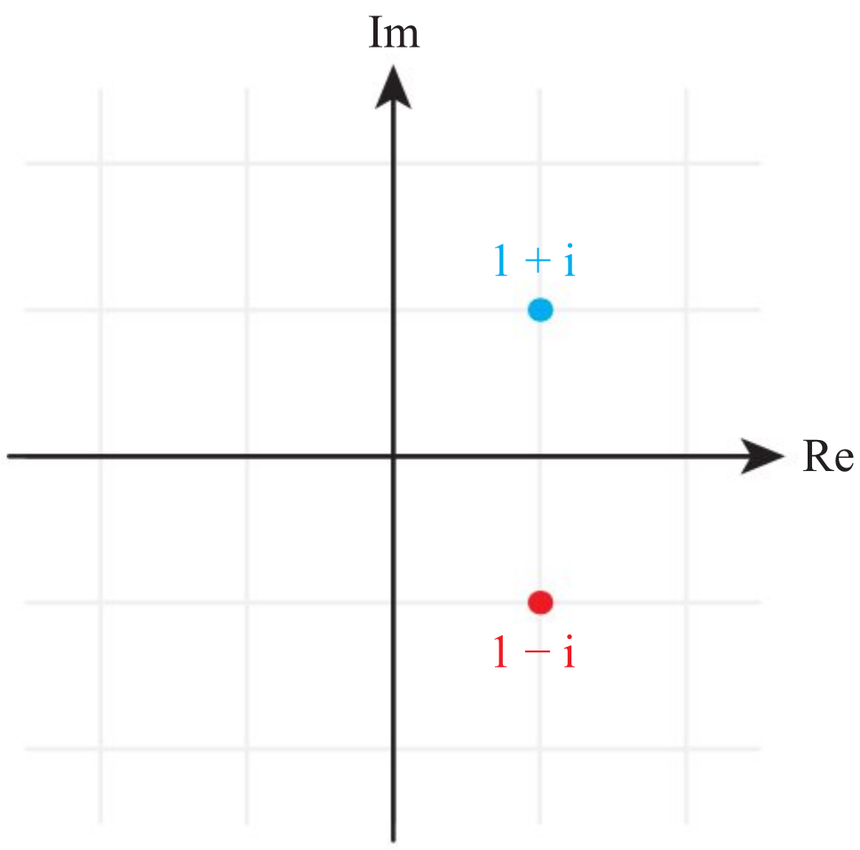
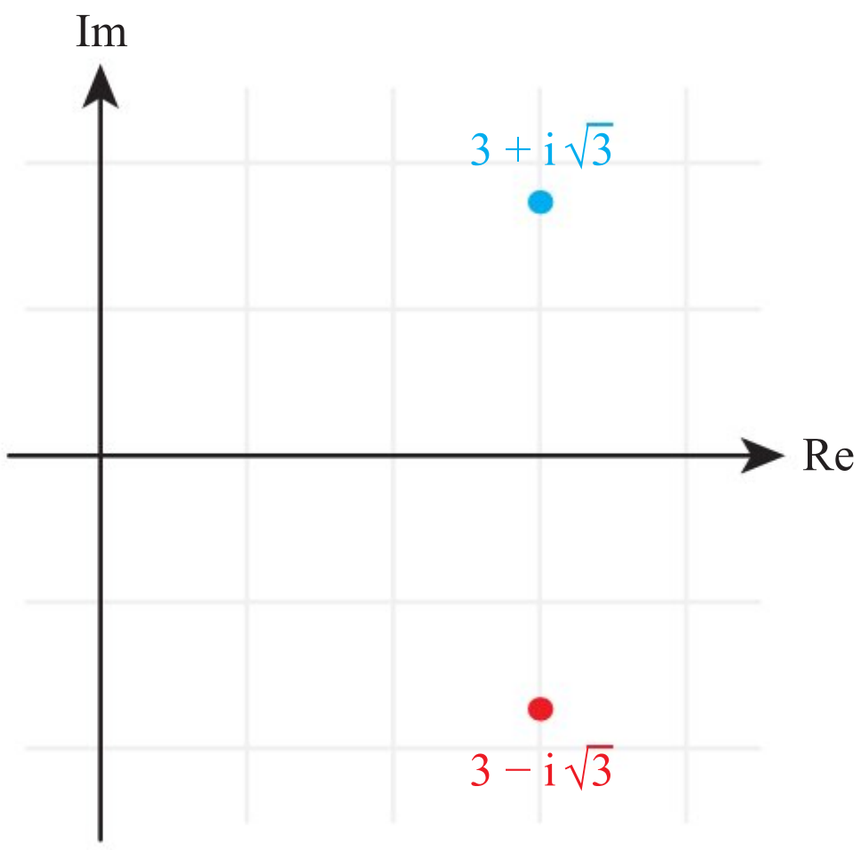
b



3 **a** $3 \pm \sqrt{3}\text{i}$

b



4 $\frac{3}{5} + \frac{1}{5}\text{i}$

5 $-0.5 - \text{i}$

6 $-\frac{\text{i}}{3}$

7 $z = -2 - 2\text{i}$

8 **a** $|z| = \sqrt{2}, \arg z = \frac{\pi}{4}$ **b** 8i

9 $16\sqrt{2}\,\text{cis}\frac{3\pi}{4}$

10 **a** $p = 3,\ q = 0.5$ **b** $p = \frac{1}{3} - \frac{2}{3}\text{i},\ q = \frac{1}{3} + \frac{2}{3}\text{i}$

11 **a** $\frac{1}{2} + \frac{\sqrt{3}}{2}\text{i}$ **b** $-\frac{\sqrt{3}}{2} + \frac{1}{2}\text{i}$

12 $\frac{a}{a^2+1} - \frac{1}{a^2+1}\text{i}$

13 $x = -\frac{\text{i}}{3}$

14 $z = 4 + 3\text{i}$

15 $1, \frac{\pi}{6}$

16 $\pm\sqrt{\frac{3}{2}}$

17 $2 \pm \text{i}$

18 $5 + 12\text{i}$

19 $b = -2,\ c = 5$

20 $3 + 4\text{i}$

21 **a** $0.387, 1.22$ **b** $0.387\cos(3t + 1.22)$

22 **a** $\sqrt{17}, 2.90$ **b** $\text{e}^{\text{i}\frac{\pi}{6}}$

c $(-3.96, -1.13)$

23 $2 \pm \text{i}\sqrt{5}$

24 $x^2 + y^2 - 2x = 3$

25 **a** $(1 - \sqrt{3}) + (1 + \sqrt{3})\text{i}$

b $z = \sqrt{2}\left(\cos\frac{\pi}{4} + \text{i}\sin\frac{\pi}{4}\right),\ w = 2\left(\cos\frac{\pi}{3} + \text{i}\sin\frac{\pi}{3}\right),$

$|zw| = 2\sqrt{2}, \arg(zw) = \frac{7\pi}{12}$

c $\frac{\sqrt{6} + \sqrt{2}}{4}$

26 **a** $2, \frac{\pi}{6}$ **b** $-128\sqrt{3}$

27 $3 + 4\text{i}$

28 1.30 radians, 2.28

29 **a** $2\sqrt{2 - \sqrt{3}}$ **b** $\frac{\pi}{12}$

Chapter 7 Prior Knowledge

1 $\begin{pmatrix} 106 & 91.8 & 67.2 & 69.4 \\ 245 & 220 & 163 & 153 \\ 222 & 203 & 149 & 136 \\ 130 & 112 & 81.9 & 83.2 \end{pmatrix}$

Exercise 7A

1 **a** Connected

b Complete, connected, simple

2 **a** Connected, simple, tree

b None

3 **a** No **b** Yes

4 **a** Yes **b** No

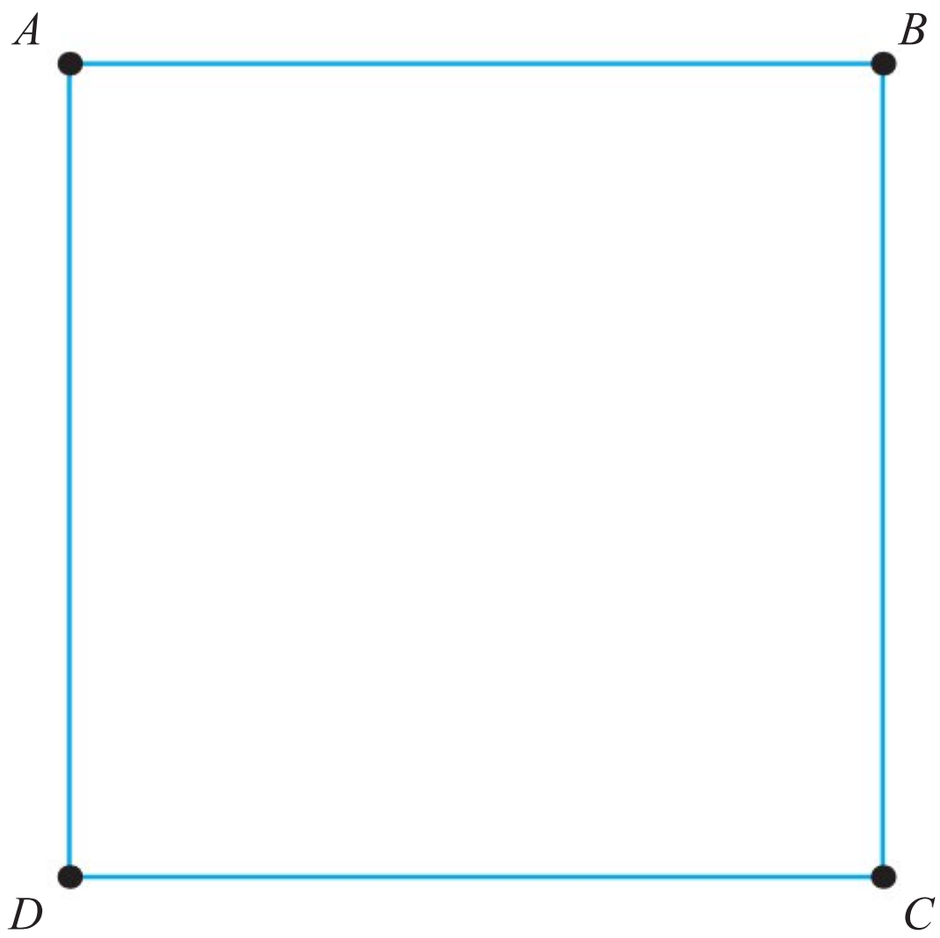
5 **a**
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

b
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

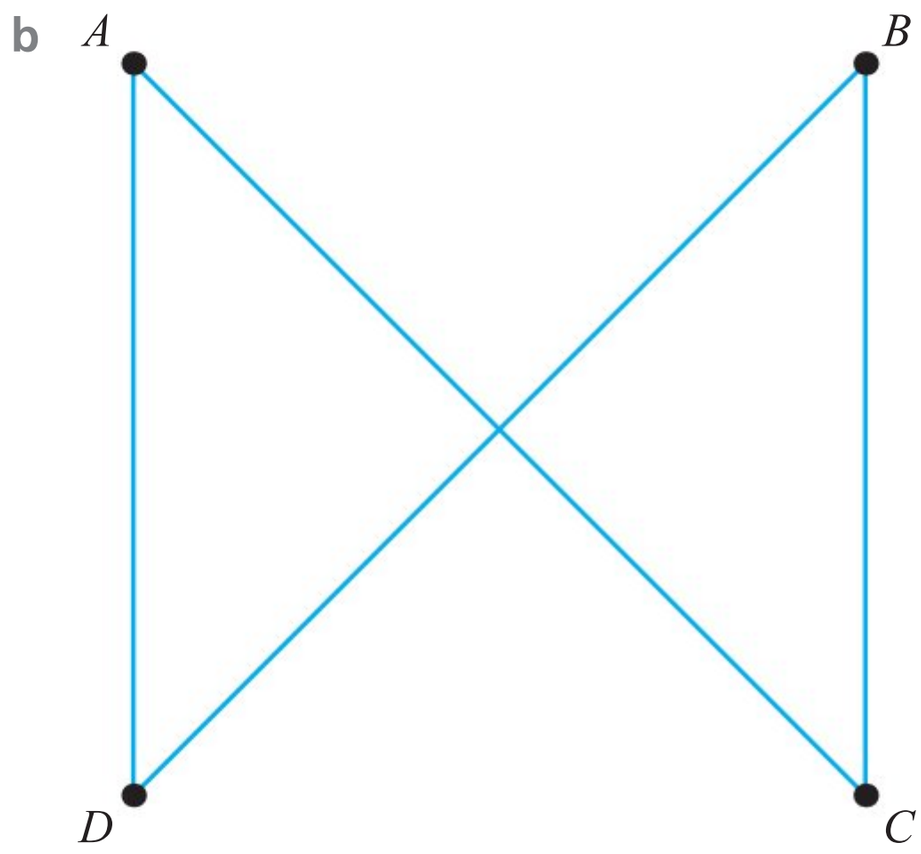
6 **a**
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

b
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 2 \end{pmatrix}$$

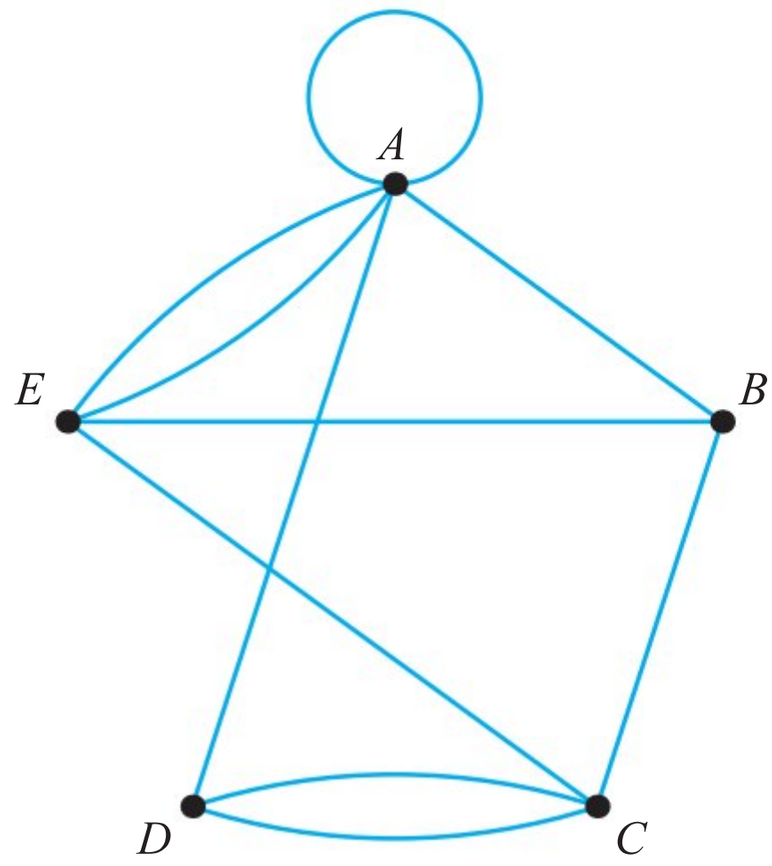
7 **a**



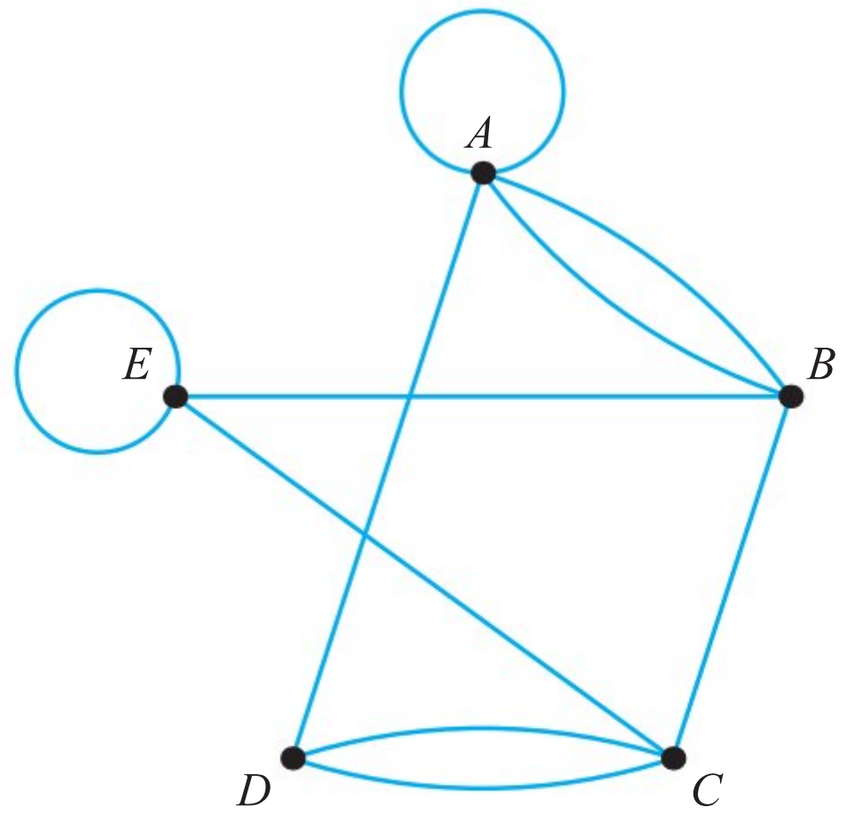
b



8 **a**



b



9 **a** **i** B, D **ii** B, E
iii 3 **iv** 2
b **i** A, C **ii** B, D, E
iii 2 **iv** 3

10 **a** **i** B, D **ii** B, D, E
iii 3 **iv** 5
b **i** C, D, E **ii** A, B, C
iii 8 **iv** 3

11 **a** **i** 2 **ii** 3
b **i** 2 **ii** 2

12 **a** **i** 4 **ii** 6
b **i** 2 **ii** 3

13 a

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

b 4; C is friends with 4 out of the other 6 people.

c Everyone is friends with the other six people.

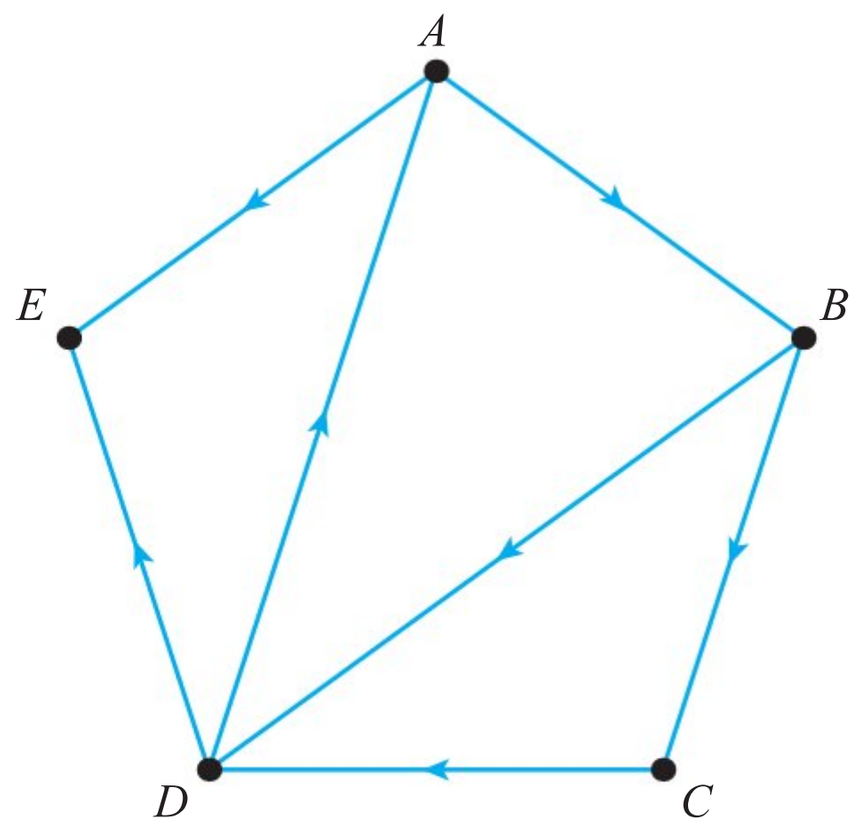
d 8

14 a There is a loop $ABDEA$.

b So that power can be supplied to each device.

c Any one of AB, BD, DE or EA.

15 a



b It is not possible to get from E to any other vertex.

c EA or ED

Exercise 7B

1 a $AEDB$, $ADCB$ **b** $CBAE$, $CDAE$

2 a $ABCDA$, $ABDEA$ **b** $CDABC$

3 a 6 **b** 7

4 a 14 **b** 19

5 a 0 **b** 1

6 a 1 **b** 1

7 a Eulerian; e.g. $ADBECABCDEA$

b Semi-Eulerian; e.g. $DCBAECA$

8 a Semi-Eulerian; e.g. $FEDCABCFD$

b Neither

9 a $ACBDEA$ (there are alternative solutions)

b $ABDCEA$

10 a $ABFECDA$

b $ADEBCFA$ (there are alternative solutions)

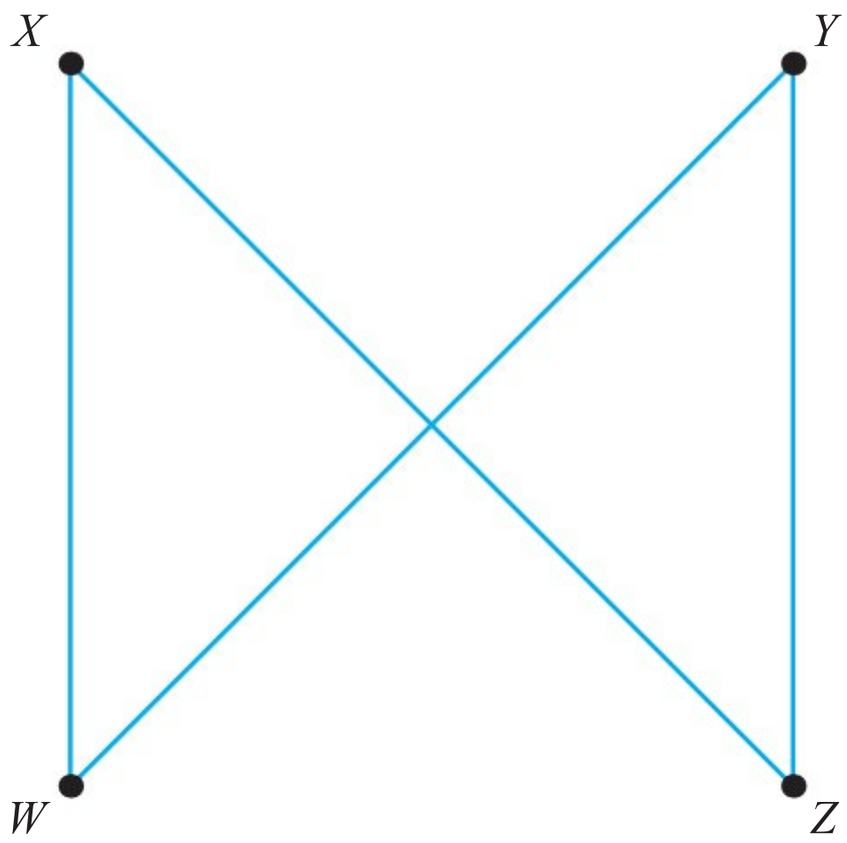
11 a Four vertices have odd degree

b $SMNPQNRQMRS$ (there are alternative solutions)

12 There are two vertices of odd degree: $UWXVYZWUVZ$ (there are alternative solutions)

13 a B and C

b



c 16

14 a e.g. Hamiltonian cycle $ABEDCA$

b 80

Exercise 7C

1 a

	A	B	C	D	E
A	–	34	20	18	–
B	34	–	20	–	25
C	20	20	–	12	–
D	18	–	12	–	15
E	–	25	–	15	–

b

	A	B	C	D	E
A	–	9	–	–	9
B	9	–	6	12	–
C	–	6	–	–	14
D	–	12	–	–	8
E	9	–	14	8	–

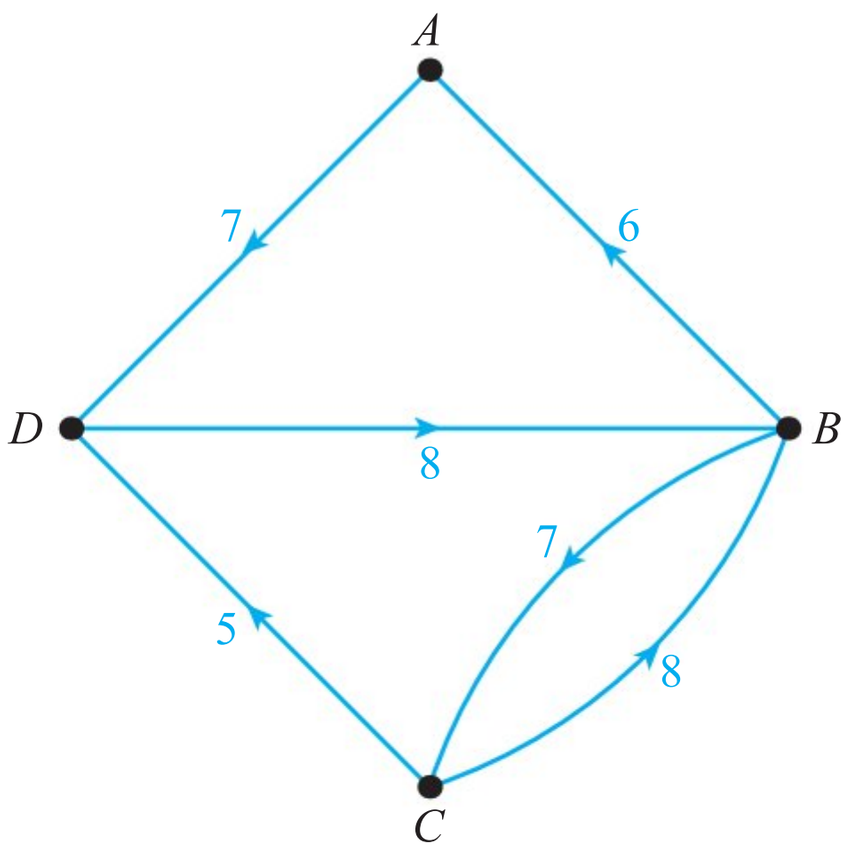
2 a

	A	*B*	*C*	*D*
A	–	6	5	7
B	8	–	10	–
C	–	–	–	–
D	7	–	6	–

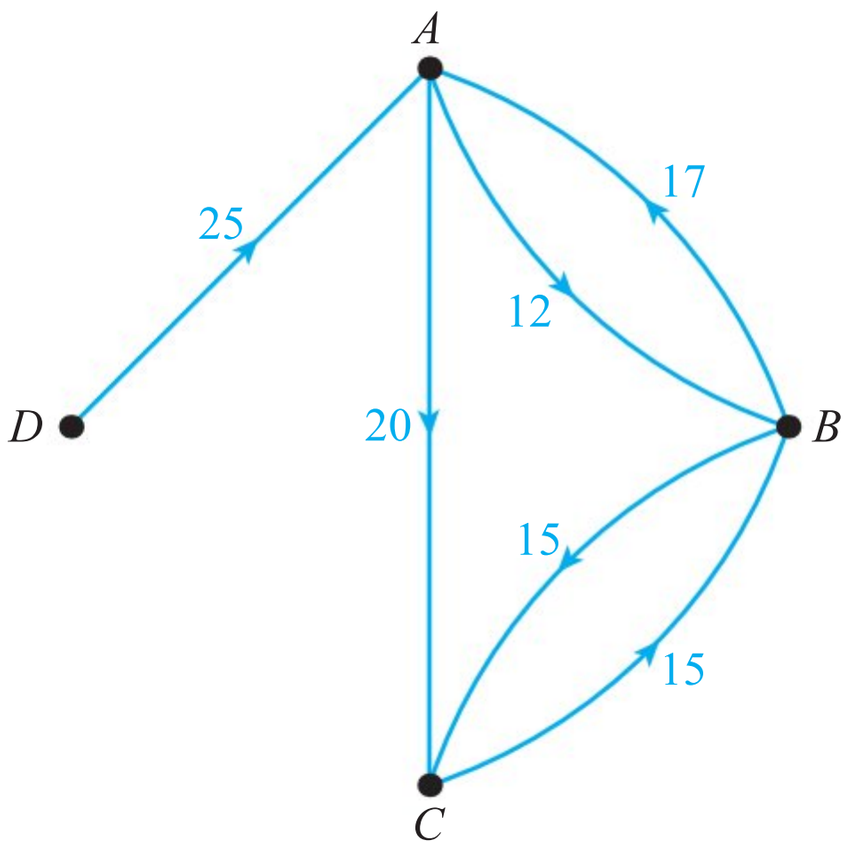
b

	A	*B*	*C*	*D*
A	–	–	8	8
B	5	–	6	–
C	10	–	–	6
D	–	–	7	–

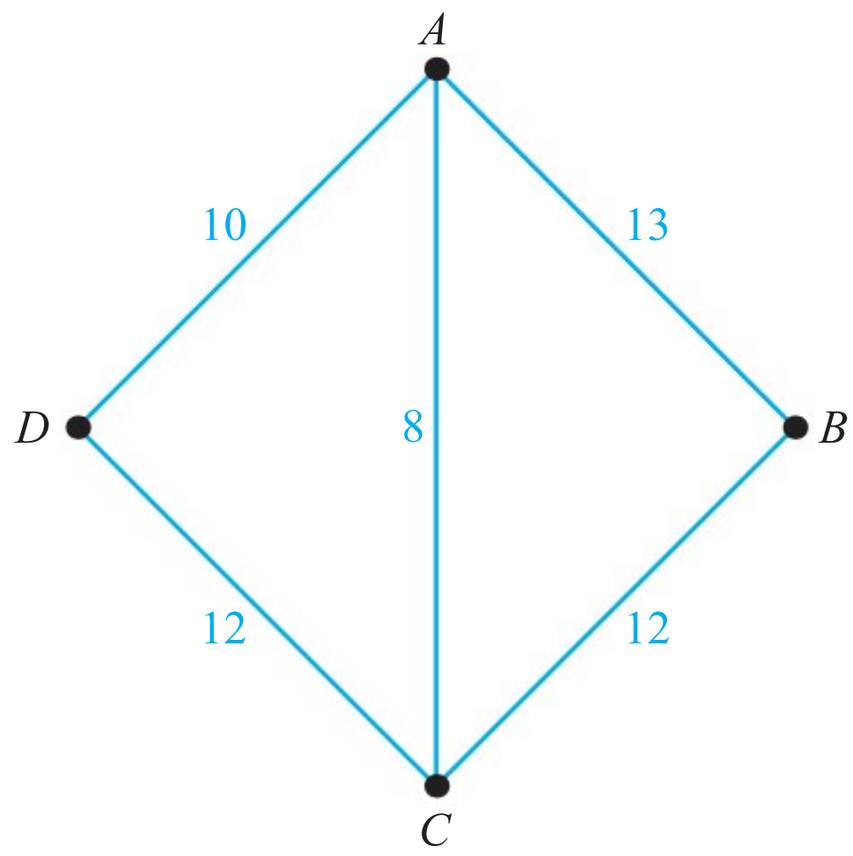
3 a



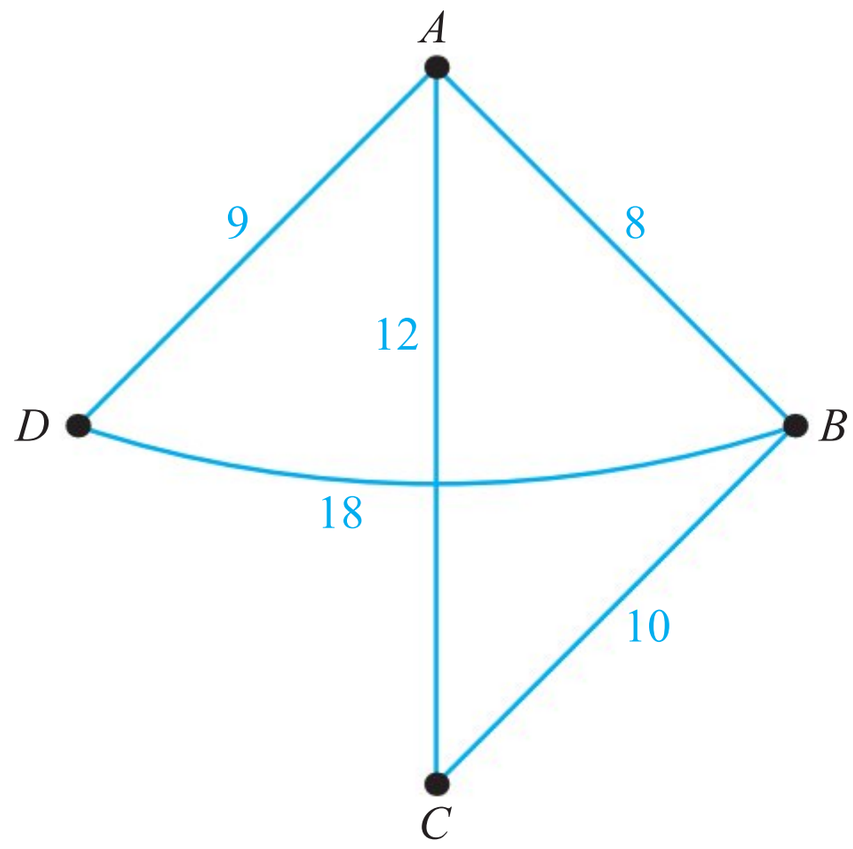
b



4 a



b



5 a

$$\begin{pmatrix} 0 & 1/2 & 0 & 1/4 & 1/3 \\ 1/3 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/4 & 1/3 \\ 1/3 & 1/2 & 1/2 & 0 & 1/3 \\ 1/3 & 0 & 1/2 & 1/4 & 0 \end{pmatrix}$$

b

$$\begin{pmatrix} 0 & 1/4 & 0 & 1/4 & 1/3 \\ 1/3 & 0 & 1/2 & 1/4 & 1/3 \\ 0 & 1/4 & 0 & 1/4 & 0 \\ 1/3 & 1/4 & 1/2 & 0 & 1/3 \\ 1/3 & 1/4 & 0 & 1/4 & 0 \end{pmatrix}$$

6 a $\begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \end{pmatrix}$

b $\begin{pmatrix} 0 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1 & 0 & 1/2 \\ 1/3 & 0 & 1 & 0 \end{pmatrix}$

7 a 0.143 **b** 0.125

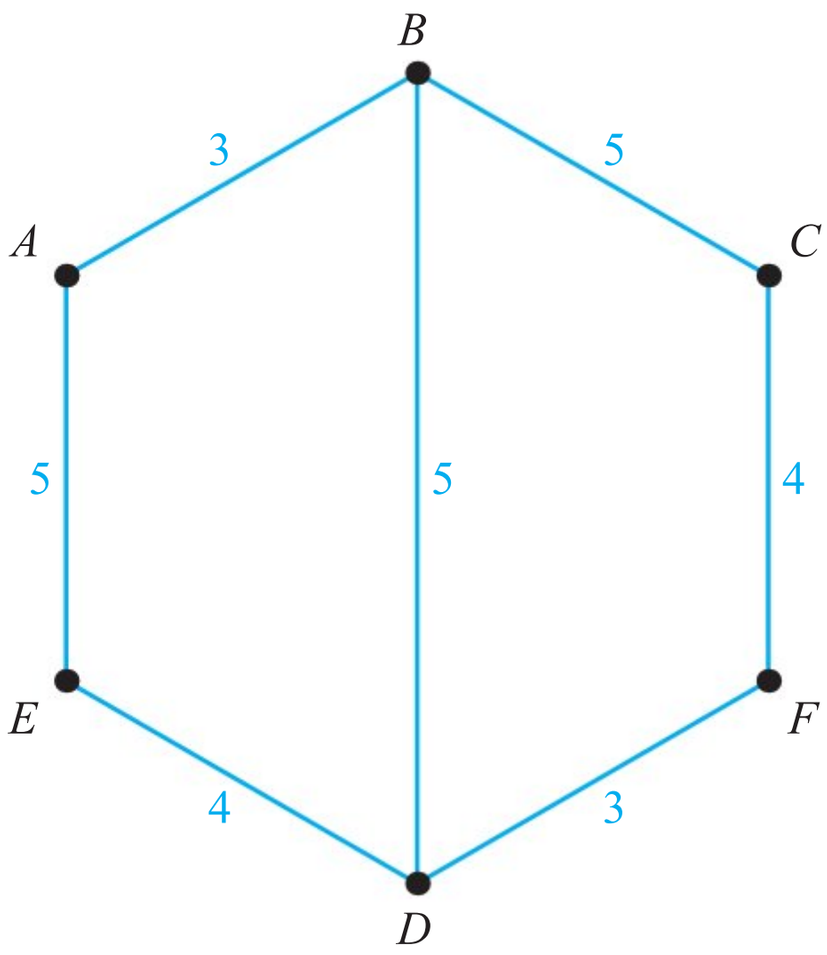
8 a 0.167 **b** 0.400

9 a B (0.352) **b** C (0.421)

10 a A and B (0.286) **b** B and D (0.333)

11 a 1500 km (via A) **b** \$350 (via E)

12 a



b ABD, 8 minutes

13 a 12 km ($SCBDT$)

b $SCBT$ or $SCDT$ (13 km)

14 a $\begin{pmatrix} 0 & 1/4 & 0 & 0 & 0 \\ 1 & 0 & 1/2 & 1/3 & 1/2 \\ 0 & 1/4 & 0 & 1/3 & 0 \\ 0 & 1/4 & 1/2 & 0 & 1/2 \\ 0 & 1/4 & 0 & 1/3 & 0 \end{pmatrix}$

b 0.145 **c** B (33.3%)

15 A(0.293), E(0.220), C(0.195), B and D (0.146)

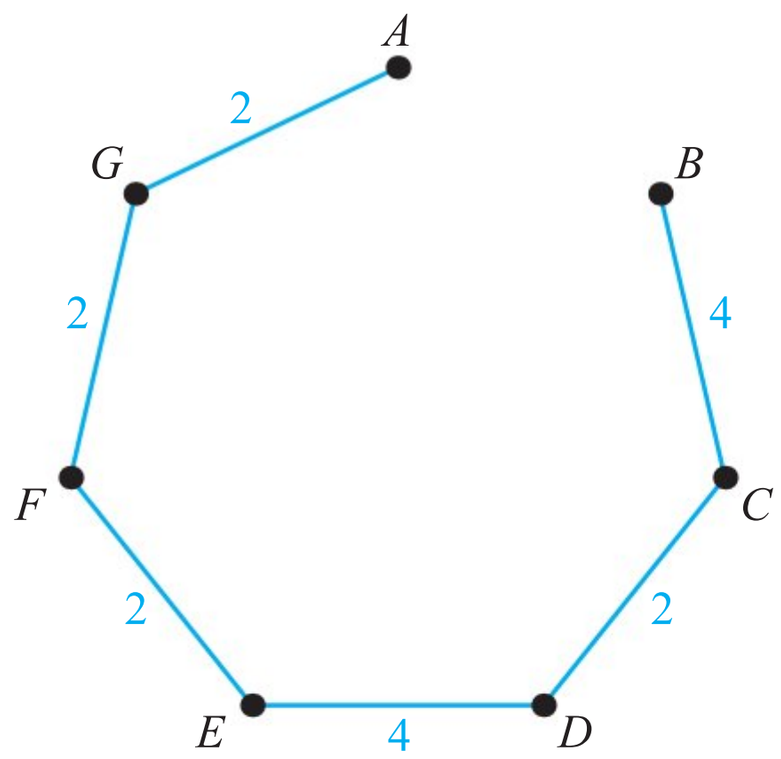
16 a D(0.387)

b $\begin{pmatrix} 0 & 0 & 0 & 0.3 & 0 \\ 0.45 & 0 & 0 & 0.3 & 0 \\ 0.45 & 0.45 & 0 & 0.3 & 0 \\ 0 & 0.45 & 0.9 & 0 & 0 \\ 0.1 & 0.1 & 0.1 & 0.1 & 1 \end{pmatrix}$

c No, relative probabilities stay the same.

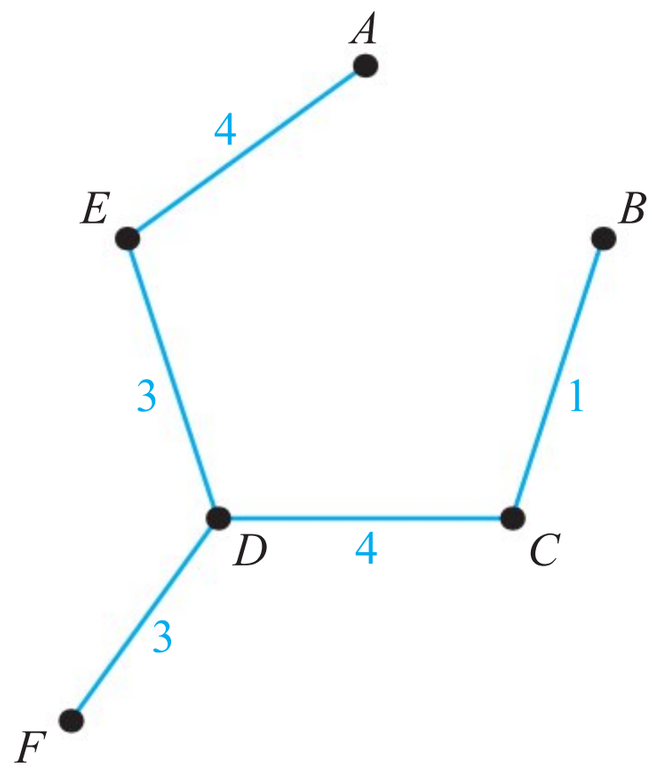
Exercise 7D

1 a



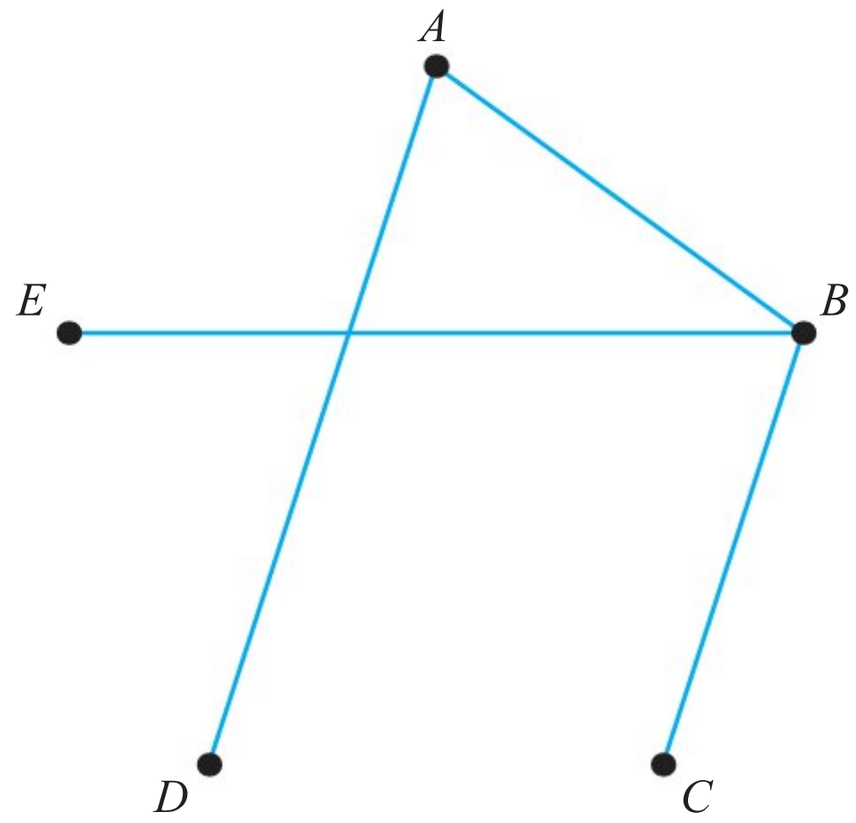
Weight = 16

b



Weight = 15

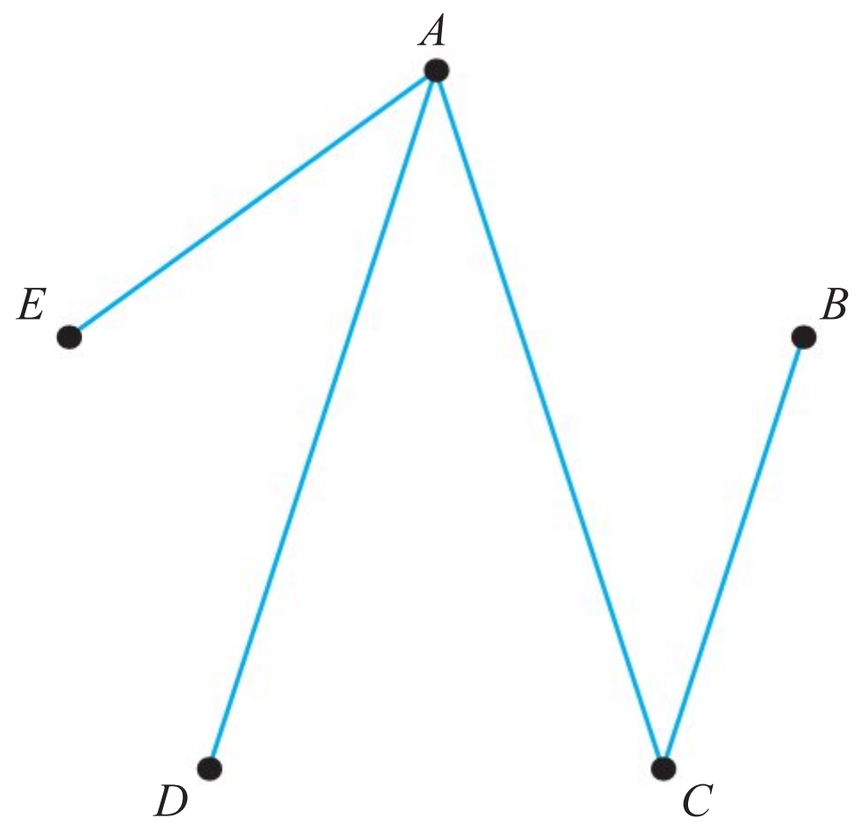
2 a



Weight = 41

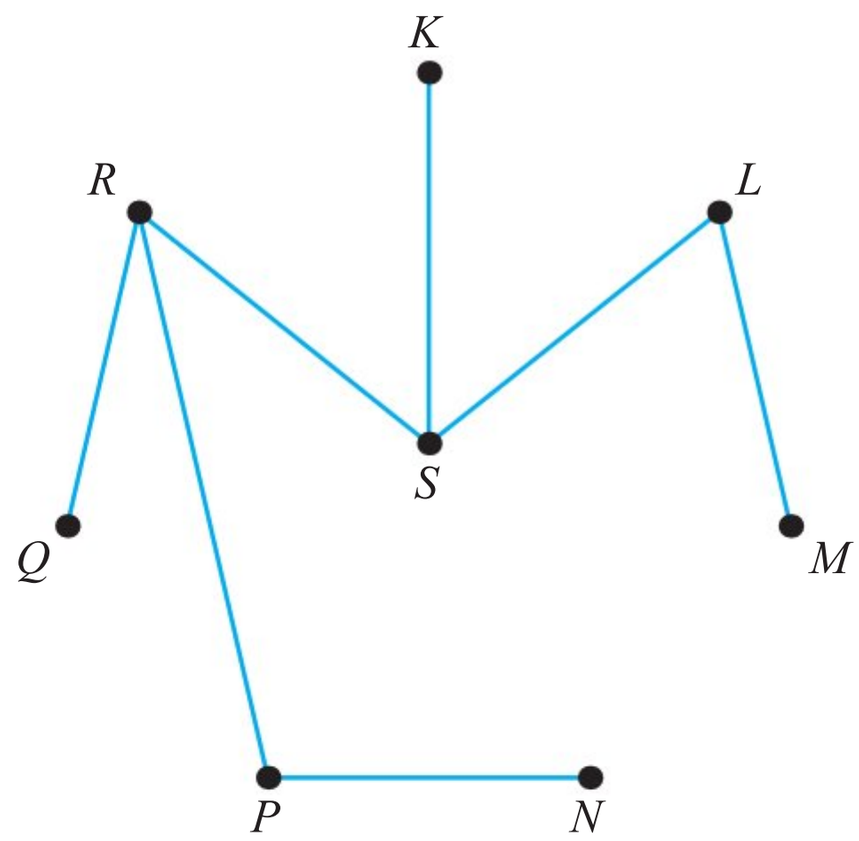
(can replace *BE* by *CE*)

b



Weight = 28

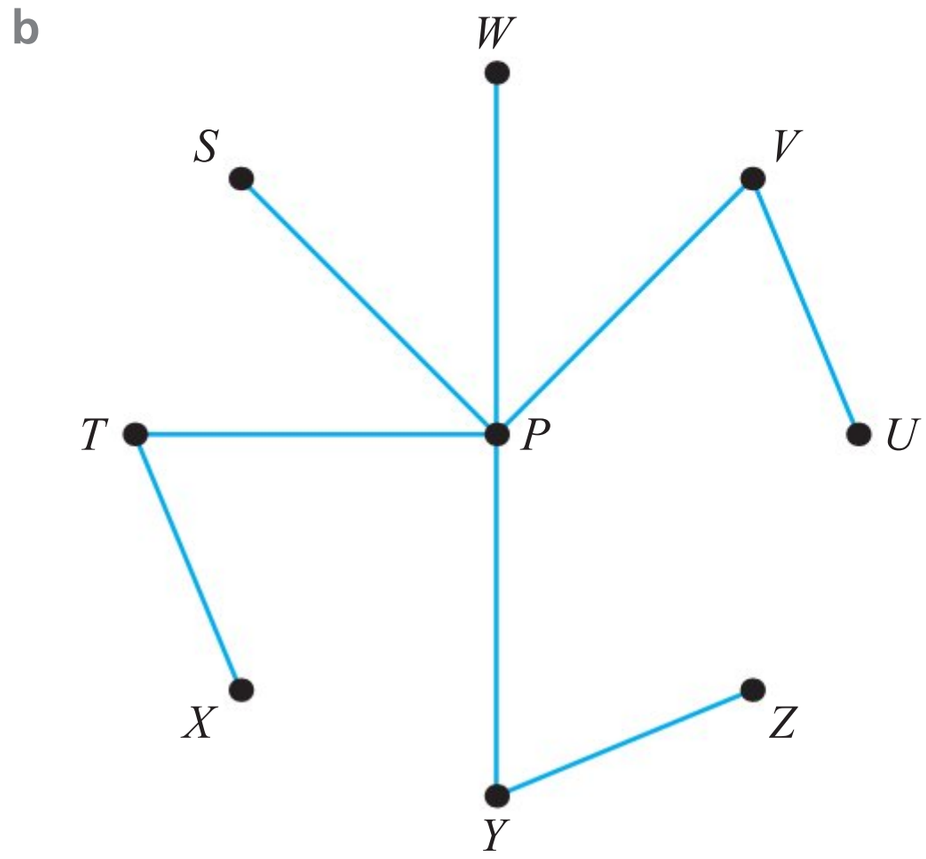
3 a



Weight = 47

(can replace *RP* by *RS*)

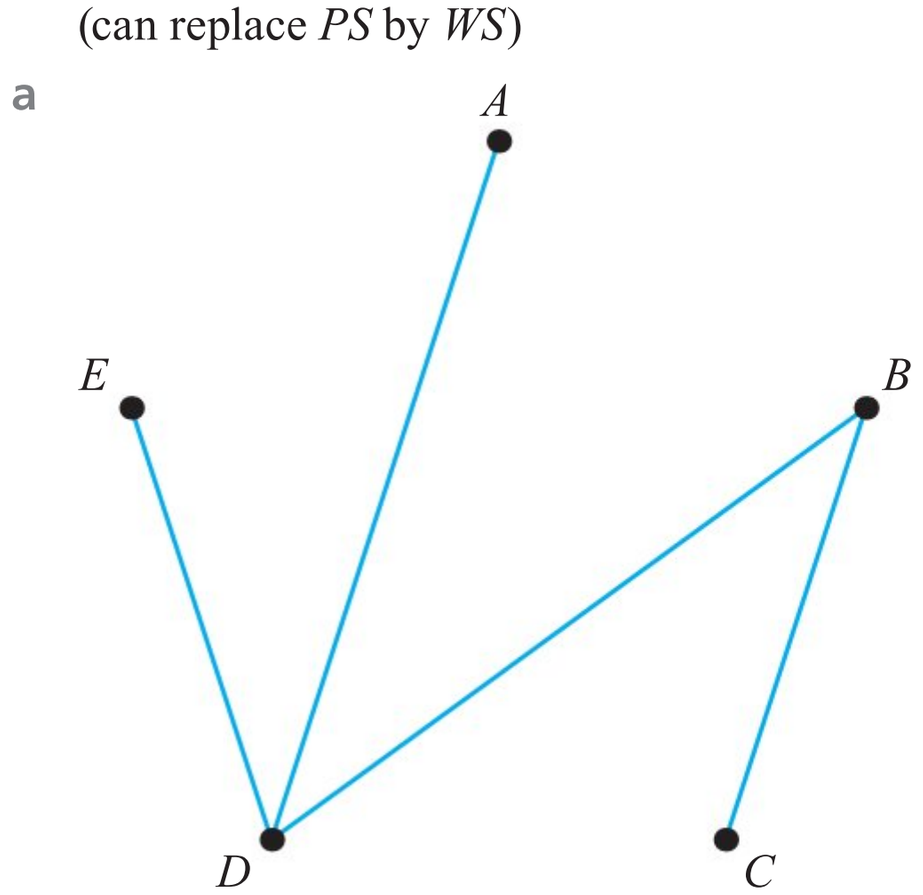
b



Weight = 92

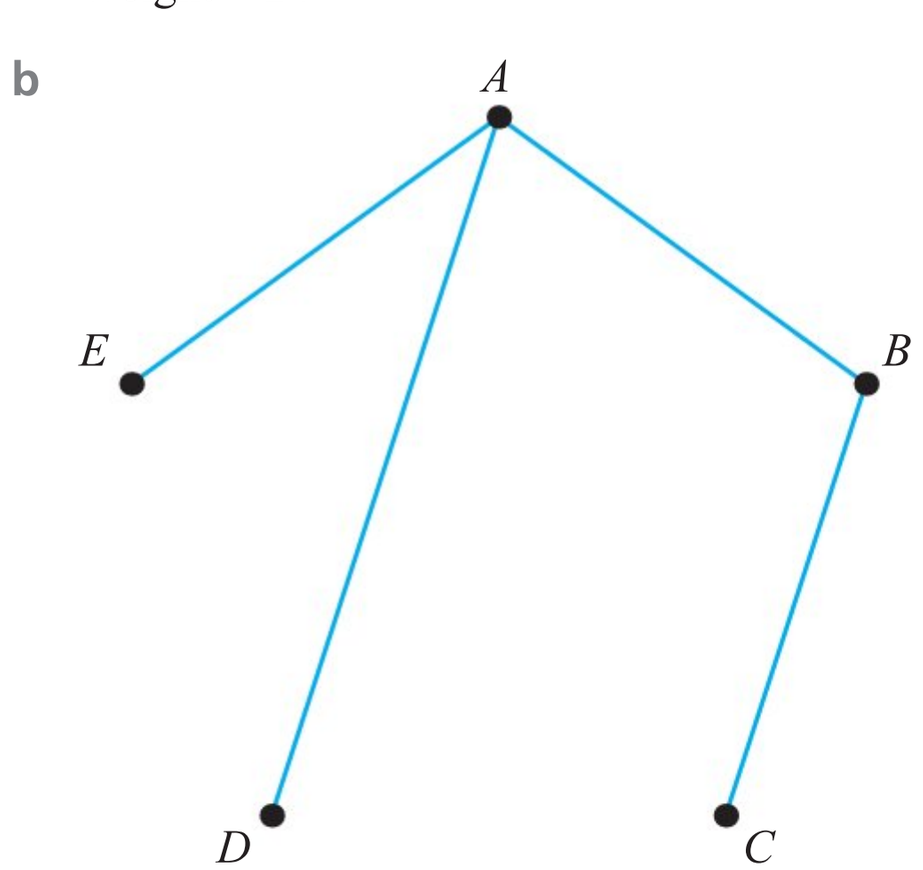
(can replace *PS* by *WS*)

4 a



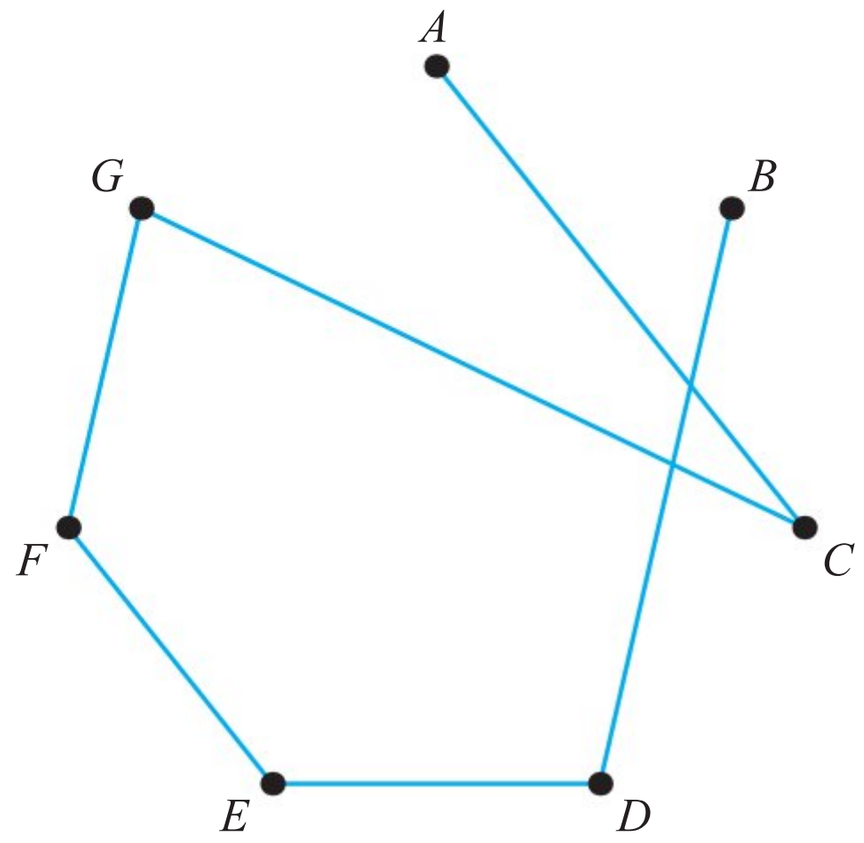
Weight = 24

b



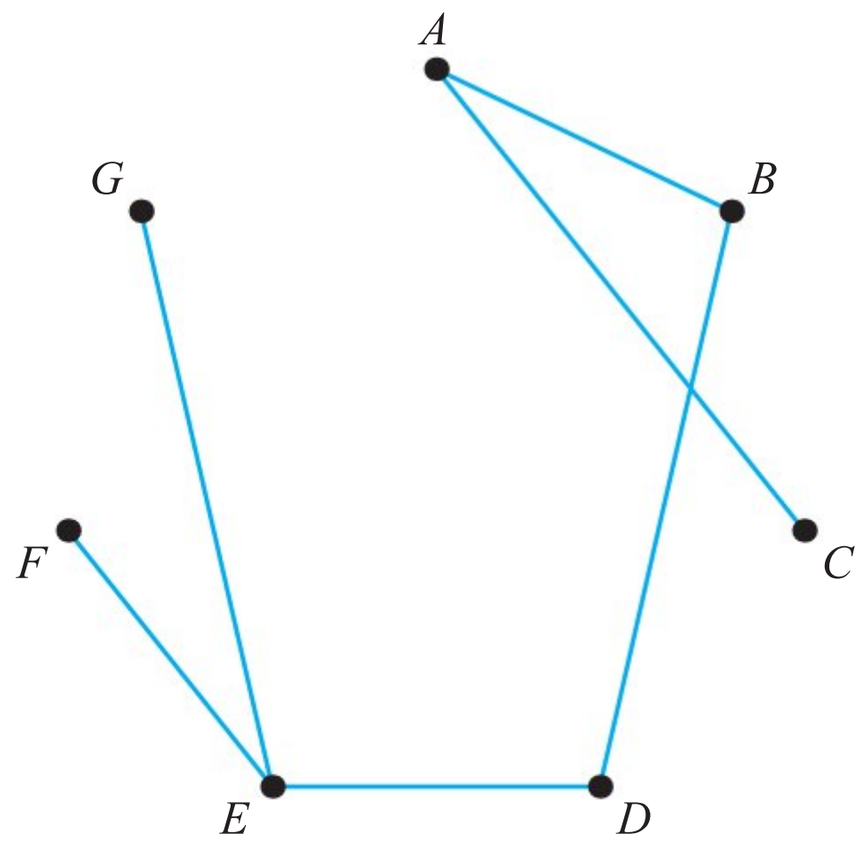
Weight = 52

5 a



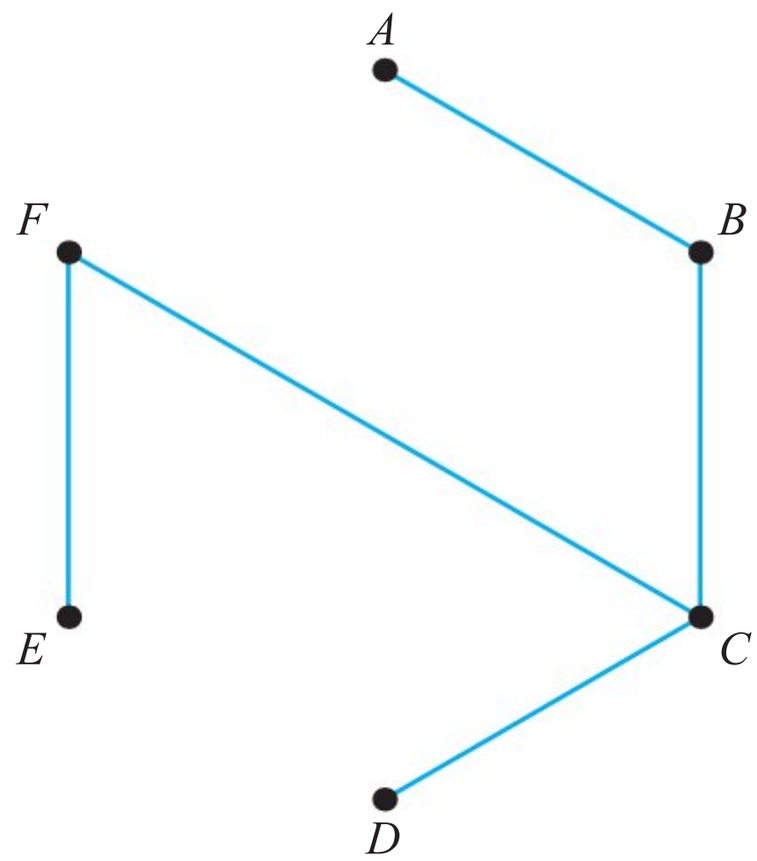
Weight = 120

b



Weight = 155

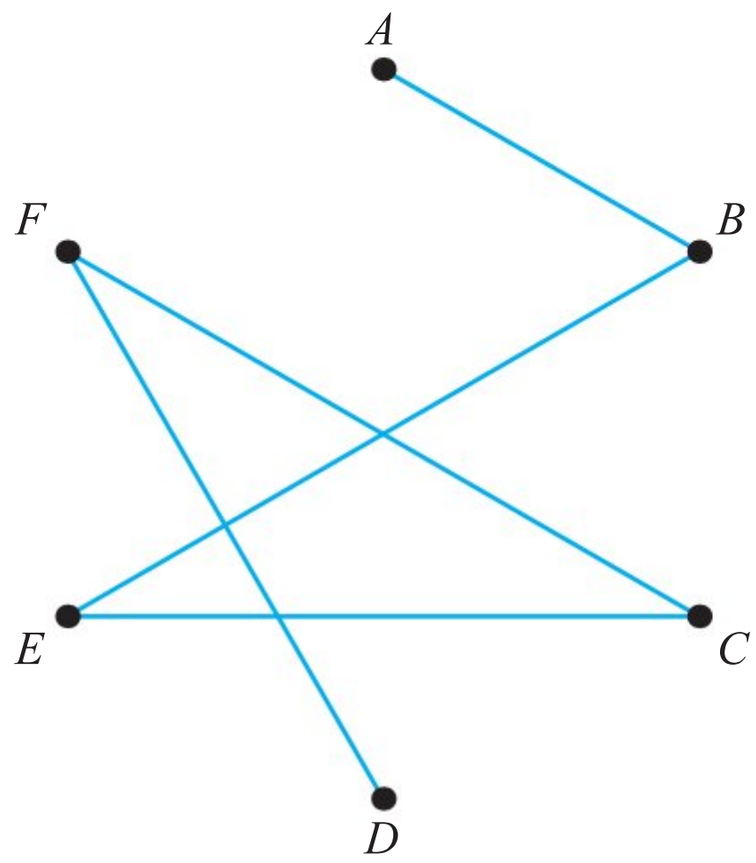
6 a



(or *DA* instead of *DC*)

Weight = 21

b



Weight = 19

7 a *AB*, *BD*, *DF*, *AC*, *CE*

b *AB*, *AD*, *DF*, *FC*, *CE*

8 a *AB*, *AE*, *ED*, *EC* (or *AC*)
or *AE*, *ED*, *AB*, *AC* (or *EC*)

b *AB*, *BC*, *CD*, *DE*
or *AE*, *AB*, *ED*, *BC*
or other combinations

9 a *AB*, *BG*, *BH*, *HE*, *HD*, *DC*, *GF*

b *AG*, *GB*, *BC*, *GD*, *DE*, *AF* (or *EF*)
or *AG*, *GD*, *DE*, *GB*, *BC*, *AF* (or *EF*)

10 a *AD*, *DB*, *BC*, *DE*

b *AD*, *AB*, *BC*, *EA*

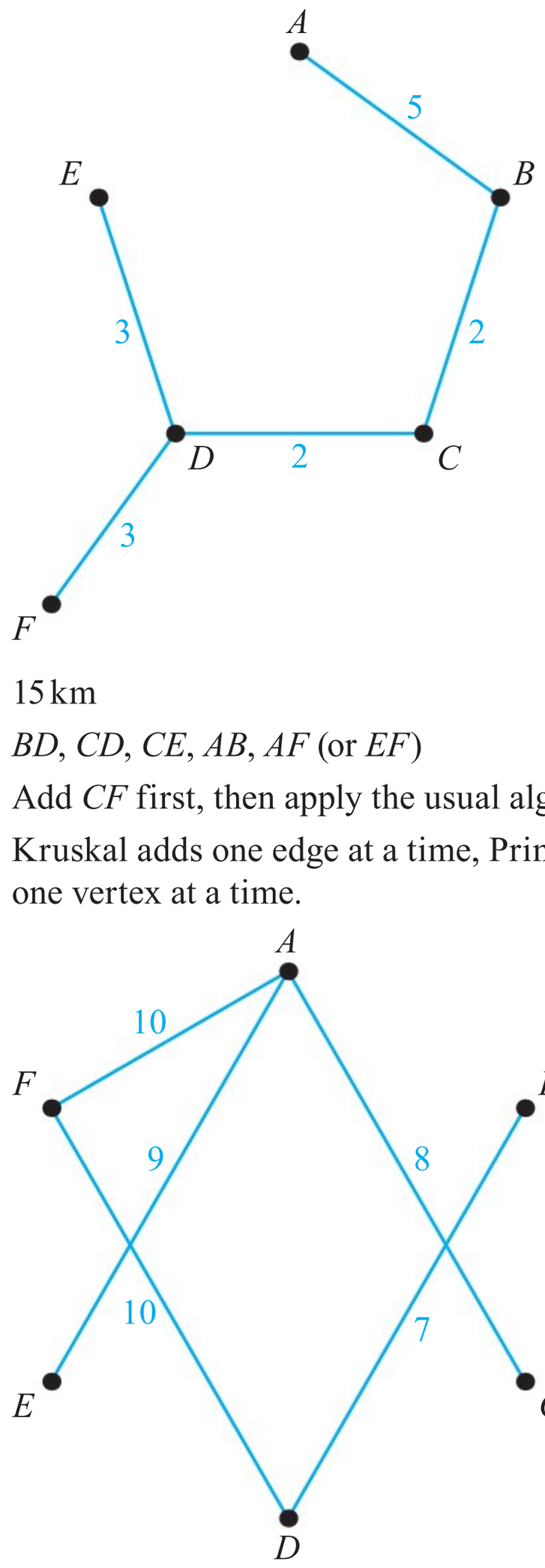
11 a *ED*, *EF*, *FG*, *GC*, *CA*, *DB*
or *ED*, *DG*, *EF*, *GC*, *CA*, *DB*

b *EF*, *ED*, *EG*, *DB*, *BA*, *AC*

12 a *CF*, *FE*, *EB*, *BA*, *DA* (or *DC*)
or *CF*, *FE*, *CB*, *BA*, *DC* (or *DA*)

b *CF*, *FD*, *EC*, *EB*, *BA*

13 **a**



b 15 km

14 **a** *BD*, *CD*, *CE*, *AB*, *AF* (or *EF*)

b Add *CF* first, then apply the usual algorithm.

15 **a** Kruskal adds one edge at a time, Prim adds one vertex at a time.

b

c 44 m

d **i** Each workstation is connected to the power supply.

ii There are no unnecessary connections.

16 **a** 49 **b** 58

17 $0 < x < 7$ or $21 < x < 28$

Exercise 7E

1 **a** *DE*, *EF* (70) **b** *DF*, *FG* (45)

2 **a** *AF*, *FE* (52) **b** *AF*, *FE* (34)

3 **a** *AB*, *EG* (78) **b** *BC*, *CD*, *AF*, *FE* (93)

4 **a** *AB*, *CF*, *FH* (137) **b** *BE*, *EF*, *DG* (107)

5 **a** *O*, *C* **b** 9

c 81 **d** *OB*, *BC*

6 **a** *F* and *D* (*FABFEDCBD*); 6800 m

b Vertices *D* and *F* have odd degrees.

c 8050 m

d *FE* and *ED*

7 e.g. *CBAJIHGFEDBDFHJCFDC*, 8.7 km

8 **a** 99 minutes (NB, I_1, SB, I_1, I_2, SB, I_1, NB)

b 130 minutes

c e.g. The time to walk from one bridge to another

9 **a** *AD* and *EH*

b 109 minutes

c 100 minutes

d e.g. *ABCDACFEDEHFGH*

10 **a** 208 m **b** *FG*, *DH*

c 208 m **d** 230 m

11 **a** *C* and *F* have odd degrees.

b *CEF* (15)

c *DEFDBACBECFECD* (180)

d **i** 165 **ii** *C* and *F*

Exercise 7F

1 **a** 36 (*ADCBA*) **b** 85 (*ACBDA*)

2 **a** 63 (*ACDEBA*) **b** 98 (*ADBECA*)

3 **a** 98 (*AECBDA*) **b** 110 (*ADCBEA*)

4 **a** 32 **b** 77

5 **a** 46 **b** 81

6 **a** 95 **b** 105

7 **a**

	A	*B*	*C*	*D*	*E*
A	–	4	10	9	5
B	4	–	6	5	9
C	10	6	–	11	15
D	9	5	11	–	4
E	5	9	15	4	–

b

	A	*B*	*C*	*D*	*E*
A	–	10	9	20	8
B	10	–	12	18	6
C	9	12	–	12	11
D	20	18	12	–	12
E	8	6	11	12	–

8 a

	A	*B*	*C*	*D*
A	–	8	10	7
B	8	–	7	9
C	10	7	–	6
D	7	9	6	–

b

	A	*B*	*C*	*D*
A	–	30	28	12
B	30	–	12	18
C	28	12	–	17
D	12	18	17	–

9 a

	A	*B*	*C*	*D*
A	–	4	7	6
B	4	–	3	8
C	7	3	–	5
D	6	8	5	–

b

	A	*B*	*C*	*D*
A	–	20	38	35
B	20	–	18	15
C	38	18	–	35
D	35	15	35	–

10 a *ABCDA*, *ABDCA*, *ACBDA*

b *ABDCA* (45)

11 a 32

b 31

c It is at most 31 km long.

12 a The cycle *ACBEDA* has length 29.

b 28

c 24

d It is 24, 25, 26, 27 or 28 minutes.

13 a *PUSR* is 40 km.

b *PUQ*, 25 km

c

	P	*Q*	*R*	*S*	*T*	*U*
P	–	**25**	40	**24**	**26**	14
Q	**25**	–	20	21	23	11
R	40	20	–	**16**	**28**	26
S	**24**	21	**16**	–	**12**	10
T	**26**	23	**28**	**12**	–	12
U	14	11	26	10	12	–

d 119 km

e 79 km

14 a 40

b 40

c The lower and upper bounds are equal.

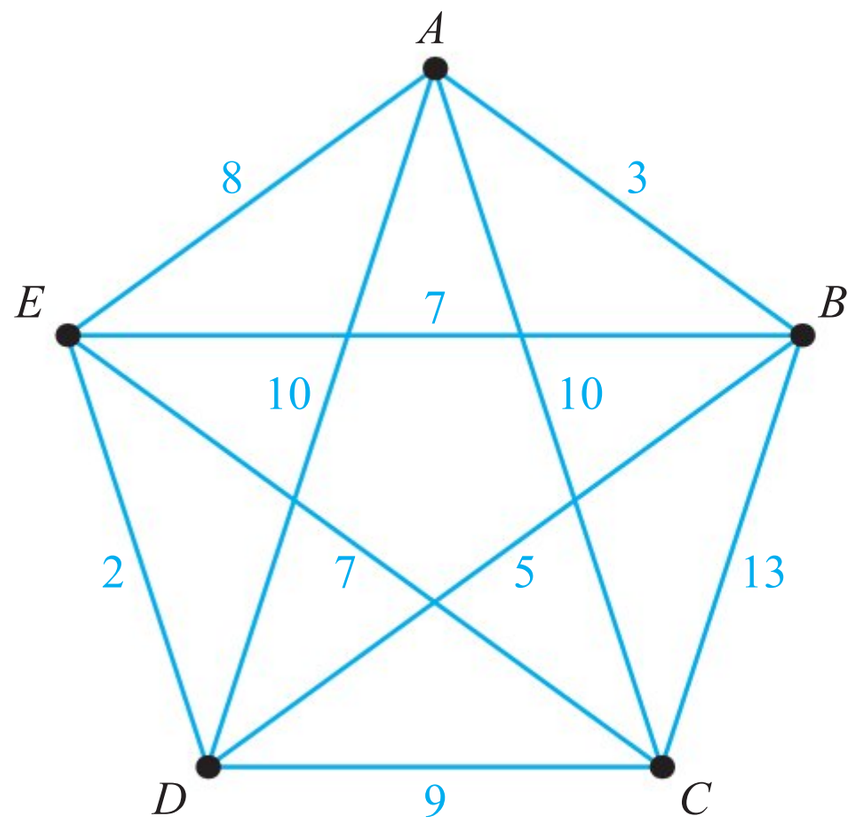
d *ABCDEFA*

15 a i *BDE* (7)

ii *BAC* (13)

iii *CED* (9)

b



c 27 (*CEDBAC*)

d 26

e The lower bound is unattainable; the paths *CD* and *CE* cannot be linked to either end of a route, nor can *CE* twice. *CD* and *CA* is the next best option, and can form a route.

f 27 minutes: *CEDBAC*

Chapter 7 Mixed Practice

1 a The second one, all vertices have even degree.

b e.g. *ABCDEBDA*

c *C* or *E*

2 a *RS*

b e.g. *PQRSUV*

c e.g. *PQRSVUP*

3 **a**

	A	B	C	D	E
A	0	0	0	1	1
B	0	0	1	1	1
C	0	1	0	1	0
D	1	1	1	0	0
E	1	1	0	0	0

b 20

c B and D

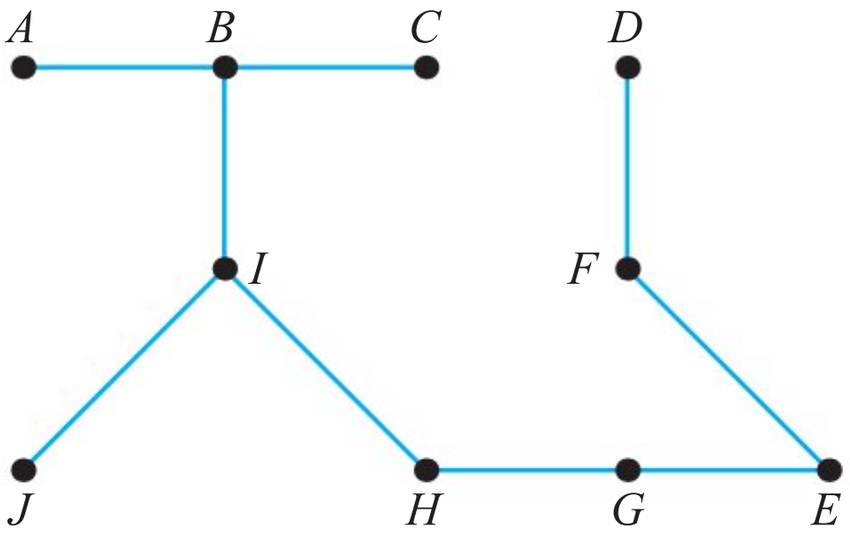
4 **a** A tree of minimum weight which includes every vertex of the graph.

b BH, NF, HN, HA, BE, CN; weight = 48

c BH, NF, HN, HA, BE, CN

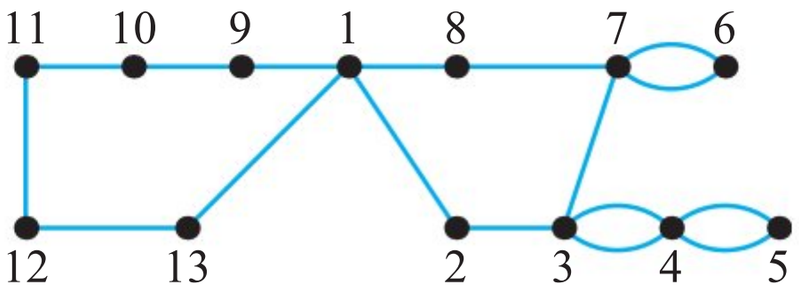
5 **a** Kruskal's adds one edges at a time, Prim's adds one vertex at a time.

b



Length = 117

6 **a** **i**



ii deg 2: 2, 5, 6, 8, 9, 10, 11, 12, 13;
deg 4: 1, 3, 4, 7

iii All vertices have even degree.

b **i** G: 15, H : 22

ii G: exactly two vertices have odd degrees (E and F).

iii Both graphs have vertices of odd degree.

7 **a** Hamiltonian cycle

b 111 ($BAFCDEB$)

c 106

d $106 \leqslant T \leqslant 111$

8 **a** $n-1$

b **i** AB, BC, AD, DE

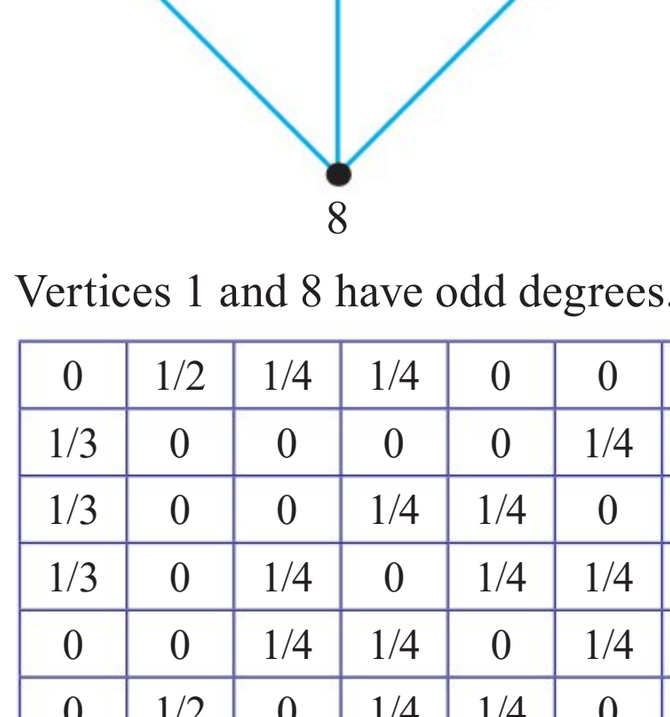
c **i** All vertices have even degree.

ii e.g. $ABCDEACEBDA$, 46

9 $C(0.293)$, $E(0.244)$, $D(0.195)$, $B(0.171)$, $A(0.098)$

10 276

11 **a**



b Vertices 1 and 8 have odd degrees.

c

0	1/2	1/4	1/4	0	0	0	0
1/3	0	0	0	0	1/4	0	0
1/3	0	0	1/4	1/4	0	1/2	0
1/3	0	1/4	0	1/4	1/4	0	0
0	0	1/4	1/4	0	1/4	0	1/3
0	1/2	0	1/4	1/4	0	0	1/3
0	0	1/4	0	0	0	0	1/3
0	0	0	0	1/4	1/4	1/2	0

d 0.215 **e** 11.5%

12 **a** A and E have odd degrees.

b 250 m **c** A and E

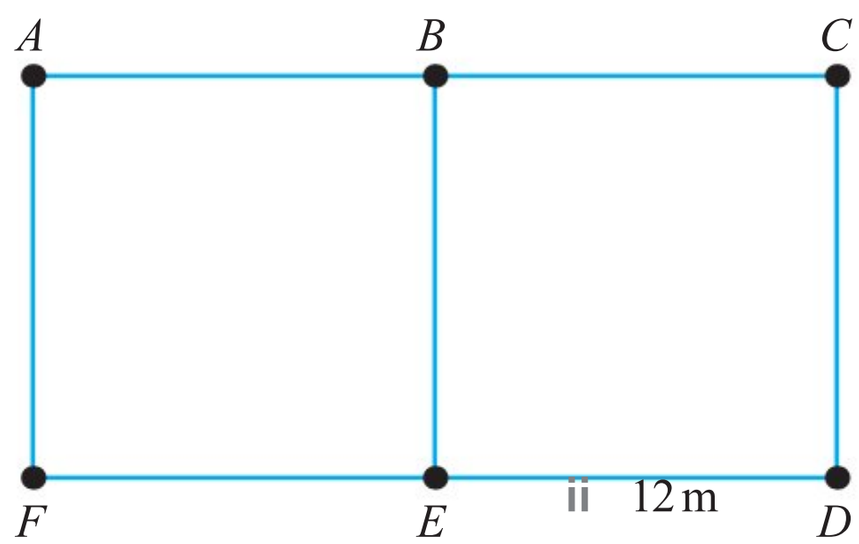
13 **b** **i** e.g $PQRSTSRTQP$

ii 34

c **i** To find a Hamiltonian cycle of least weight.

ii There is no cycle containing vertex P.

14 **a**



ii 12 m

b The number of walks of length 2 from a vertex to itself

c 8

d *ABEDC*, *AFEBC*, *AFEDC*

15 a 63

b 63

c *ADCBEA* is a solution to the travelling salesman problem.

16 a i There are vertices of odd degree.

ii No, there are more than two vertices of odd degree.

b i All degrees have doubled, so all are even

ii 306 km

17 \$92, repeat *AE*, *EB*, *DG*, *GC*

18 a

	A	*B*	*C*	*D*	*E*	*F*
A	–	8	8	16	18	18
B	8	–	7	15	17	17
C	8	7	–	8	10	10
D	16	15	8	–	6	8
E	18	17	10	6	–	9
F	18	17	10	8	9	–

b 52 **c** 56

d $52 \leqslant T \leqslant 56$ **e** *CBACDEFC*

19 a *CD*, *AB*, *BC*, *BF*, *CE*

b i 231

ii 239

iii 239

c $239 \leqslant L \leqslant 253$

20 a 33

b There is a cycle of length 33 (*ABCDEA*).

Chapter 8 Prior Knowledge

1 a 0.4 **b** 2.5

2 0.345

3 0.174

4 a 0.12 **b** 0.833

5 $\begin{pmatrix} 1319 \\ 1082 \\ 2541 \end{pmatrix}$

6 $x = -10.25,\ y = 7.25,\ z = 10.75$

7 a $\lambda = 1, 0.1,\ v = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

b $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.1 \end{pmatrix}\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^{-1}$

Exercise 8A

1 a $\mathrm{E}(Y) = 21,\ \mathrm{Var}(Y) = 20$

b $\mathrm{E}(Y) = 31,\ \mathrm{Var}(Y) = 80$

2 a $\mathrm{E}(Y) = -4,\ \mathrm{Var}(Y) = \frac{5}{9}$

b $\mathrm{E}(Y) = 4.9,\ \mathrm{Var}(Y) = 0.05$

3 a $\mathrm{E}(Y) = -25,\ \mathrm{Var}(Y) = 45$

b $\mathrm{E}(Y) = -3,\ \mathrm{Var}(Y) = 5$

4 a $\mathrm{E}(Y) = 24,\ \mathrm{Var}(Y) = 20$

b $\mathrm{E}(Y) = 21,\ \mathrm{Var}(Y) = 45$

5 a $\mathrm{E}(Y) = 1,\ \mathrm{Var}(Y) = 0.2$

b $\mathrm{E}(Y) = 5,\ \mathrm{Var}(Y) = 1.25$

6 a $\mathrm{E}(Z) = 30,\ \mathrm{Var}(Z) = 35$

b $\mathrm{E}(Z) = -37,\ \mathrm{Var}(Z) = 107$

7 a $\mathrm{E}(Z) = -37.75,\ \mathrm{Var}(Z) = 48.5$

b $\mathrm{E}(Z) = \frac{7}{6},\ \mathrm{Var}(Z) = \frac{7}{3}$

8 a $\mathrm{E}(Z) = 28.5,\ \mathrm{Var}(Z) = 45$

b $\mathrm{E}(Z) = -74,\ \mathrm{Var}(Z) = 224$

9 a $\mathrm{E}(Z) = 4.7,\ \mathrm{Var}(Z) = 1.16$

b $\mathrm{E}(Z) = 3,\ \mathrm{Var}(Z) = \frac{11}{16}$

10 a $\mathrm{E}(Z) = 1.5,\ \mathrm{Var}(Z) = 6$

b $\mathrm{E}(Z) = 0,\ \mathrm{Var}(Z) = 6$

11 a $\mathrm{E}(Z) = 17,\ \mathrm{Var}(Z) = 14$

b $\mathrm{E}(Z) = 17,\ \mathrm{Var}(Z) = 14$

12 a $\mathrm{E}(\bar{X}) = 5,\ \mathrm{Var}(\bar{X}) = 0.171$

b $\mathrm{E}(\bar{X}) = 6,\ \mathrm{Var}(\bar{X}) = 0.208$

13 a $\mathrm{E}(\bar{X}) = -4.7,\ \mathrm{Var}(\bar{X}) = 0.04$

b $\mathrm{E}(\bar{X}) = -15.1,\ \mathrm{Var}(\bar{X}) = 0.0467$

14 a $\mathrm{E}(\bar{X}) = 12,\ \mathrm{Var}(\bar{X}) = 0.9$

b $\mathrm{E}(\bar{X}) = 8,\ \mathrm{Var}(\bar{X}) = 0.0208$

15 a $\mathrm{E}(\bar{X}) = 3,\ \mathrm{Var}(\bar{X}) = 0.15$

b $\mathrm{E}(\bar{X}) = 3.6,\ \mathrm{Var}(\bar{X}) = 0.315$

16 a 4.2 b 13.6
c 44.64
17 a $\frac{8}{15}$ b 13.8
18 a $\frac{25}{8}$ b 34.25
c 85.9
19 a 15 b 17
20 6.6 cm, 0.6 cm
21 198.8 g, 4.16 g
22 102 g, 1.92 g
23 $\mathrm{E}(B) = 3.1,\ \mathrm{Var}(B) = 0.3$
24 $\mathrm{E}(V) = -6,\ \mathrm{Var}(V) = 2.56$
25 1076 kg, 30.2 kg
26 0, 1.41
27 0, 35.4
28 68.3 g
29 65 minutes, 8.06 minutes
30 a 19 b 10

Exercise 8B

1 a $W \sim \mathrm{N}(21, 16)$ b $W \sim \mathrm{N}(42, 67)$
2 a $W \sim \mathrm{N}(7, 111)$ b $W \sim \mathrm{N}(-1, 7)$
3 a $W \sim \mathrm{N}(7, 1.48)$ b $W \sim \mathrm{N}(-5.5, 6.91)$
4 a $W \sim \mathrm{N}(0, 8)$ b $W \sim \mathrm{N}(15, 9)$
5 a $W \sim \mathrm{N}(10, 0.5)$ b $W \sim \mathrm{N}(5, 0.3)$
6 a $\bar{X} \sim \mathrm{N}(30, 1.25)$ b $\bar{X} \sim \mathrm{N}(30, 0.5)$
7 a Cannot say b Cannot say
8 a $\sum_{i=1}^{50} X_i \sim \mathrm{N}(1500, 5000)$
b $\sum_{i=1}^{40} X_i \sim \mathrm{N}(1200, 4000)$
9 a 0.543 b 0.996
10 a 0.331 b 0.137
11 a $\mathrm{N}(91.3, 16.3)$ b 0.0156
12 a 0.3 s, 0.721 s b 0.339
c 0.166
13 a 65 cm, 0.005 cm^2 b 0.00235
14 a 0.252 b 0.0175
15 0.0352
16 0.0336
17 0.0820
18 a 0.4 kg, 0.223 kg^2
b 0.198
c 0.0209
19 a 0.196
b 0.0211
20 0.272 m
21 0.0281
22 a 2500 g, 1.79 g
b 0.995
c CLT shows that the distribution of the mass of a ream is approximately normal.
23 a 0.321
b The distribution of the times for each company were unknown.
24 a 0.208
b 0.196
25 $\mu = 7.33$ mm, $\sigma = 0.525$ mm
26 a 0.0228
b i 0.868
ii 0.315
iii 0.868
c 0.691
d 0.645
27 a 0.0173
b The mean of normal variables is normal.
28 44

Exercise 8C

1 a No, $X \sim \mathrm{B}(60, 0.801)$
b No, $X \sim \mathrm{B}(5, 0.195)$
2 a Yes, $X \sim \mathrm{Po}(3)$
Conditions well met.
b Yes, $X \sim \mathrm{Po}(4)$
But occurrences of fish unlikely to be independent of each other as live in shoals.
3 a No – this is a continuous distribution.
b No – this is a continuous distribution.

4 a No – not Poisson as not a number of events in a given period; not binomial as n not fixed.

b No – not Poisson as not a number of events in a given period; not binomial as n not fixed.

5 a 0.132 b 0.108

6 a 0.0789 b 0.666

7 a 0.774 b 0.399

8 a 0.843 b 0.990

9 a 0.085 b 0.601

10 a 0.507 b 0.458

11 a 0.138 b 0.590

12 a 0.249 b 0.213

13 a 3.1 b 3.1

14 a 2.30 b 5.3

15 a $X \sim \text{Po}(36)$ b $X \sim \text{Po}(24)$

16 a $X \sim \text{Po}(2.4)$ b $X \sim \text{Po}(3)$

17 a $X \sim \text{Po}(54)$ b $X \sim \text{Po}(9.6)$

18 a $X \sim \text{Po}(6.5)$ b $X \sim \text{Po}(8.9)$

19 a $X \sim \text{Po}(14)$ b $X \sim \text{Po}(7)$

20 0.0116

21 a 0.0595 b 0.0548

22 a 0.298 b 0.973

23 a 0.868

b The rate of arrival of messages is unlikely to be constant – there will probably be more at some times of the day than others. Within each distribution messages are not likely to be independent as they may occur as part of a conversation. The two distributions are also probably not independent of each other, as times when more emails arrive might be similar to times when more texts might arrive.

24 a 0.916 b 0.0656

25 a 0.430

b No – likely to be more outbreaks at certain times of the year so not constant rate, and outbreaks not independent since contagious.

26 a i 0.161 ii 0.0514

b 0.0132

27 a 0.0537 b 0.321

28 a 0.475 b 1 c 0.00413

29 a 0.273 b 0.143 c 201

30 a 0.747 b 78

Exercise 8D

1 a $\begin{pmatrix} 0.25 & 0.7 \\ 0.75 & 0.3 \end{pmatrix}$ b $\begin{pmatrix} 0.45 & 0.9 \\ 0.55 & 0.1 \end{pmatrix}$

2 a $\begin{pmatrix} 0.82 & 0.08 & 0.22 \\ 0.06 & 0.73 & 0.11 \\ 0.12 & 0.19 & 0.67 \end{pmatrix}$

b $\begin{pmatrix} 0.64 & 0 & 0.45 \\ 0.02 & 0.57 & 0 \\ 0.34 & 0.43 & 0.55 \end{pmatrix}$

3 a $\begin{pmatrix} 0.71 & 0 & 0.24 & 0.45 \\ 0.29 & 0.88 & 0 & 0 \\ 0 & 0.05 & 0.57 & 0.07 \\ 0 & 0.07 & 0.19 & 0.48 \end{pmatrix}$

b $\begin{pmatrix} 0.35 & 0.15 & 0.1 & 0.2 \\ 0.1 & 0.4 & 0.05 & 0.2 \\ 0.25 & 0.2 & 0.5 & 0.05 \\ 0.3 & 0.25 & 0.35 & 0.55 \end{pmatrix}$

4 a $\begin{pmatrix} 0.14 & 0.23 \\ 0.86 & 0.77 \end{pmatrix}$ b $\begin{pmatrix} 0.5 & 1 \\ 0.5 & 0 \end{pmatrix}$

5 a $\begin{pmatrix} 0.3 & 0 & 0.4 \\ 0.1 & 1 & 0.4 \\ 0.6 & 0 & 0.2 \end{pmatrix}$

b $\begin{pmatrix} 0.22 & 0.5 & 0.12 \\ 0.33 & 0.29 & 0.8 \\ 0.45 & 0.21 & 0.08 \end{pmatrix}$

6 a $\begin{pmatrix} 0 & 0.55 & 0.05 & 0.3 \\ 0.3 & 0 & 0.7 & 0.1 \\ 0.35 & 0.25 & 0 & 0.6 \\ 0.35 & 0.2 & 0.25 & 0 \end{pmatrix}$

b $\begin{pmatrix} 0.84 & 0 & 0.31 & 0.34 \\ 0 & 0 & 0.13 & 0.22 \\ 0 & 0.5 & 0.56 & 0.44 \\ 0.16 & 0.5 & 0 & 0 \end{pmatrix}$

7 a 0.666 b 0.590

8 a 0.316 b 0.378

9 a 0.112 b 0.0426

10 a $\begin{pmatrix} 0.610 \\ 0.390 \end{pmatrix}$ b $\begin{pmatrix} 0.503 \\ 0.497 \end{pmatrix}$

11 a $\begin{pmatrix} 0.277 \\ 0.325 \\ 0.398 \end{pmatrix}$ b $\begin{pmatrix} 0.264 \\ 0.365 \\ 0.371 \end{pmatrix}$

12 a $\begin{pmatrix} 0.260 \\ 0.254 \\ 0.267 \\ 0.220 \end{pmatrix}$ b $\begin{pmatrix} 0.153 \\ 0.448 \\ 0.237 \\ 0.162 \end{pmatrix}$

13 a $\begin{pmatrix} 0.591 \\ 0.409 \end{pmatrix}$ b $\begin{pmatrix} 0.517 \\ 0.483 \end{pmatrix}$

14 a $\begin{pmatrix} 0.209 \\ 0.579 \\ 0.213 \end{pmatrix}$ b $\begin{pmatrix} 0.333 \\ 0.333 \\ 0.333 \end{pmatrix}$

15 a $\begin{pmatrix} 0.253 \\ 0.259 \\ 0.261 \\ 0.227 \end{pmatrix}$ b $\begin{pmatrix} 0.153 \\ 0.444 \\ 0.239 \\ 0.164 \end{pmatrix}$

16 a $\begin{pmatrix} 13/22 \\ 9/22 \end{pmatrix}$ b $\begin{pmatrix} 61/118 \\ 57/118 \end{pmatrix}$

17 a $\begin{pmatrix} 49/235 \\ 136/235 \\ 10/47 \end{pmatrix}$ b $\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$

18 a $\begin{pmatrix} 249/985 \\ 51/197 \\ 257/985 \\ 224/985 \end{pmatrix}$ b $\begin{pmatrix} 9/59 \\ 4/9 \\ 127/531 \\ 29/177 \end{pmatrix}$

19 a 0.35 b $\begin{pmatrix} 0.875 & 0.875 \\ 0.125 & 0.125 \end{pmatrix}$

c 12.5%

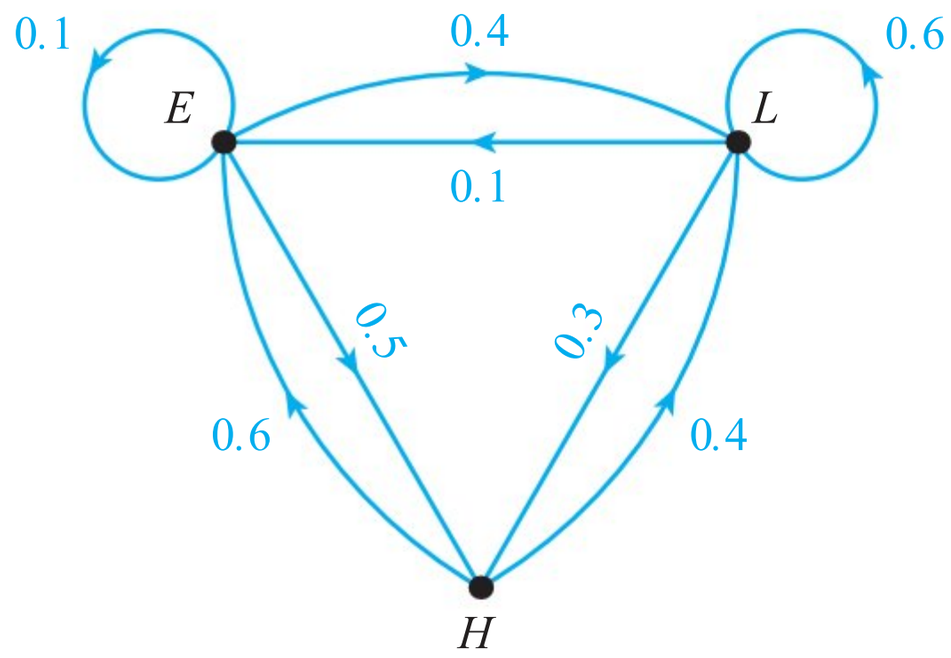
20 a $\begin{pmatrix} 0.95 & 0.8 \\ 0.05 & 0.2 \end{pmatrix}$ b 0.938

c $\begin{pmatrix} 0.941 \\ 0.059 \end{pmatrix}$

21 a $\begin{pmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{pmatrix}$ b $\begin{pmatrix} 433 \\ 357 \\ 210 \end{pmatrix}$

c $\begin{pmatrix} 450 \\ 350 \\ 200 \end{pmatrix}$

22 a



b 0.319 c 0.5

23 a $\begin{pmatrix} 0.85 & 0.1 & 0.25 \\ 0.05 & 0.75 & 0.25 \\ 0.1 & 0.15 & 0.5 \end{pmatrix}$

b 0.278

c Bull 51.5%, Bear 29.4%, Stagnant 19.1%

24 a $\begin{pmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix}$ b $\frac{2}{3}$

25 a $\begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}$ b 0.561

c $\begin{cases} 0.7a + 0.4b = a \\ 0.3a + 0.6b = b \\ a + b = 1 \end{cases}$ d $a = \frac{4}{7}, b = \frac{3}{7}$

26 a 0.125

b $\begin{cases} 0.5d + 0.25h = d \\ 0.5d + 0.5h + 0.5r = h \\ 0.25h + 0.5r = r \\ d + h + r = 1 \end{cases}$

c $d = 0.25, h = 0.5, r = 0.25$

27 **a** $\begin{pmatrix} 0.9 & 0.15 \\ 0.1 & 0.85 \end{pmatrix}$

b $\lambda = 1, \frac{3}{4}$

$v_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

c $\mathbf{P} = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{3}{4} \end{pmatrix}$

d $0.6 + 0.05\left(0.75^n\right)$

e 0.6

28 **a** $\begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$

b $\lambda = 1, \frac{1}{5}$

$v_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

c $\mathbf{P} = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{5} \end{pmatrix}$

d $\frac{1}{4} - \frac{1}{4}\left(\frac{1}{5}\right)^n$

e $\frac{1}{4}$

Chapter 8 Mixed Practice

1 **a** 3.2 **b** −7.6 **c** 14.0

2 **a** 0.1 **b** 4.2 **c** 0.6 **d** 4.28

3 **a**

x	1	2	3	4
$\mathbf{P}(X = x)$	$\frac{1}{13}$	$\frac{5}{26}$	$\frac{4}{13}$	$\frac{11}{26}$

b $\frac{40}{13}$ **c** 22.9

4 **a** **i** 6.4 kg **ii** $0.52\,\text{kg}^2$

b Masses of individual components are independent.

5 **a** 20 m, $1.57\,\text{m}^2$ **b** 0.0553

6 0.123

7 **a** 45, 6.71

b Number of tweets on any given day independent from other days.

8 **a** Independent events; constant rate of success

b 0.731

9 **a** The average rate must be constant. However, we might expect it to vary over different times of the day and with different weather conditions. Birds must arrive independently, but we know they may come in flocks.

b 0.156

10 **a** 0.04 **b** $\begin{pmatrix} 24.6\% \\ 42.6\% \\ 32.8\% \end{pmatrix}$

c $\begin{pmatrix} 16.2\% \\ 33.7\% \\ 50.1\% \end{pmatrix}$

11 **a** **i** $2\mu, 2\sigma^2$ **ii** $3\mu, 9\sigma^2$

iii $\mu, 3\sigma^2$ **iv** $\mu, \frac{\sigma^2}{n}$

12 0.00537

13 **a** 0.110 **b** 0, 168 **c** 0.350

14 0.882

15 **a** $X \sim \text{Po}(45)$, calls arrive independently and at a constant rate.

b 0.204 **c** 0.0207

16 **a** 0.547 **b** 3

17 **a** 0.362 **b** 0.279

c 0.659 **d** 0.0464

e 0.247 **f** 0.641

18 **a** 0.189 **b** 0.372 **c** 0.208

19 **a**

$$\begin{array}{c} \begin{matrix} R & F & S \end{matrix} \\ \begin{pmatrix} 0.6 & 0.4 & 0.5 \\ 0.1 & 0.4 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{pmatrix} \begin{matrix} R \\ F \\ S \end{matrix} \end{array}$$

b 0.526

c **i** $\begin{cases} 0.6r + 0.4f + 0.5s = r \\ 0.1r + 0.4f + 0.2s = f \\ 0.3r + 0.2f + 0.3s = s \\ r + f + s = 1 \end{cases}$

ii $\begin{pmatrix} 38/71 \\ 13/71 \\ 20/71 \end{pmatrix}$

20 **a** 0.785 **b** 0.122 **c** 0.00976