Chapter 4

Loans and annuities

Contents:

OPENING PROBLEM

Diane is about to retire at the age of 68. Over her career, she has accumulated £600 000 in savings which she places in an *annuity* fund paying 4% p.a. interest compounded monthly. Her plan is to make monthly withdrawals from the account to live on for the next 25 years.

Things to think about:

a What is meant by an annuity fund? How does it differ from the standard compound interest investments we have studied previously?

b Diane reasons that for her money to last for 25 years or 300 months, she should be able to withdraw at least $\frac{£600\,000}{300} = £2000$ each month. Is Diane correct? Explain your answer.

c If Diane's investment is to be completely used up by regular withdrawals over 25 years, how much can she withdraw each month?

Loans and **annuities** play an important role in modern life. Understanding these processes allows you to make informed financial decisions.

HISTORICAL NOTE

In 14th century Italy, merchants would set up stalls in local markets and lend money to customers. Interest was applied, and the borrower was expected to repay the money in regular intervals. The word "bank" is derived from the Italian word *banca*, which describes the benches on which the merchants sat.

If a banker suffered a series of bad loans and was unable to repay his own creditors, the creditors would break up his bench, showing that the banker was no longer in business. This was known as *banca rotta*, meaning "broken bench". Over time, the Italian word for "broken" was replaced with the Latin word *rupta*, giving *banca rupta*, the basis of the modern term "bankrupt".

A LOANS

A common way to borrow money to finance larger purchases such as houses, cars, renovations, education expenses, and share portfolios, is to take out a **personal loan**. The financial institution lends an amount of money to the borrower, who repays the amount plus interest by a series of regular repayments over a given time period.

The process of repaying a loan with a series of regular repayments is called **amortisation**.

Interest is calculated on the **reducing balance** of the loan, so the interest gradually reduces as the loan is repaid.

Because of the interest, the total repayment will be greater than the amount originally borrowed. The difference between these values is the interest paid.

interest paid = total repayments − amount borrowed

CALCULATING REPAYMENTS

We can use a **table of repayments** to work out the repayments required for a loan. The table below shows the amount you need to repay each month for every $1000 borrowed.

Table of monthly repayments per $1000 borrowed

Loan term (years)	*Annual interest rate* 4%	4.5%	5%	5.5%	6%	6.5%	7%
1	85.1499	85.3785	85.6075	85.8368	86.0664	86.2964	86.5267
1.5	57.3314	57.5557	57.7805	58.0059	58.2317	58.4581	58.6850
2	43.4249	43.6478	43.8714	44.0957	44.3206	44.5463	44.7726
3	29.5240	29.7469	29.9709	30.1959	30.4219	30.6490	30.8771
4	22.5791	22.8035	23.0293	23.2565	23.4850	23.7150	23.9462
5	18.4165	18.6430	18.8712	19.1012	19.3328	19.5661	19.8012
10	10.1245	10.3638	10.6066	10.8526	11.1021	11.3548	11.6108
15	7.39688	7.64993	7.90794	8.17083	8.43857	8.71107	8.98828
20	6.05980	6.32649	6.59956	6.87887	7.16431	7.45573	7.75299

We can also calculate the monthly repayments using the TVM solver on a calculator.

In this case, you are receiving the amount loaned, so PV is positive. PMT is the amount to be repaid each time period, so it is negative.

GRAPHICS CALCULATOR INSTRUCTIONS

Example 1 Self Tutor

Erica takes out a personal loan of $16 500 to buy a car. She negotiates a term of 4 years at 5.5% p.a. interest.

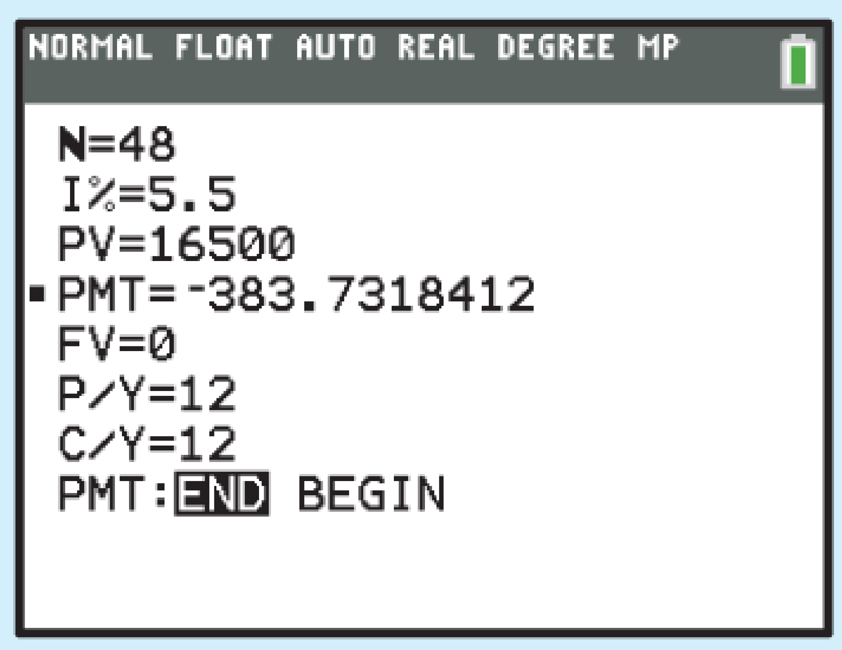
a Calculate the monthly repayments using the table. Check your answer using technology.

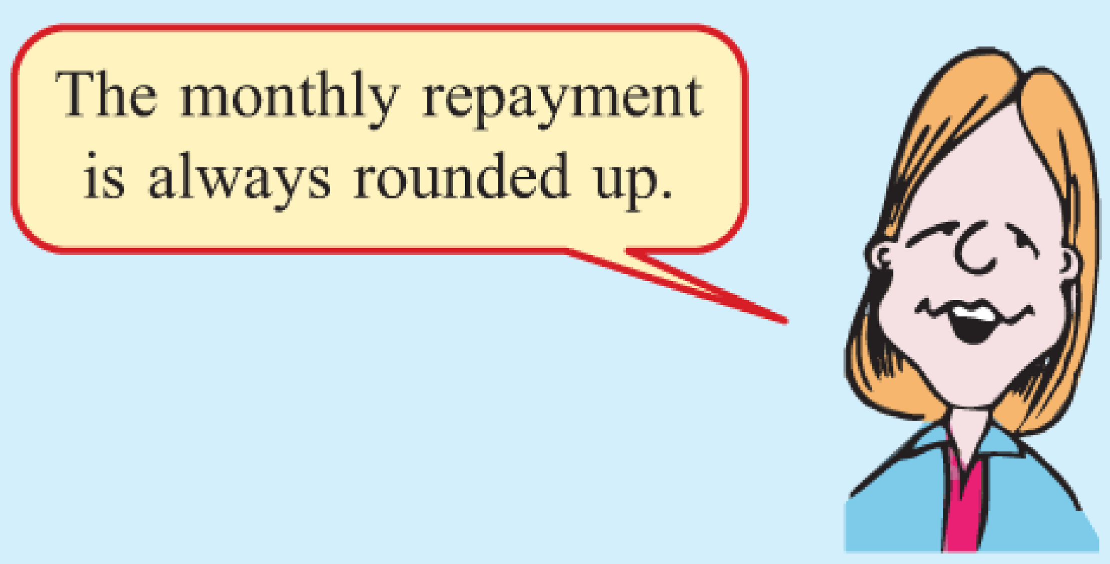
b Hence calculate: **i** the total repayment **ii** the total interest charged.

a From the table, the monthly repayments on each $1000 for 4 years at 5.5% p.a. = $23.2565

$\therefore$ repayments on $16 500
= $23.2565 × 16.5 {16.5 lots of $1000}
= $383.732

The repayments are $383.74 per month.





b **i** Total repayment
= monthly repayment × number of months
= $383.74 × 48
= $18 419.52

ii Interest
= total repayment − amount borrowed
= $18 419.52 − $16 500
= $1919.52

In the following Exercise, calculate monthly repayments using your calculator. Where appropriate, check your answers using the table.

EXERCISE 4A

1 Don takes out a personal loan for $12 000 to fund his wedding. He will repay it over 5 years at 6% p.a. Calculate the:

a monthly repayments **b** total repayment **c** interest charged.

2 Jay and Penni need £9500 to fund house renovations. They take out a personal loan over 3 years at 4.5% p.a. Calculate the:

a monthly repayments **b** total repayment **c** interest charged.

3 Binh-vu needs $15 000 to buy a boat. His bank offers him a personal loan at 6.5% p.a. Calculate the total interest he will pay if he repays the loan with monthly repayments over:

a 2 years **b** 5 years.

4 Mimi takes out a personal loan of €10 000. She will repay the loan over 4 years at 6% p.a. She is charged a €150 application fee and a €10 monthly service fee.
Calculate the total cost of taking out the loan.

5 The spreadsheet alongside shows the progress of a $30 000 loan taken out over 5 years at 8% p.a. interest compounded monthly.

a Check that the monthly repayment has been correctly calculated.

b What is the account balance after six months?

c How much interest is paid in:

i month 1 **ii** month 6?

Explain the differences between these amounts.

	A	B	C	D	E
1	**LOAN SPREADSHEET**				
2					
3	**Loan amount**	$30,000.00			
4	**Number of years**	5			
5	**Rate p.a.**	8.00%			
6	**Periods p.a.**	12			
7	**Rate per period**	0.667%			
8	**Repayment**	$608.30			
9					
10	**Month**	**Amount**	**Interest**	**Repayment**	**Balance**
11	1	$30,000.00	$200.00	$608.30	$29,591.70
12	2	$29,591.70	$197.28	$608.30	$29,180.68
13	3	$29,180.68	$194.54	$608.30	$28,766.92
14	4	$28,766.92	$191.78	$608.30	$28,350.40
15	5	$28,350.40	$189.00	$608.30	$27,931.10
16	6	$27,931.10	$186.21	$608.30	$27,509.01

d Click on the spreadsheet icon.
Fill row 16 down to the end of the 60th month.

i How much interest is paid in the 60th month?

ii How much interest is paid in total?

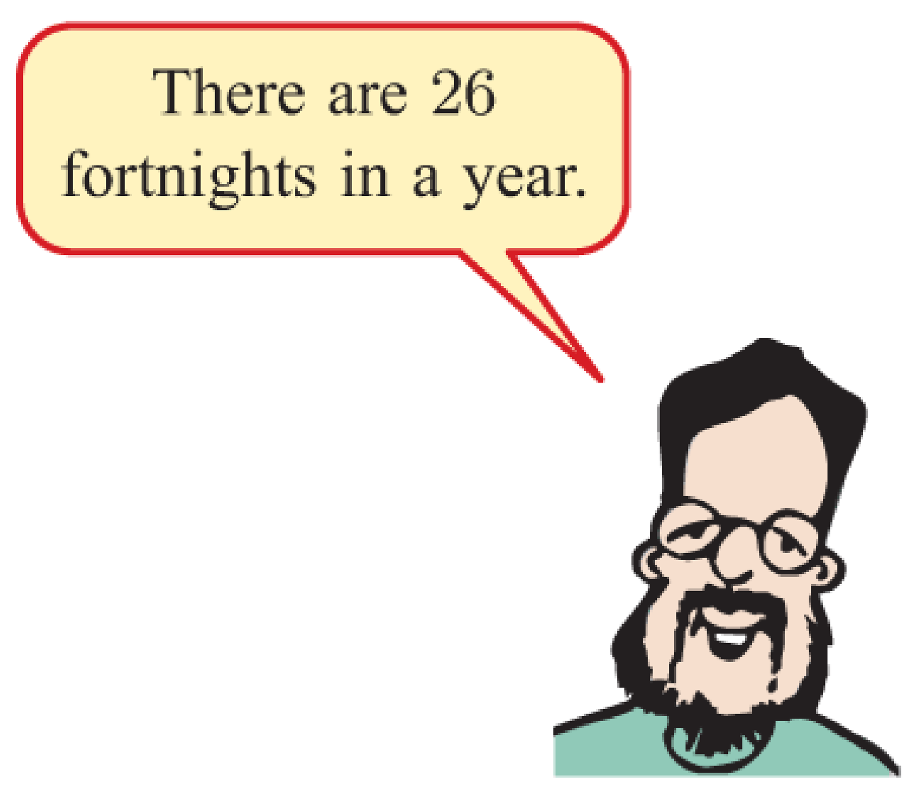
SPREADSHEET

e Explain why the final payment is slightly less.

6 Grace takes out a loan of $7000 to buy a jet ski. She will repay the loan over 2 years at 9.9% p.a. interest compounded fortnightly.

a Calculate Grace's fortnightly repayments.

b Find the total amount of interest charged on the loan.



Example 2 — Self Tutor

Ryan takes out a $10 000 loan to be repaid over three years at 9.6% p.a. interest compounded monthly.

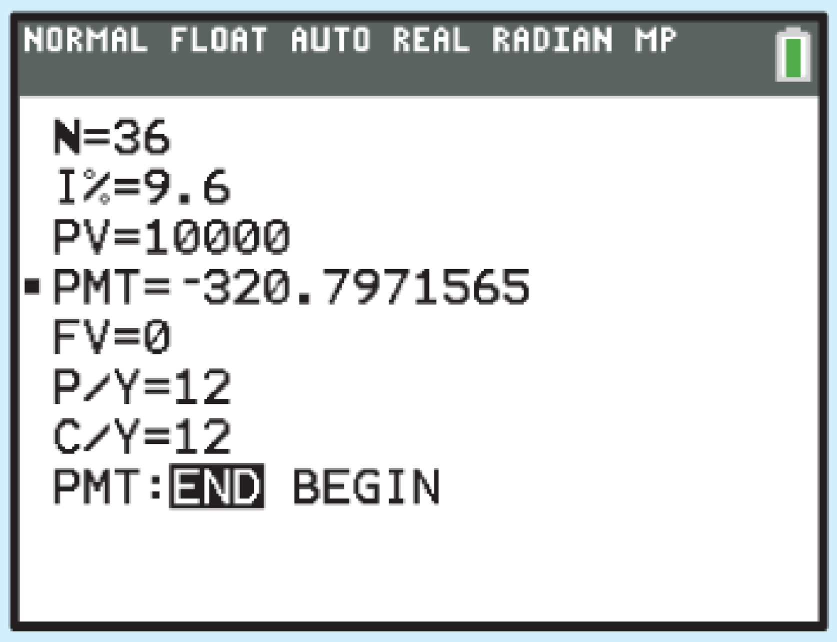
a Calculate the monthly repayments.

b Find the outstanding balance on the loan after 1 year of repayments.

a $N = 3 \times 12 = 36$, $I\% = 9.6$, $PV = 10\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$\therefore \quad PMT \approx -320.80$

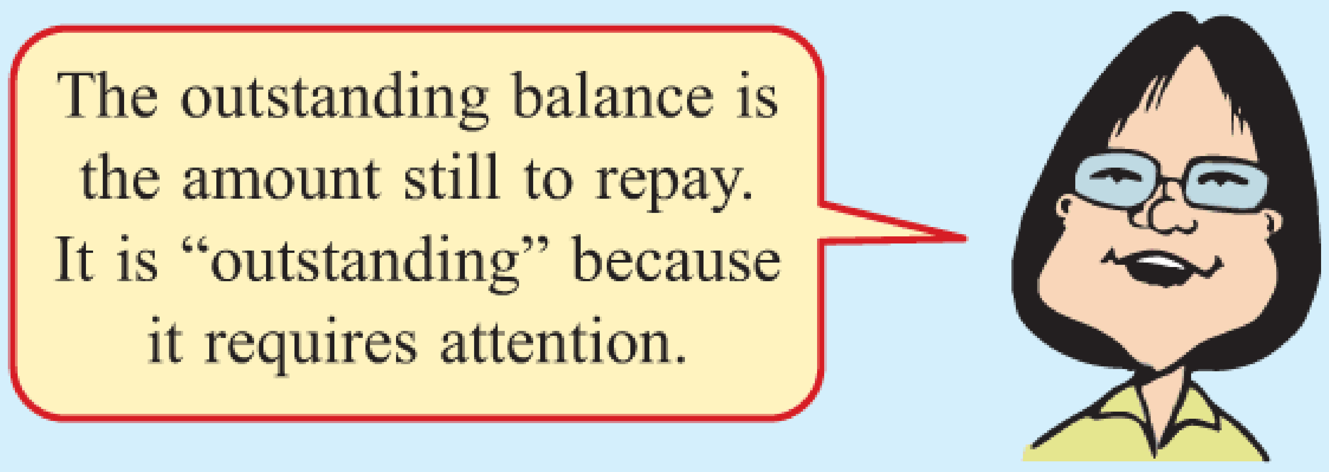
The monthly repayment is $320.80 .

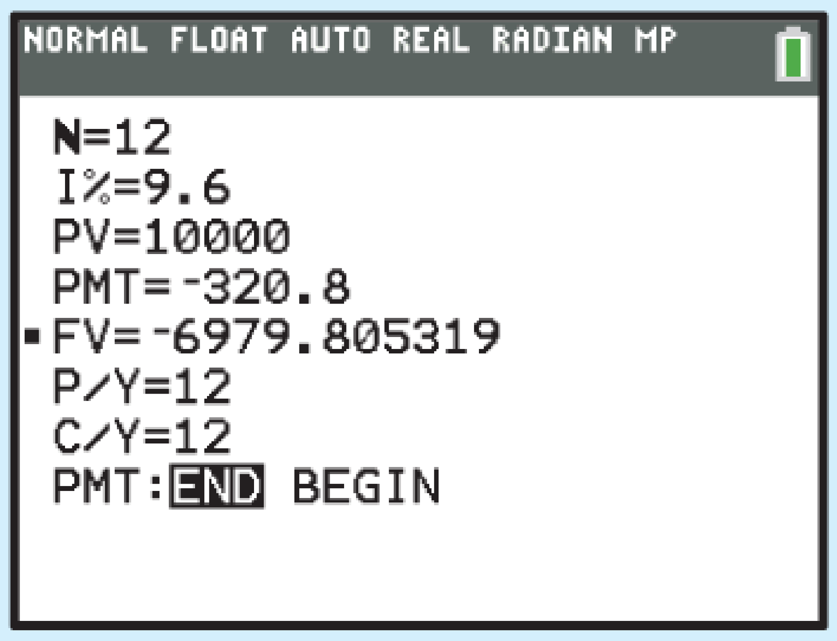


b $N = 1 \times 12 = 12$, $I\% = 9.6$, $PV = 10\,000$, $PMT = -320.80$, $P/Y = 12$, $C/Y = 12$

$\therefore \quad FV \approx -6979.81$

The outstanding balance on the loan after 1 year is $6979.81 .

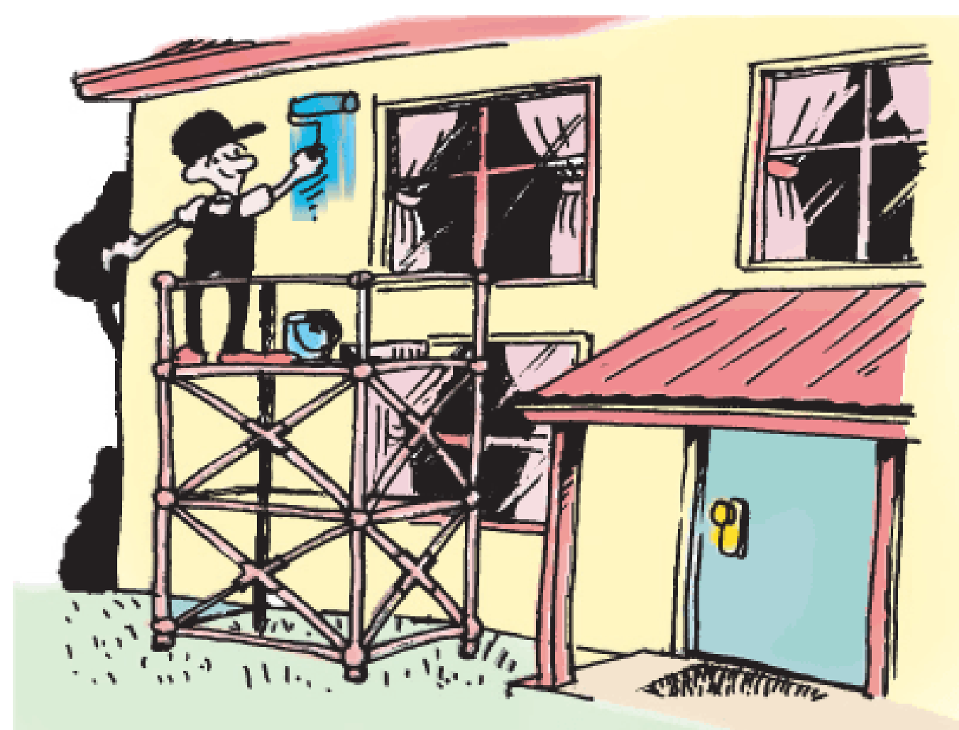




7 Pieter takes out a loan of £20 000 to renovate his home. He agrees to repay the loan over 4 years at 8.25% p.a. interest compounded monthly. Find:

a Pieter's monthly repayments

b the outstanding balance on the loan after 1 year of repayments.



8 Jacob wants to buy a piano valued at $12 000. He has $3500 in savings, and will borrow the remaining money.

a How much will Jacob borrow?

b Jacob will repay the loan over 5 years at 7.9% p.a. interest compounded quarterly.

  i Calculate Jacob's quarterly repayments.

  ii Calculate the total interest charged on the loan.

  iii Find the outstanding balance on the loan after 2 years.

9 Simon took out a loan of $7000 to pay for a holiday. He will repay the loan over 4 years with monthly repayments of $165.36 .

a What annual interest rate, compounded monthly, is being charged?

b Calculate the total interest Simon will pay in the first year.

10 Consider a loan of $25 000 at 8.5% p.a. interest compounded monthly.

- **a** Calculate the monthly loan repayments if the loan is taken out for:
  - **i** 3 years **ii** 5 years **iii** 7 years.
- **b** Which loan charges the least interest in total? Explain your answer.

11 Ally takes out an €18 000 car loan over 5 years at 10.5% p.a. interest compounded monthly.

- **a** Calculate the monthly repayments.
- **b** Calculate the total interest Ally will pay on the loan.
- **c** Find the outstanding loan balance after $2\frac{1}{2}$ years of repayments.
- **d** Explain why Ally still has more than half the loan to pay off when half the loan period has passed.

12 Shane and Julie have taken out a $250 000 loan to purchase an apartment. They will repay the loan over 20 years at 6.25% p.a. fixed interest compounded monthly.

- **a** Calculate the monthly repayments.
- **b** Calculate the total interest charged.
- **c** Find the outstanding balance on the loan after 10 years.
- **d** After 10 years, the couple can afford to increase their monthly repayments. They aim to completely repay the loan over the next 5 years.
  - **i** Calculate their monthly repayments for the next 5 years.
  - **ii** Calculate the total interest that would be charged over the 15 years.
  - **iii** How much interest will the couple save by repaying their loan sooner?

13 A young couple borrows $435 000 to buy their home at a starting rate of 6.25% p.a. interest compounded monthly over 30 years.

- **a** Calculate the monthly repayments for this starting interest rate.
- **b** Find the outstanding debt after 3 years of repayments.
- **c** After 3 years the interest rate drops to 5.5% p.a.
  - **i** Calculate the new monthly repayments.
  - **ii** If the couple is able to keep paying the original repayments, how much earlier will the loan be paid off? State any assumptions you make.

ACTIVITY 1

Interest rates have changed a great deal over time. They have a big impact on what homes a family can afford.

MONTHLY REPAYMENTS

PRINTABLE TABLE

What to do:

1 Suppose you have saved $50 000 as deposit for a new house. The house is valued at $450 000 (including all fees), so you will need a loan for $400 000.

Copy and complete the tables on the next page with the monthly repayments needed, and the total interest paid, for different interest rates and loan durations. You will need to click on the "Monthly repayments" icon and use the table of monthly repayments given. Consult the websites of some banks to find the current interest rates.

Monthly repayments

	4%	6%	8%	10%	12%	*Current interest rates*
5 years						
10 years						
15 years						
20 years						

Total interest paid

	4%	6%	8%	10%	12%	*Current interest rates*
5 years						
10 years						
15 years						
20 years						

2 Discuss the impact of higher interest rates on the total interest that needs to be paid over the course of a loan. Include a discussion of the benefit of paying off a loan over a shorter time period, and what effect interest rates have on a family's ability to do this.

ACTIVITY 2 THE LOAN REPAYMENTS FORMULA

In the previous Exercise we used technology and a table of values to find the necessary repayments for a loan. In this Activity we explore the calculations leading to these values. We will use our knowledge of geometric sequences and series.

Consider a loan for which PV is the amount originally borrowed
n is the number of repayment periods
i is the interest rate *per period*, and
p is the repayment per period.

After each time period, interest is charged on the loan by multiplying the balance by $(1+i)$. The repayment p is then subtracted from the balance.

So, the balance after:

- 0 repayment periods $= PV$
- 1 repayment period $= PV(1+i) - p$
- 2 repayment periods $= [PV(1+i)-p](1+i) - p = PV(1+i)^2 - p(1+i) - p$

What to do:

1 Show that the balance of the loan after:

a 3 repayment periods $= PV(1+i)^3 - p(1+i)^2 - p(1+i) - p$

b 4 repayment periods $= PV(1+i)^4 - p(1+i)^3 - p(1+i)^2 - p(1+i) - p$

2 Following the pattern in the formulae, show that the balance of the loan after n repayment periods will be

$$\underbrace{PV(1+i)^n}_{\text{compound interest formula for balance if no repayments were made at all.}} - \underbrace{\sum_{k=0}^{n-1} p(1+i)^k}_{\text{reduction due to regular repayments}} = PV(1+i)^n - p\,\frac{(1+i)^n - 1}{i}.$$

3 After n repayments, the loan must be completely repaid. Hence show that the regular repayment must be $p = \dfrac{PV \times i \times (1+i)^n}{(1+i)^n - 1}$.

4 By setting $PV = 16\,500$, $i = \dfrac{0.055}{12}$, and $n = 4 \times 12 = 48$, verify the result in **Example 1**.

5 Use the formula $p = \dfrac{PV \times i \times (1+i)^n}{(1+i)^n - 1}$ to check the repayments you calculated in **Exercise 4A**.

6 Jade takes a loan of $20 000 to be repaid over 5 years at 5% p.a. interest compounded monthly. Use the formula to find her monthly repayments. Check your answer using technology.

7 Apply the formula in a spreadsheet to generate the table of values for loan repayments shown on page **79**.

8 Are there any other advantages to knowing a formula to find an answer, rather than relying on a calculator's built-in solver?

DISCUSSION

Do you think it is better to *borrow* money to purchase an item you want, or to *save* your money until you can afford to purchase the item?

In your discussion, consider:

- the costs associated with borrowing
- the benefits associated with having the item sooner rather than later
- the effect of inflation on the cost of the item.

B ANNUITIES

An **annuity** is an investment where an individual makes a lump-sum deposit, and then makes regular withdrawals over a fixed time period. The investment earns interest according to the balance of the annuity each time period.

In effect, an annuity is the reverse of a personal loan. The person loans the bank their money, and the bank makes regular repayments over time. As the balance in the annuity fund decreases, so does the interest it earns. However, the regular repayments remain the same.

Annuities are most commonly used as a source of regular income for people who have retired from work. Money saved during their working life is "rolled over" into an annuity fund. The regular withdrawals provide money for the person to live on.

DISCUSSION

In some countries, all workers receive **superannuation** payments as part of their salaries. They do not have access to this money until they reach retirement age, so it is a form of compulsory saving.

Why might a government make superannuation compulsory?

When performing calculations with annuities:

- The person is depositing a lump-sum with the bank, so PV is negative.
- The person receives regular repayments, so PMT is positive.

Example 3 **Self Tutor**

Heather has just retired at age 65. She has $900 000 in her savings fund. She "rolls over" the money into an annuity fund which returns 4% p.a. compounded monthly.

a If Heather withdraws $5400 per month to live on, how long will it take for the money in the fund to run out?

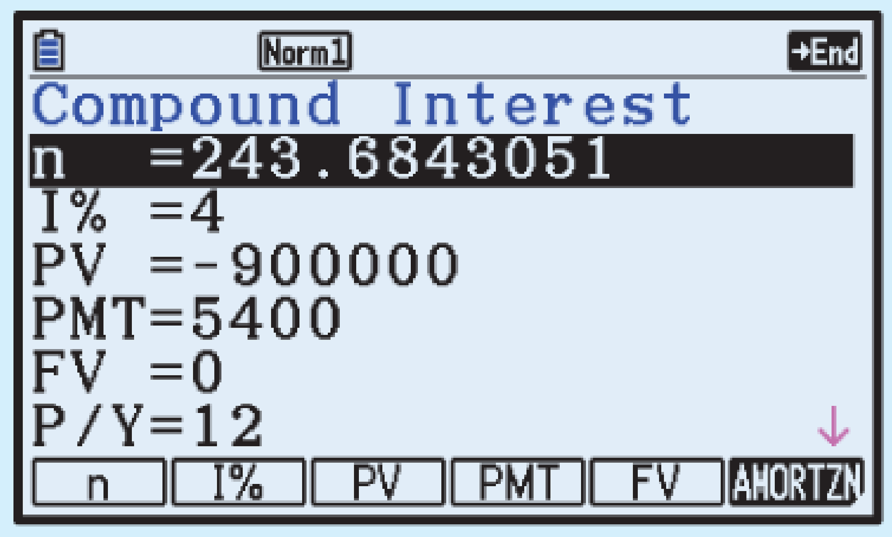
b If Heather wants the money to last until she is 90 years old, how much can she afford to withdraw each month?

a We need to find how long it will take for the future value to fall to $0.

$I\% = 4$, $PV = -900\,000$, $PMT = 5400$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$\therefore \quad N \approx 243.7$

It will take 244 months (or 20 years 4 months) for the money in the fund to run out.

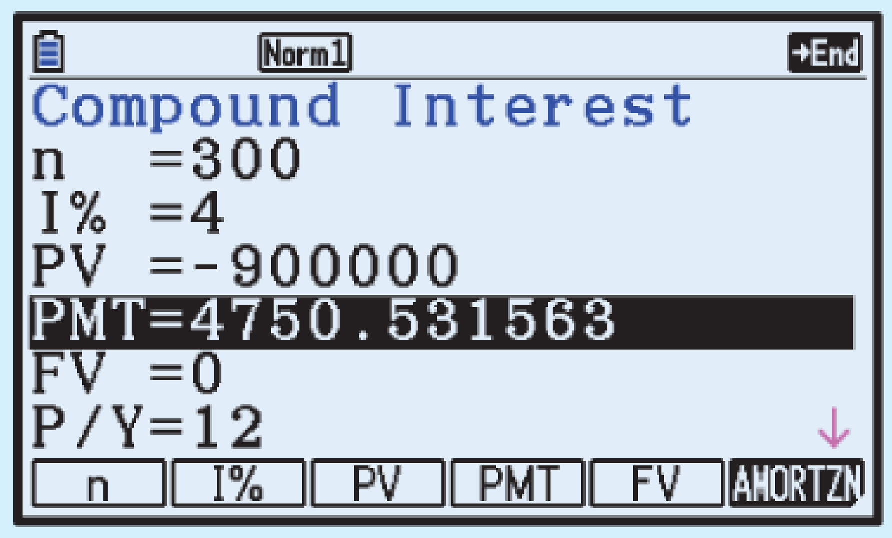


b Heather wants the money to last $90 - 65 = 25$ years.

$N = 25 \times 12 = 300$, $I\% = 4$, $PV = -900\,000$, $FV = 0$, $P/Y = 12$, $C/Y = 12$

$\therefore \quad PMT \approx 4750.53$

Heather can afford to withdraw $4750.53 each month.



EXERCISE 4B

1 Sue has achieved her target of $700 000 in her savings fund for her retirement at age 55. She rolls the money into an annuity account earning 4.85% p.a. compounded monthly.

- **a** How long will Sue's money last if she withdraws $4000 per month?
- **b** If Sue wants her money to last for 30 years, how much can she afford to withdraw per month?

2 Terrence has $800 000 in his savings fund. He rolls his money into an annuity fund which earns 4.7% p.a. compounded monthly. He wants to withdraw $7000 each month to live on.

- **a** How long will his money last?
- **b** How much *longer* would his money last if he only withdrew $6000 each month?

3 Henry has retired at age 68 with £830 000 in his savings fund. He rolls the money into an annuity fund that earns 4% p.a. compounded monthly.

- **a** If Henry wants the money to last until he is 85, how much can he withdraw each month?
- **b** Henry currently lives on £6000 per month. Will Henry be able to maintain his current standard of living? Explain your answer.

4 Tamsyn invests $750 000 in a savings account which pays 3.8% p.a. interest compounded quarterly.

- **a** How much money will be in the account when she retires in 10 years' time?
- **b** When she retires, Tamsyn rolls the money into an annuity fund which pays 4.9% p.a. interest compounded monthly. Tamsyn would like the money to last for 15 years.
  - **i** How much can Tamsyn withdraw each month?
  - **ii** How long will it take for the balance of the fund to fall to $300 000?

5 After retiring at age 55, Danny rolls his £600 000 into an annuity account earning 5.9% p.a. compounded quarterly.

- **a** If he wants his money to last for 25 years, how much can he withdraw each quarter?
- **b** How much money will be left in Danny's annuity account when he is 68?
- **c** How much *more* could Danny withdraw each quarter if his money only needed to last 20 years?

6 When Maggie retires at 70, she will deposit her savings in an annuity account which pays 6.2% p.a. interest compounded monthly. She wants to withdraw $4500 per month from the annuity account until she is 90.

- **a** Show that Maggie will withdraw a total of $1 080 000 from her annuity account.
- **b** Explain why Maggie does not need $1 080 000 in savings at the time when she retires.
- **c** Calculate the amount Maggie will need in savings when she retires.

7 Abby has carefully saved $1 000 000 during her career. She rolls it into an annuity fund which pays 5% p.a. interest compounded monthly. If Abby withdraws $4000 each month, how long will Abby's money last?

Hint: Find the balance of the fund after 1 year.

8 Igor has saved €550 000. He wants to put the money in an annuity fund where he can withdraw €5000 per month for the next 15 years.

- **a** What interest rate, compounded monthly, will Igor require?
- **b** The best interest rate Igor can find is 4.9% p.a. compounded monthly.
  - **i** If he still wants to withdraw €5000 per month, how much *less* time will the money last?
  - **ii** If he still wants the money to last 15 years, how much *less* must his withdrawals be each month?

9 Luke rolls his $700 000 of savings into an annuity fund which earns 4.5% p.a. interest compounded monthly. He wants the money to last for 16 years.

- **a** How much money can Luke withdraw each month?
- **b** Find the outstanding balance of the fund after 10 years.
- **c** After 10 years, Luke receives an inheritance of $100 000 which he adds to the annuity fund. How much is Luke now able to withdraw each month for the remaining 6 years?

10 Célia deposits €500 000 in an annuity fund which earns 4.5% p.a. interest compounded monthly. Célia wants the money to last for 20 years.

- **a** How much can Célia afford to withdraw each month?
- **b** Find the outstanding balance of her fund after 5 years.
- **c** After 5 years, Célia transfers the balance of her fund to an account which earns 5.5% p.a. interest compounded monthly.
  - **i** How much can Célia afford to withdraw each month for the remaining 15 years?
  - **ii** How long will it take for the balance of the fund to fall to €200 000?

ACTIVITY 3 PERPETUITIES

A **perpetuity** is a special type of annuity in which the regular payments continue indefinitely. This is achieved by ensuring the interest earned by the investment matches the payments taken out.

The **price** we would expect to pay for a perpetuity is therefore the principal required to give interest equal to the regular repayment.

So, the price or **present value** of a perpetuity is given by

$PV = \frac{PMT}{r}$ where PMT is the payment received each year and
r is the annual interest rate.

For example, for a perpetuity which pays \$2000 per year indefinitely, with an interest rate of 4% p.a., you would expect to pay $PV = \frac{PMT}{r} = \frac{\$2000}{0.04} = \$50\,000$.

What to do:

1 Find the price you would expect to pay for a perpetuity of:

a \$300 per year with an interest rate of 5% p.a.

b \$4000 per year with an interest rate of 3.2% p.a.

c \$18 000 per year with an interest rate of 4.5% p.a.

2 Find the annual payment you would expect to receive from a perpetuity which has:

a present value €1000 and interest rate 3.5% p.a.

b present value €25 000 and interest rate 4.8% p.a.

3 **a** Suppose you had \$500 000 in savings, and used it to purchase a perpetuity with interest rate 4% p.a. What payment would you expect to receive each year?

b Suppose \$500 000 is deposited in a pension fund which pays 4% p.a. interest compounded annually. How much can be withdrawn from the fund each year so that the money lasts for:

i 10 years **ii** 20 years **iii** 50 years **iv** 200 years **v** 1000 years?

c Comment on your answers to **a** and **b**.

ACTIVITY 4 GROWING ANNUITIES

In the previous Exercise, we have assumed that the amount withdrawn each time period does not change over time.

This is unrealistic for several reasons:

- The prices of goods and services increase over time with inflation, so the cost of living at the same standard increases.
- As we age, we inevitably need more health care and domestic services, so these expenses increase.

In a **growing annuity**, the amount withdrawn increases by a certain percentage, called the **growth rate**, each year.

What to do:

1 **a** Click on the icon to open a spreadsheet for a growing annuity fund. \$300 000 is deposited into the fund which earns 6% p.a. interest compounded monthly for 20 years. We assume a growth rate of 2% p.a.
The initial withdrawal is calculated as \$1819.92.

	A	B	C	D	E
1	Growth Annuities				
2					
3	Amount		\$300,000		
4	Number of years		20		
5	Rate p.a.		0.06		
6	Periods p.a.		12		
7	Rate per period		0.005		
8	Growth rate p.a.		0.02		
9	Growth rate per period		0.00167		
10					
11	Initial withdrawal		\$1,819.92		
12					
13	Period	Amount	Interest	Withdrawal	Balance
14	1	\$300,000.00	\$1,500.00	\$1,819.92	\$299,680.08
15	2	\$299,680.08	\$1,498.40	\$1,822.95	\$299,355.53
16	3	\$299,355.53	\$1,496.78	\$1,825.99	\$299,026.32
17	4	\$299,026.32	\$1,495.13	\$1,829.03	\$298,692.42
18	5	\$298,692.42	\$1,493.46	\$1,832.08	\$298,353.80

b Discuss what is happening to the withdrawals over time.

c Find the amount withdrawn for the final time period.

d Use your calculator to find the regular withdrawal you would make for this investment. Compare this value with the initial and final withdrawals in the growing annuity.

e How could you calculate the regular withdrawal found in **d** by adjusting the spreadsheet?

2 Suppose \$500 000 is deposited into a growing annuity fund which earns 4% p.a. interest compounded monthly for 25 years. Assume a growth rate of 3% p.a.

a Use the spreadsheet to find:

i the initial monthly withdrawal **ii** the total amount withdrawn over 25 years.

b Hence find the total interest earned by the annuity fund. Check your answer by finding the sum of the interest payments in the spreadsheet.

3 Predict the effect on the initial withdrawal of increasing:

a the amount originally deposited **b** the duration of the annuity

c the interest rate **d** the growth rate.

Use the spreadsheet to check your answers.

4 For a growing annuity over n periods with initial deposit PV, interest rate i per period, and growth rate g per period, the initial withdrawal w is given by $w = \dfrac{PV \times (i-g) \times (1+i)^n}{(1+i)^n - (1+g)^n}$.

a Verify that this formula gives the correct initial withdrawals for the scenarios in **1** and **2**.

b What happens to this formula when $g = 0$? Compare your answer with the formula found in **Activity 2** on page **83**.

REVIEW SET 4A

1 Alberto takes out a personal loan for \$23 000 at 7% p.a. over 5 years. Calculate:

a his monthly repayments **b** the total of the repayments

c the total interest charged.

2 Alexandra takes out a loan of €2000 to pay for an emergency vet bill. She will repay the loan over 6 months at 8.12% p.a. interest compounded fortnightly. Calculate:

a Alexandra's fortnightly repayments

b the outstanding balance on the loan after 6 fortnights.

3 Simone borrows $410 000 to purchase a house. She agrees to repay the loan over 25 years at 6.95% p.a. interest compounded monthly.

a Find Simone's minimum monthly repayments.

b Show that the total interest Simone will pay is greater than the amount she originally borrowed.

4 Yasmin retires at age 60 with $500 000 in her savings fund. She rolls the money into an annuity fund earning 5.25% p.a. interest compounded monthly.

a How long will Yasmin's money last if she withdraws $6000 per month?

b If Yasmin wants her money to last for 25 years, how much can she afford to withdraw per month?

5 When Scott retires at 68, he will deposit his savings in an annuity fund which pays 5.4% p.a. interest compounded monthly. He wants to be able to withdraw $6000 per month from the fund until he is 85.

a Calculate the amount Scott will need to have in savings when he retires.

b How much money will be left in Scott's account when he is 80?

6 After retiring at age 65, Vasili rolls €1 400 000 of savings into an annuity account earning 5.4% p.a. interest compounded monthly. He wants his money to last for 30 more years.

a How much can Vasili withdraw per month?

b How long will it take for the balance of the fund to fall below €1 000 000?

c How much of Vasili's annuity will be left after 20 years?

d After 20 years, Vasili must withdraw €20 000 from the account to pay for an operation.

i If Vasili continues to make the same monthly withdrawals he made for the first 20 years, how long will his money last?

ii How much is Vasili now able to withdraw each month if his money is to last for the remaining 10 years?

REVIEW SET 4B

1 Nicola and Hamish take out a personal loan of $12 000 to pay for their wedding. The loan is to be repaid over 4 years at 5.5% p.a. Calculate:

a their monthly repayments

b the total interest charged on the loan.

2 A loan of 500 000 pesos is taken out at 6% p.a. interest compounded monthly.

a Calculate the monthly loan repayments if the loan is taken out for:

i 4 years

ii 6 years.

b Which loan charges the least total interest? Explain your answer.

3 Peter has saved \$4500 towards buying a car. The car he wants to buy is valued at \$22 000, so he will borrow the remaining money. He is able to get a loan for 4 years at 6.9% p.a. interest, compounded quarterly.

- **a** How much will Peter borrow?
- **b** Calculate Peter's quarterly repayments.
- **c** Find the outstanding balance after 2 years.
- **d** After 2 years the interest rate for the loan increases to 7.5% p.a. compounded quarterly.
  - **i** Calculate the new quarterly repayments for the remaining 2 years.
  - **ii** How much *additional* interest must Peter pay as a result of the interest rate rise?

4 Answer the **Opening Problem** on page **78**.

5 Pia retires at age 62 with €350 000 in her savings fund. She rolls this money into an annuity fund earning 5.8% p.a. interest compounded monthly.

- **a** How much will she be able to withdraw each month if her money is to last another 2 decades?
- **b** How much *more* will she be able to withdraw each month if her money was to only last 15 years?

6 Harold rolls his £800 000 of savings into an annuity account. He wants the money to last for 15 years.

- **a** Given that Harold can withdraw £6284.75 each month, find the annual interest rate, compounded monthly, of the account.
- **b** How much longer would Harold's money last if he withdrew only £5000 each month?

4 **a** \$590, $\approx 22.8\%$ **b** 0.109 cm, $\approx 0.417\%$
c 386 people, $\approx 8.80\%$
5 $\approx 0.0811\%$
6 **a** ≈ 6.16 m^2 **b** ≈ 7.07 m^2 **c** ≈ 0.911 m^2, $\approx 14.8\%$

REVIEW SET 3B

1 **a** $14.85 \text{ s} < t < 14.95 \text{ s}$ **b** $6.66 \text{ m s}^{-1} < s < 6.77 \text{ m s}^{-1}$
2 $267.5 \text{ cm}^2 < A < 355.5 \text{ cm}^2$ **3** $57.9 \text{ km} < D < 61.9 \text{ km}$
4 **a** ≈ 175 cm **b** ≈ 2.87 cm, $\approx 1.61\%$
5 **a** **i** 2 m **ii** 2.24 m **iii** 2.236 m
b 2.24 m and 2.236 m
6 **a** $\approx 4.51\%$ **b** $\approx 1.32\%$ **c** $\approx 0.0507\%$
d $\approx 0.0402\%$ **e** $\approx 8.49 \times 10^{-6}\,\%$
7 **a** ≈ 38.5 cm^2 **b** $37.4 \text{ cm}^2 < A < 39.6 \text{ cm}^2$
c $\approx 2.92\%$

EXERCISE 4A

1 **a** \$232 **b** \$13 920 **c** \$1920
2 **a** £282.60 **b** £10 173.60 **c** £673.60
3 **a** \$1036.80 **b** \$2610 **4** €1903.28
5 **b** \$27 509.01
c **i** \$200 **ii** \$186.21
The balance of the loan is less in month 6 which means the interest paid will also be less.
d **i** \$4.02 **ii** \$6497.40 (\$6498 using technology)
e The monthly repayment was rounded up, so every month the payments have reduced the balance by a little extra.
6 **a** \$148.64 **b** \$729.28
7 **a** £490.61 **b** £15 598.73
8 **a** \$8500 **b** **i** \$518.58 **ii** \$1871.60 **iii** \$5492.57
9 **a** 6.30% p.a. **b** \$395.65
10 **a** **i** \$789.19 **ii** \$512.92 **iii** \$395.92
b The 3 year loan charges the least interest of \$3410.84 as more is paid off each month and therefore less interest is charged.
11 **a** €386.90 **b** €5214 **c** €10 169.13
d Ally pays more interest in the first $2\frac{1}{2}$ years than in the second $2\frac{1}{2}$ years.
12 **a** \$1827.33 **b** \$188 559.20 **c** \$162 745.03
d **i** \$3165.28 **ii** \$159 196.40 **iii** \$29 362.80
13 **a** \$2678.37 **b** \$418 703.37
c **i** \$2483.49
ii 4 years earlier; we assume the new rate is fixed for the remainder of the loan period.

EXERCISE 4B

1 **a** 25 years 4 months **b** \$3693.84
2 **a** 12 years 7 months **b** 3 years 1 month longer
3 **a** £5614.06
b No, he can only afford to spend £5614.06 per month. Otherwise his money will run out before he turns 84.
4 **a** \$1 094 748.09
b **i** \$8600.27 **ii** 11 years 11 months
5 **a** £11 512.29 **b** £394 007.62 **c** £1312.64
6 **a** $\$4500 \times 12 \times 20 = \$1\,080\,000$
b Maggie will earn interest on the money in the annuity account as she makes her regular withdrawals.
c \$618 117.53
7 The money will last forever.
8 **a** 7.19% **b** **i** 2 years 10 months **ii** €679.24
9 **a** \$5121.03 **b** \$322 605.07 **c** \$6708.44
10 **a** €3163.24 **b** €413 500.41
c **i** €3378.64 **ii** 9 years 3 months

REVIEW SET 4A

1 **a** \$455.43 **b** \$27 325.80 **c** \$4325.80
2 **a** €157.24 **b** €1086.93
3 **a** \$2884.74
b Total repayments $= \$2884.74 \times 12 \times 25 = \$865\,422$
Total interest charged $= \$865\,422 - \$410\,000 = \$455\,422$
4 **a** 8 years 7 months **b** \$2996.23
5 **a** \$799 813.28 **b** \$314 877.35
6 **a** €7861.43 **b** 14 years 3 months **c** €727 698.90
d **i** 9 years 7 months **ii** €7645.37

REVIEW SET 4B

1 **a** \$279.08 **b** \$1395.84
2 **a** **i** 11 742.52 pesos **ii** 8286.45 pesos
b The 4 year loan charges the least interest of 63 640.96 pesos as more is paid off each month and therefore less interest is charged.
3 **a** \$17 500 **b** \$1260.97 **c** \$9347.67
d **i** \$1269.19 **ii** \$65.76
4 **a** An annuity fund is an investment where an individual makes a lump-sum deposit, and then makes regular *withdrawals* from the account. We have previously considered compound interest investments that make regular *deposits* into an account.
b Diane is technically correct, but she will be able to withdraw more than £2000 per month since the money in the fund will earn interest.
c £3167.02
5 **a** €2467.29 **b** €448.52
6 **a** 4.90% p.a. **b** 6 years 7 months

EXERCISE 5A

1 **a** We have assumed that the cyclist travelled at a constant speed of 30 km h^{-1} the entire time. This is not realistic, the cyclist will travel at different speeds uphill, downhill, and on flat ground.
b 60 km
2 **a** Briony constructed her model by finding the equation of the line through (0, 8) and (12, 23). Briony has assumed that the laptop will charge at a constant rate, and indefinitely. These assumptions are not realistic, but they may be satisfactory for this problem.
b $0 \leqslant C \leqslant 100$, $t \geqslant 0$
Briony's model suggests that it is possible to have charge greater than 100%, which is not possible.
c 73.6 minutes
d **i** The laptop likely charges at a faster rate earlier on, then at a slower rate as it approaches a full charge.
ii Yes, 73.6 minutes was a useful estimate.
3 **a** $t = \frac{3}{20}d$ seconds **b** 75 seconds
c Rick will take a longer time to run 500 m than our prediction. He will not be able to run 500 m at the same pace that he runs 100 m.
4 **a** **C**, it is usually coldest at dawn, and warmest in the afternoon. Daily temperature should be roughly periodic.
b $\approx 28°$C