuity

Tomasz invests 150 000 PLN into an annuity that pays a fixed amount of money per month. The annual interest on the annuity is 1.5$ and it is compounded monthly. Calculate how much money can Tomasz receive each month, if the annuity is to last for 15 year.

TVM on the next slide.

Annuity

Our inputs:

n = 180 (we have 180 months in 15 years)
I% = 1.5 (annual interest rate)
PV = - 150 000 (negative, he invests it)
PMT = ? (should be positive, as he gets this money)
FV = 0 (no more money on the investment)
P/Y = 12 (12 payments per year)
C/Y = 12 (compounded 12 times per year)

And we get $PMT = 931.11$. Tomasz can withdraw 931.11 PLN each month for 15 years.

Annuity

Our inputs:

$$
\begin{aligned}
\text{n} &= 180 \text{ (we have 180 months in 15 years)} \\
\text{I\%} &= 1.5 \text{ (annual interest rate)} \\
\text{PV} &= \text{- 150 000 (negative, he invests it)} \\
\text{PMT} &= \text{? (should be positive, as he gets this money)} \\
\text{FV} &= 0 \text{ (no more money on the investment)} \\
\text{P/Y} &= 12 \text{ (12 payments per year)} \\
\text{C/Y} &= 12 \text{ (compounded 12 times per year)}
\end{aligned}
$$

And we get $PMT = 931.11$. Tomasz can withdraw 931.11 PLN each month for 15 years.

Annuity

Our inputs:

n = 180 (we have 180 months in 15 years)
I% = 1.5 (annual interest rate)
PV = - 150 000 (negative, he invests it)
PMT = ? (should be positive, as he gets this money)
FV = 0 (no more money on the investment)
P/Y = 12 (12 payments per year)
C/Y = 12 (compounded 12 times per year)

And we get $PMT = 931.11$. Tomasz can withdraw 931.11 PLN each month for 15 years.

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n=..., i% = ..., etc.). But make sure you write down the answer explicitly, for example: "Tomasz needs to invest 666 PLN".

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n=..., i% = ..., etc.). But make sure you write down the answer explicitly, for example: "Tomasz needs to invest 666 PLN".

Now we move on to some more complicated examples. These are the types of examples you may expect to appear on the exam.

Quick note on writing out the solution. If you use TVM on GDC, then as your solution you should write down your GDC input (n=..., i% = ..., etc.). But make sure you write down the answer explicitly, for example: "Tomasz needs to invest 666 PLN".

Exam-style question 1

Maria takes a loan of 300 000 PLN. The annual interest on the loan is 3.5% and it is compounded monthly.
(a) Calculate the monthly repayments if the loan is to be repaid in 20 years.
(b) Calculate the total interest Maria would pay for her loan.
(c) After 10 years Maria is able to increase the monthly repayments to 2500 PLN. Calculate how long it will take to repay the whole loan.

Exam-style question 1a

Part (a) is easy:

n = 240 (we have 240 months in 20 years)
I% = 3.5 (annual interest rate)
PV = 300 000 (positive, she gets this money)
PMT = ? (should be negative as she repays it)
FV = 0 (loan is repaid)
P/Y = 12 (12 payments per year)
C/Y = 12 (compounded 12 times per year)

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years.

Exam-style question 1a

Part (a) is easy:

$$\begin{aligned} n &= 240 \text{ (we have 240 months in 20 years)} \\ I\% &= 3.5 \text{ (annual interest rate)} \\ PV &= 300\,000 \text{ (positive, she gets this money)} \\ PMT &= ? \text{ (should be negative as she repays it)} \\ FV &= 0 \text{ (loan is repaid)} \\ P/Y &= 12 \text{ (12 payments per year)} \\ C/Y &= 12 \text{ (compounded 12 times per year)} \end{aligned}$$

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years.

Exam-style question 1a

Part (a) is easy:

$$\begin{aligned} \text{n} &= 240 \text{ (we have 240 months in 20 years)} \\ \text{I\%} &= 3.5 \text{ (annual interest rate)} \\ \text{PV} &= 300\,000 \text{ (positive, she gets this money)} \\ \text{PMT} &= ? \text{ (should be negative as she repays it)} \\ \text{FV} &= 0 \text{ (loan is repaid)} \\ \text{P/Y} &= 12 \text{ (12 payments per year)} \\ \text{C/Y} &= 12 \text{ (compounded 12 times per year)} \end{aligned}$$

And we get $PMT = -1739.88$. Maria needs to pay 1739.88 PLN each month to repay her loan in 20 years.

Exam-style question 1b

In part (b) we first need to calculate how much she paid back. She paid 1739.88 per month for 240 months. So she paid a total of

$$1739.88 \cdot 240 = 417571.2$$

The loan was 300 000, so the total interest was:

$$417571.2 - 300000 = 117571.2$$

Exam-style question 1b

In part (b) we first need to calculate how much she paid back. She paid 1739.88 per month for 240 months. So she paid a total of

$$1739.88 \cdot 240 = 417571.2$$

The loan was 300 000, so the total interest was:

$$417571.2 - 300000 = 117571.2$$

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

n = 120 (we have 120 months in 10 years)

I% = 3.5 (annual interest rate)

PV = 300 000 (positive, she gets this money)

PMT = −1739.88 (should be negative as she repays it)

FV = ? (how much she still owes)

P/Y = 12 (12 payments per year)

C/Y = 12 (compounded 12 times per year)

And we get $FV = -175948.09$. So she still needs to repay 175948.09PLN.

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

$$
\begin{aligned}
n &= 120 \text{ (we have 120 months in 10 years)}\\
I\% &= 3.5 \text{ (annual interest rate)}\\
PV &= 300\,000 \text{ (positive, she gets this money)}\\
PMT &= -1739.88 \text{ (should be negative as she repays it)}\\
FV &= ? \text{ (how much she still owes)}\\
P/Y &= 12 \text{ (12 payments per year)}\\
C/Y &= 12 \text{ (compounded 12 times per year)}
\end{aligned}
$$

And we get $FV = -175948.09$. So she still needs to repay 175948.09PLN.

Exam-style question 1c

Part (c) is done in 2 parts. First we need to see how much she still owes after 10 years:

$$\begin{aligned}
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\text{PV} &= 300\,000 \text{ (positive, she gets this money)}\\
\text{PMT} &= -1739.88 \text{ (should be negative as she repays it)}\\
\text{FV} &= \text{? (how much she still owes)}\\
\text{P/Y} &= 12 \text{ (12 payments per year)}\\
\text{C/Y} &= 12 \text{ (compounded 12 times per year)}
\end{aligned}$$

And we get $FV = -175948.09$. So she still needs to repay 175948.09PLN.

Exam-style question 1c

Now we increase the payments:

$$
\begin{aligned}
n &= ? \text{ (unknown number of months)} \\
I\% &= 3.5 \text{ (annual interest rate)} \\
PV &= 175948.09 \text{ (positive, she gets this money)} \\
PMT &= -2500 \text{ (increased repayments)} \\
FV &= 0 \text{ (loan is to be repaid)} \\
P/Y &= 12 \text{ (12 payments per year)} \\
C/Y &= 12 \text{ (compounded 12 times per year)}
\end{aligned}
$$

And we get $n = 78.9$. So she needs another 79 months to repay the loan. The total time to repay the loan is then 199 months ($120 + 79$).

Exam-style question 1c

Now we increase the payments:

$$
\begin{aligned}
n &= \text{? (unknown number of months)} \\
I\% &= 3.5 \text{ (annual interest rate)} \\
PV &= 175948.09 \text{ (positive, she gets this money)} \\
PMT &= -2500 \text{ (increased repayments)} \\
FV &= 0 \text{ (loan is to be repaid)} \\
P/Y &= 12 \text{ (12 payments per year)} \\
C/Y &= 12 \text{ (compounded 12 times per year)}
\end{aligned}
$$

And we get $n = 78.9$. So she needs another 79 months to repay the loan. The total time to repay the loan is then 199 months $(120 + 79)$.

Exam-style question 1c

Now we increase the payments:

$$\begin{aligned} \text{n} &= \text{? (unknown number of months)} \\ \text{I\%} &= 3.5 \text{ (annual interest rate)} \\ \text{PV} &= 175948.09 \text{ (positive, she gets this money)} \\ \text{PMT} &= -2500 \text{ (increased repayments)} \\ \text{FV} &= 0 \text{ (loan is to be repaid)} \\ \text{P/Y} &= 12 \text{ (12 payments per year)} \\ \text{C/Y} &= 12 \text{ (compounded 12 times per year)} \end{aligned}$$

And we get $n = 78.9$. So she needs another 79 months to repay the loan. The total time to repay the loan is then 199 months $(120 + 79)$.

Exam-style question 2

Maria invests 50 000 PLN into savings accounts that pays 1.5% annual interest rate compounded monthly. Additional she puts another 1000 PLN into the account at the end of each month. Tomasz also invests a certain sum into a savings account that pays 1.6% annual interest rate compounded quarterly. He makes no further payments into the account. Calculate how much he needs to invest to have the same amount of money in the account in 5 years.

We will do this in two parts. First we will calculate the amount in Maria's account in 5 years.

Exam-style question 2

Maria invests 50 000 PLN into savings accounts that pays 1.5% annual interest rate compounded monthly. Additional she puts another 1000 PLN into the account at the end of each month. Tomasz also invests a certain sum into a savings account that pays 1.6% annual interest rate compounded quarterly. He makes no further payments into the account. Calculate how much he needs to invest to have the same amount of money in the account in 5 years.

We will do this in two parts. First we will calculate the amount in Maria's account in 5 years.

Exam-style question 2

We input:

n = 60 (60 months in 5 years)

I% = 1.5 (annual interest rate)

PV = −50000 (negative, she invests this)

PMT = −1000 (also negative)

FV = ? (unknown)

P/Y = 12 (12 payments per year)

C/Y = 12 (compounded 12 times per year)

And we get $FV = 116158.62$.

Exam-style question 2

We input:

$$
\begin{aligned}
\text{n} &= 60 \text{ (60 months in 5 years)}\\
\text{I\%} &= 1.5 \text{ (annual interest rate)}\\
\text{PV} &= -50000 \text{ (negative, she invests this)}\\
\text{PMT} &= -1000 \text{ (also negative)}\\
\text{FV} &= ? \text{ (unknown)}\\
\text{P/Y} &= 12 \text{ (12 payments per year)}\\
\text{C/Y} &= 12 \text{ (compounded 12 times per year)}
\end{aligned}
$$

And we get $FV = 116158.62$.

Exam-style question 2

We input:

$$\begin{aligned} \text{n} &= 60 \text{ (60 months in 5 years)} \\ \text{I\%} &= 1.5 \text{ (annual interest rate)} \\ \text{PV} &= -50000 \text{ (negative, she invests this)} \\ \text{PMT} &= -1000 \text{ (also negative)} \\ \text{FV} &= ? \text{ (unknown)} \\ \text{P/Y} &= 12 \text{ (12 payments per year)} \\ \text{C/Y} &= 12 \text{ (compounded 12 times per year)} \end{aligned}$$

And we get $FV = 116158.62$.

Exam-style question 2

Now we look at Tomasz:

n = 60 (60 months in 5 years)

$I\%$ = 1.6 (annual interest rate)

PV = ? (unknown, will be negative)

PMT = 0 (no payments)

FV = 116158.6181 (what Maria should have)

P/Y = 4 (4 periods per year)

C/Y = 4 (compounded 4 times per year)

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Exam-style question 2

Now we look at Tomasz:

$$
\begin{aligned}
\text{n} &= 60 \text{ (60 months in 5 years)}\\
\text{I\%} &= 1.6 \text{ (annual interest rate)}\\
\text{PV} &= ? \text{ (unknown, will be negative)}\\
\text{PMT} &= 0 \text{ (no payments)}\\
\text{FV} &= 116158.6181 \text{ (what Maria should have)}\\
\text{P/Y} &= 4 \text{ (4 periods per year)}\\
\text{C/Y} &= 4 \text{ (compounded 4 times per year)}
\end{aligned}
$$

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Exam-style question 2

Now we look at Tomasz:

$$\begin{aligned}
\text{n} &= 60 \text{ (60 months in 5 years)} \\
\text{I\%} &= 1.6 \text{ (annual interest rate)} \\
\text{PV} &= ? \text{ (unknown, will be negative)} \\
\text{PMT} &= 0 \text{ (no payments)} \\
\text{FV} &= 116158.6181 \text{ (what Maria should have)} \\
\text{P/Y} &= 4 \text{ (4 periods per year)} \\
\text{C/Y} &= 4 \text{ (compounded 4 times per year)}
\end{aligned}$$

And we get $PV = -91417.36$. Tomasz needs to invest 91417.36 PLN to have the same amount as Maria in 5 years.

Exam-style question 3

Tomasz takes a loan of 200 000 PLN. The interest on the loan is 3.2% compounded quarterly. The loan is to be repaid in monthly payments of 1477.74 PLN per month.

(a) How long does it take to repay the loan?

(b) Calculate the total interest on the loan.

(b) After 5 years the bank decreases the interest on the loan to 3% how much quicker will Tomasz repay his loan given that the repayments are not changed?

This is an unusual question, because $P/Y \neq C/Y$. Compounding are quarterly, but the payments are monthly.

Exam-style question 3

Tomasz takes a loan of 200 000 PLN. The interest on the loan is 3.2% compounded quarterly. The loan is to be repaid in monthly payments of 1477.74 PLN per month.

(a) How long does it take to repay the loan?

(b) Calculate the total interest on the loan.

(b) After 5 years the bank decreases the interest on the loan to 3% how much quicker will Tomasz repay his loan given that the repayments are not changed?

This is an unusual question, because $P/Y \neq C/Y$. Compounding are quarterly, but the payments are monthly.

Exam-style question 3a

Our input:

n = ? (unknown number of months)
I% = 3.2 (annual interest rate)
PV = 200 000 (positive, he gets the money)
PMT = −1477.74 (negative, he gives the money back)
FV = 0 (loan is repaid)
P/Y = 12 (12 payments per year)
C/Y = 4 (compounded 4 times per year)

And we get $n = 168$. So the loan will be repaid in 168 months or 14 years.

Exam-style question 3a

Our input:

$$
\begin{aligned}
\text{n} &= ?\ \text{(unknown number of months)}\\
\text{I\%} &= 3.2\ \text{(annual interest rate)}\\
\text{PV} &= 200\,000\ \text{(positive, he gets the money)}\\
\text{PMT} &= -1477.74\ \text{(negative, he gives the money back)}\\
\text{FV} &= 0\ \text{(loan is repaid)}\\
\text{P/Y} &= 12\ \text{(12 payments per year)}\\
\text{C/Y} &= 4\ \text{(compounded 4 times per year)}
\end{aligned}
$$

And we get $n = 168$. So the loan will be repaid in 168 months or 14 years.

Exam-style question 3a

Our input:

$$\begin{aligned} \text{n} &= ?\ \text{(unknown number of months)} \\ \text{I\%} &= 3.2\ \text{(annual interest rate)} \\ \text{PV} &= 200\,000\ \text{(positive, he gets the money)} \\ \text{PMT} &= -1477.74\ \text{(negative, he gives the money back)} \\ \text{FV} &= 0\ \text{(loan is repaid)} \\ \text{P/Y} &= 12\ \text{(12 payments per year)} \\ \text{C/Y} &= 4\ \text{(compounded 4 times per year)} \end{aligned}$$

And we get $n = 168$. So the loan will be repaid in 168 months or 14 years.

Exam-style question 3b

Total interest is:

$$168 \cdot 1477.74 - 200000 = 48260.32$$

Because he pays 1477.74 PLN for 168 months for a loan of 200 000 PLN.

Exam-style question 3b

Total interest is:

$$168 \cdot 1477.74 - 200000 = 48260.32$$

Because he pays 1477.74 PLN for 168 months for a loan of 200 000 PLN.

Exam-style question 3c

We first calculate how much he still owes after 5 years:

$$
\begin{aligned}
n &= 60 \text{ (60 months in 5 years)}\\
I\% &= 3.2 \text{ (annual interest rate)}\\
PV &= 200\,000 \text{ (positive, he gets the money)}\\
PMT &= -1477.74 \text{ (negative, he gives the money back)}\\
FV &= ? \text{ (unknown)}\\
P/Y &= 12 \text{ (12 payments per year)}\\
C/Y &= 4 \text{ (compounded 4 times per year)}
\end{aligned}
$$

And we get $FV = -138560.3338$.

Exam-style question 3c

We first calculate how much he still owes after 5 years:

$$
\begin{aligned}
\text{n} &= 60 \text{ (60 months in 5 years)} \\
\text{I\%} &= 3.2 \text{ (annual interest rate)} \\
\text{PV} &= 200\,000 \text{ (positive, he gets the money)} \\
\text{PMT} &= -1477.74 \text{ (negative, he gives the money back)} \\
\text{FV} &= ? \text{ (unknown)} \\
\text{P/Y} &= 12 \text{ (12 payments per year)} \\
\text{C/Y} &= 4 \text{ (compounded 4 times per year)}
\end{aligned}
$$

And we get $FV = -138560.3338$.

Exam-style question 3c

We first calculate how much he still owes after 5 years:

$$
\begin{aligned}
\text{n} &= 60 \text{ (60 months in 5 years)} \\
\text{I\%} &= 3.2 \text{ (annual interest rate)} \\
\text{PV} &= 200\ 000 \text{ (positive, he gets the money)} \\
\text{PMT} &= -1477.74 \text{ (negative, he gives the money back)} \\
\text{FV} &= ? \text{ (unknown)} \\
\text{P/Y} &= 12 \text{ (12 payments per year)} \\
\text{C/Y} &= 4 \text{ (compounded 4 times per year)}
\end{aligned}
$$

And we get $FV = -138560.3338$.

Exam-style question 3c

Now we decrease the interest rate:

$$
\begin{aligned}
n &= ?\ \text{(unknown)}\\
I\% &= 3\ \text{(new interest rate)}\\
PV &= 138560.3338\ \text{(he still owes this)}\\
PMT &= -1477.74\ \text{(repayments are not changed)}\\
FV &= 0\ \text{(loan is repaid)}\\
P/Y &= 12\ \text{(12 payments per year)}\\
C/Y &= 4\ \text{(compounded 4 times per year)}
\end{aligned}
$$

And we get $n = 107$. So the loan takes a total of 167 months to be repaid. He repays it only one month earlier.

Exam-style question 3c

Now we decrease the interest rate:

$$
\begin{aligned}
\text{n} &= \text{? (unknown)} \\
\text{I\%} &= \text{3 (new interest rate)} \\
\text{PV} &= \text{138560.3338 (he still owes this)} \\
\text{PMT} &= -1477.74 \text{ (repayments are not changed)} \\
\text{FV} &= \text{0 (loan is repaid)} \\
\text{P/Y} &= \text{12 (12 payments per year)} \\
\text{C/Y} &= \text{4 (compounded 4 times per year)}
\end{aligned}
$$

And we get $n = 107$. So the loan takes a total of 167 months to be repaid. He repays it only one month earlier.

Exam-style question 3c

Now we decrease the interest rate:

$$\begin{aligned} \text{n} &= ?\ \text{(unknown)} \\ \text{I\%} &= 3\ \text{(new interest rate)} \\ \text{PV} &= 138560.3338\ \text{(he still owes this)} \\ \text{PMT} &= -1477.74\ \text{(repayments are not changed)} \\ \text{FV} &= 0\ \text{(loan is repaid)} \\ \text{P/Y} &= 12\ \text{(12 payments per year)} \\ \text{C/Y} &= 4\ \text{(compounded 4 times per year)} \end{aligned}$$

And we get $n = 107$. So the loan takes a total of 167 months to be repaid. He repays it only one month earlier.

In case of any questions you can email me at t.j.lechowski@gmail.com or message me via Librus.