Practice exam papers

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set A

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
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- Answer **all** questions. Answers must be written within the answer boxes provided.
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- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
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1 *[Maximum mark: 5]*

a The radius of the Earth is found to be 6.4×10^6 m correct to two significant figures. Find the upper bound on the possible value of the radius. *[1]*

b Hence find the upper bound on the surface area of the Earth, modelling it as a perfect sphere. *[2]*

c A textbook states that the area of the Earth is 5.10×10^{14} m^2. Find the percentage error if the upper bound found in part **b** had been used as an estimate. *[2]*

2 *[Maximum mark: 6]*

A right pyramid has a square base with sides of length 4 cm. It has a height of 6 cm.

a Find the volume of the pyramid. *[2]*

b Find the acute angle between the sloping edge of the pyramid and the base. *[4]*

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3 *[Maximum mark: 6]*

A biologist investigated the characteristics of a group of fruit flies. Her results are shown below:

	Red eyes	**Black eyes**
Wings	54	156
Wingless	12	34

Use the results to estimate

a the probability of having red eyes and wings *[2]*

b the probability of not having red eyes *[2]*

c the probability of having red eyes if the fly is wingless. *[2]*

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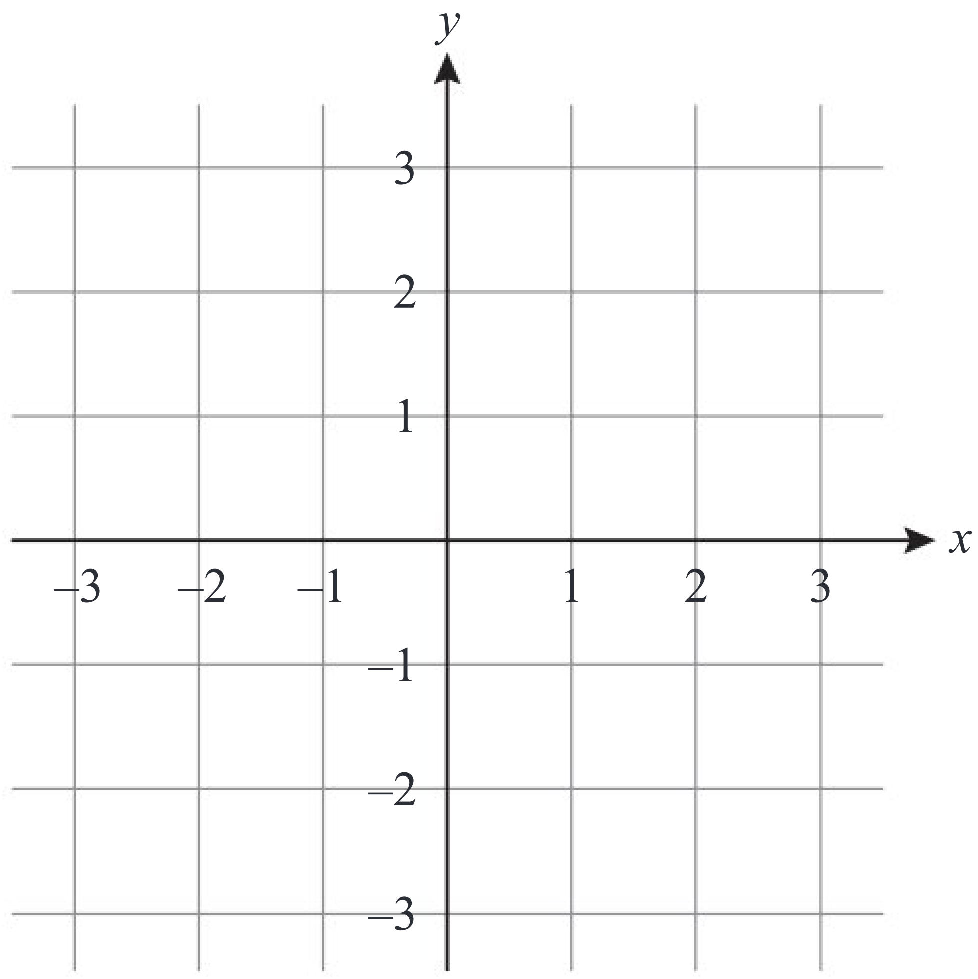
4 *[Maximum mark: 5]*

a Evaluate $\ln 3$. *[1]*

b On the axes below, sketch the graph of $y = \ln x$ *[2]*



c **i** The graph $y = \ln x$ is reflected in the line $y = x$. Sketch the new graph on the axes above.

ii Write down the equation of the new graph. *[2]*

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5 *[Maximum mark: 6]*

A geographer is investigating the link between the population of a city (P million) and the number of doctors found there (D hundred). She finds the following data:

P	1.2	1.9	2.1	2.5	2.5	3.4	5.6	6.8
D	3.1	3.4	3.3	4.8	4.5	4.1	7.2	7.7

a Find the Spearman's rank correlation coefficient between P and D. *[3]*

b The one-tailed critical value at 5% significance is 0.643. Does the sample provide evidence of a positive association between the population and the number of doctors? *[1]*

c The number of doctors in the largest city was found to be underestimated. How would the value of the Spearman's rank correlation coefficient be affected if the true value were used? Justify your answer. *[2]*

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6 *[Maximum mark: 6]*

a Find the maximum value of the function $f(x) = 8 + 2x^2 - x^4$. *[2]*

b Find the area enclosed by the curve $y = 8 + 2x^2 - x^4$ and the x-axis. *[4]*

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7 *[Maximum mark: 6]*

A gene in cats can be expressed in three different ways, depending upon the genetic material in the cell. These are described as HH, Hh and hh. A geneticist believes that this should be found in the ratio 1:2:1. He investigates a sample of 150 cats and finds the following data:

Gene expression	HH	Hh	hh
Frequency	31	77	42

Determine, at the 5% significance level, if there is evidence that the 1:2:1 prediction is incorrect. *[6]*

8 *[Maximum mark: 8]*

The graph of y against x has a gradient at any point equal to $3x^2 + 2$. It passes through the point (0,1).

a Find the equation of the graph. *[4]*

b Find the equation of the normal to the graph passing through (0,1) in the form $ay + bx = c$ where a, b and c are integers. *[4]*

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9 *[Maximum mark: 5]*

The sum of the first n terms of an arithmetic sequence is given by $9n - 2n^2$. Find the first term and the common difference. *[5]*

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10 *[Maximum mark: 8]*

For the points $A(0, 4)$, $B(2, 8)$ and $C(1, 3)$:

a Find the line perpendicular to AB through C. *[5]*

b Hence find the point on the line through A and B that is closest to C. *[3]*

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11 *[Maximum mark: 6]*

A ship leaves a dock and travels 10 km on a bearing of 040°. It then travels 20 km on a bearing of 170°.

a Find the distance of the ship from the dock. *[3]*

b On what bearing should it travel to return to the dock? *[3]*

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12 *[Maximum mark: 6]*

The cost of a mobile phone contract depends on the number of minutes spent talking each month.

Contract A has a fixed cost of \$8 then charged at a rate of \$1 per 20 minutes.

Contract B has a cost of \$10 for the first 100 minutes, then charged at a rate of \$2 per 10 minutes.

Joanne wants to investigate the cost, \$$C$, of spending x minutes talking each month on both contracts.

a Find a model to describe C in terms of x under contract A. *[1]*

b Find a piecewise linear model to describe C in terms of x under contract B. *[2]*

c Hence find values of x for which contract B is cheaper than contract A. *[3]*

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13 *[Maximum mark: 7]*

Three mobile phone masts are located, in arbitrary units, at $A(1, 2)$, $B(3, 4)$ and $C(5, 0)$. Sanjay's phone picks up the signal from all three simultaneously. Assuming the land is flat and all three signals travel at the same speed without obstruction, find the coordinates of Sanjay's phone. *[7]*

Mathematics: applications and interpretation
Standard level
Paper 2
Practice Set A

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1 hour 30 minutes

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Answer **all** questions.

Do **not** write solutions on this page.

Answer all questions in an answer booklet. Please start each question on a new page.

1 *[Maximum mark: 13]*

The number of people (in thousands) subscribed to a website x weeks after it is launched is modelled by

$$f(x) = x^3 - 6x^2 + 9x + 4, \ x \geqslant 0$$

a Find the initial number of subscribers when the website launches. *[1]*

b Find $f'(x)$. *[2]*

c Interpret $f'(x)$ in context. *[1]*

d Find all solutions of $f'(x) = 0$. *[2]*

e Find the values of x for which $f(x)$ is increasing. *[2]*

f Sketch $y = f(x)$. *[3]*

g How long does it take the website to reach 10 000 subscribers? *[2]*

2 *[Maximum mark: 12]*

A fair four-sided dice is rolled twice. S is the sum of the scores.

a Copy and complete the probability distribution of S. *[2]*

s	2	3	4	5	6	7	8
$P(S = s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$			

b Find the expected value of S. *[2]*

c Given that the total is more than 4, find the probability that it is more than 6. *[3]*

Eric plays a game where he rolls a fair four-sided dice twice. If the score is four or less, he loses and pays \$1. If he scores 5 or more, he receives \$$k$.

d Find the value of k if the game is fair. *[5]*

3 *[Maximum mark: 14]*

Kwami keeps a record of his best 5000 m time (y minutes) for each week (x) in the 13 weeks after he starts training for a competition season. The results are shown here:

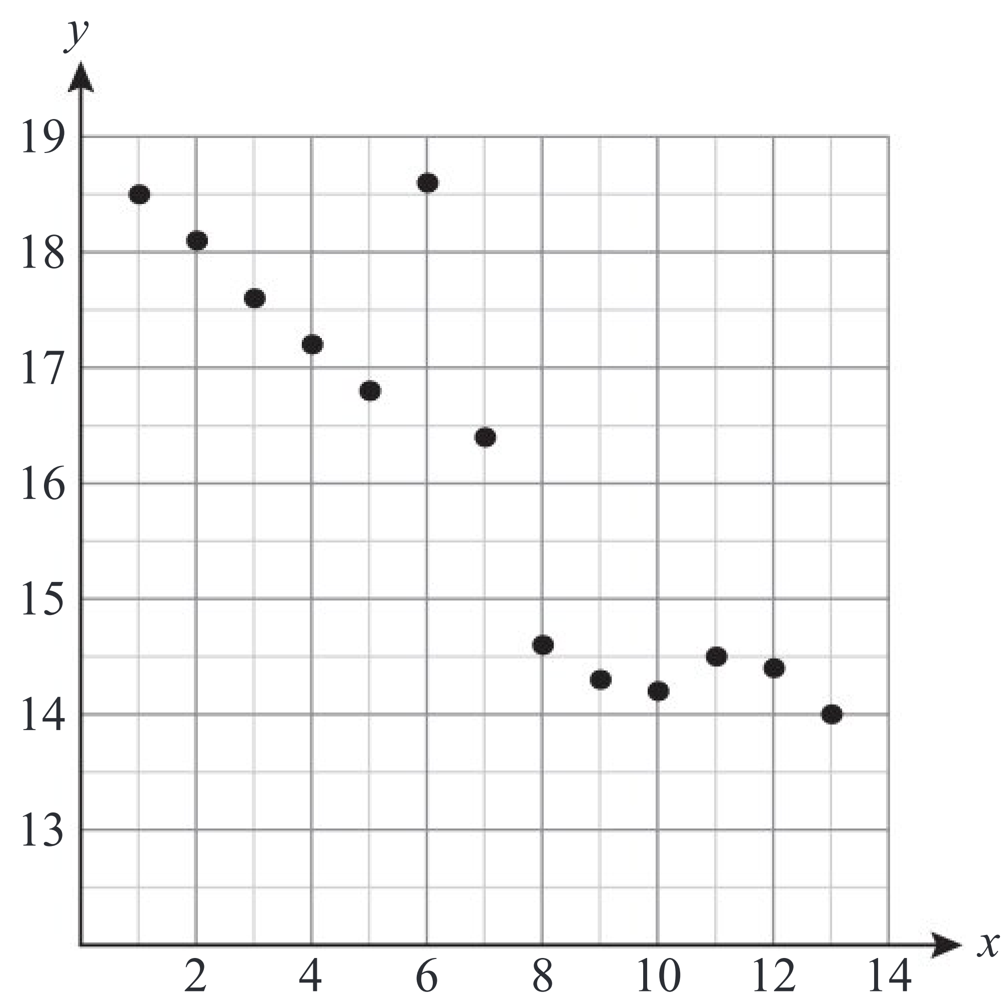
x	1	2	3	4	5	6	7	8	9	10	11	12	13
y	18.5	18.1	17.6	17.2	16.8	18.6	16.4	14.6	14.3	14.2	14.5	14.4	14

a **i** Find the mean of Kwami's best times each week.

ii Find the standard deviation in Kwami's best times each week.

iii Find Pearson's product moment correlation coefficient for these data. What type of correlation is suggested by this value? *[4]*

The results are illustrated in the following scatter diagram.



b **i** Competitions occurred every week from week n until week 12. Athletes generally have improved performance in competitions. Use the graph to suggest the value of n.

ii During one of the weeks before competitions began, Kwami was ill. Use the scatter graph to suggest which week this was. *[2]*

For the rest of this question, the result from the week where Kwami was ill should be excluded.

c **i** Create a piecewise linear model to predict y for a given x.

ii Compare and contrast, in context, the coefficients of x in each part of the linear model. *[5]*

d Use your model to predict the time Kwami would have achieved in the week he was ill if he had not been ill. *[2]*

e Explain why it would not be valid to use this model to predict Kwami's times in the following season. *[1]*

4 *[Maximum mark: 13]*

Almira is considering two different savings schemes. Both schemes involve an initial investment of \$1000 in an account.

In scheme A, at the end of each year \$50 is added to the account.

In scheme B, at the end of each year 4% compound interest is added to the account.

a How much will be in Almira's account at the end of the fifth year after investment in

i Scheme A

ii Scheme B. Give your answer correct to two decimal places. *[4]*

b What annual compound interest rate would achieve the same outcome for Almira as investing in scheme A for five complete years? *[2]*

c Almira wants to invest for n complete years. For what values of n would Almira be better off investing in scheme B? *[3]*

d Almira estimates that there is 2.5% depreciation each year. How long would Almira need to save in scheme B to use her savings to purchase something currently valued at \$1400? *[4]*

5 *[Maximum mark: 15]*

The results in an intelligence test are normally distributed with a mean of 100 and a standard deviation of 30.

a Find the probability that a randomly chosen individual will have a score above 150. *[1]*

b Only 10% of people have a score above k. Find the value of k. *[2]*

To enter a high intelligence society, people need to have a score of at least 150. Five people are chosen at random to take the test.

c Find the probability that at least two of them qualify to enter the high intelligence society. *[4]*

d Find the probability that the fifth person to take the test is the second person to attain a score of at least 150. *[3]*

People with a score of more than 170 in the test are allowed to enter a merit stream within the society.

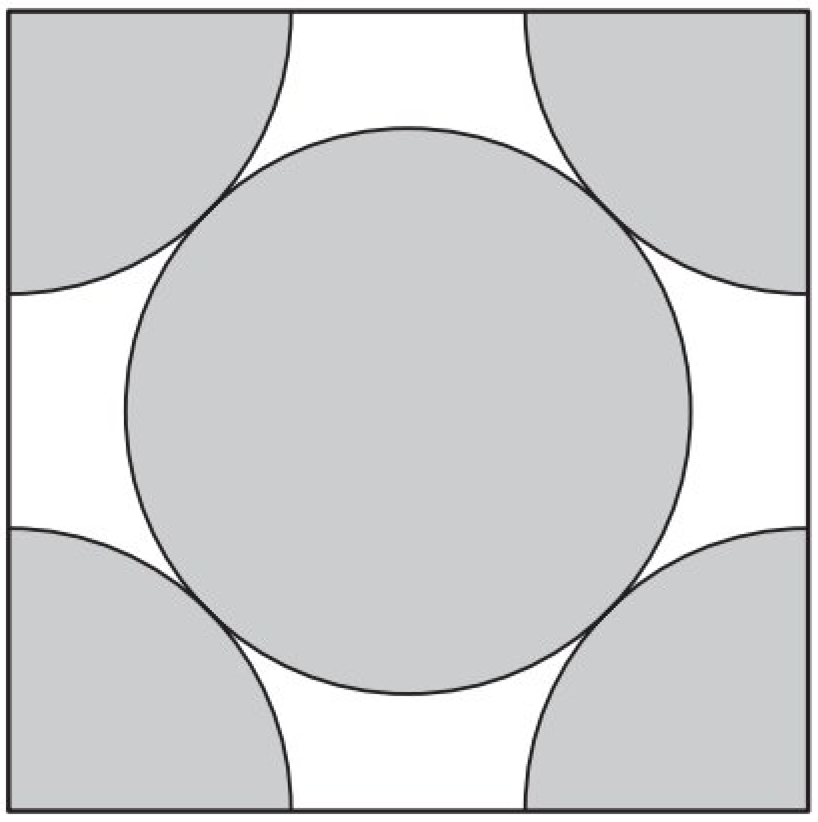
e What percentage of the society are members of the merit stream? *[4]*

f State one assumption required in your answer to part **e**. *[1]*

6 *[Maximum mark 13]*

Metal rods are modelled as perfect cylinders with radius 1 cm. They are packed into a box in two different ways.

In method 1, the repeating unit is shown below:



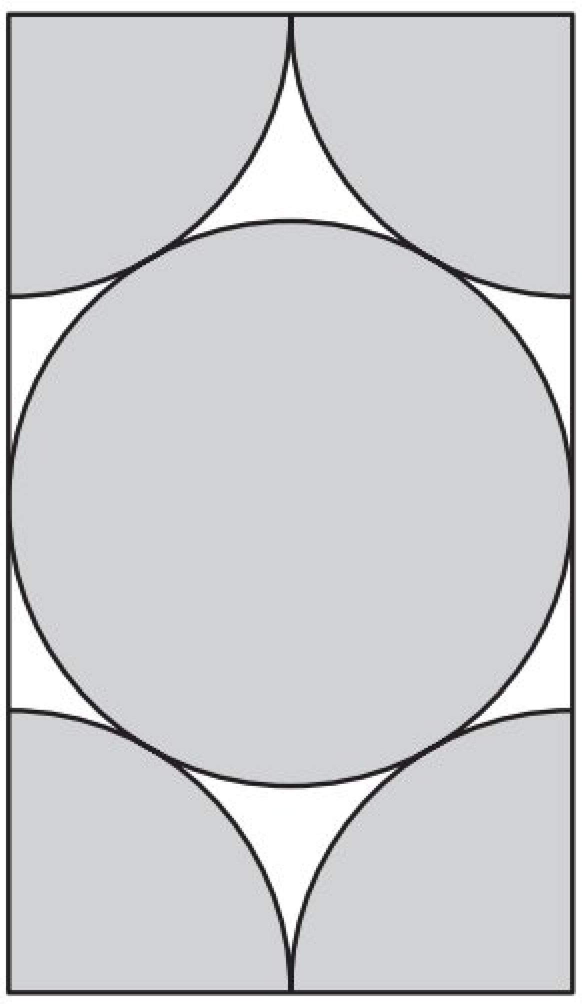
The repeating unit contains four quarter circles and one full circle.

a Explain why the diagonal of the square has length 4 cm. *[1]*

b Find the proportion of the box that is filled with metal. *[4]*

c State one assumption required in your answer to part **b**. *[1]*

In method 2, the repeating unit is shown below:



d Find the proportion of the box that contains metal in method 2. *[5]*

e Determine, with justification, whether method 1 or method 2 packs more rods into the same box. *[2]*

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set B

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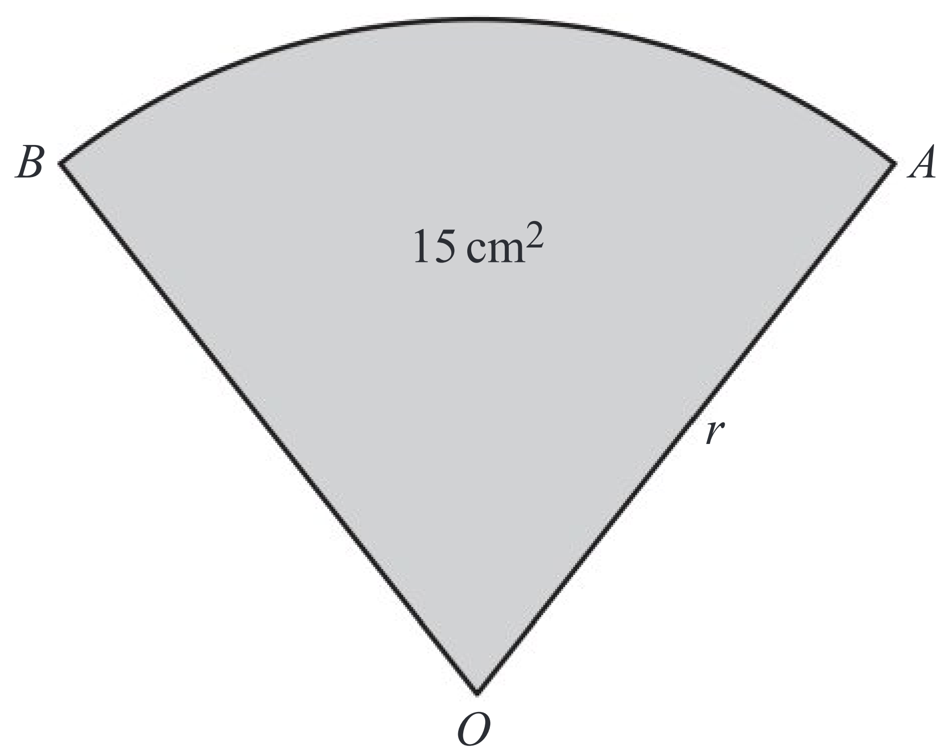
1 *[Maximum mark: 4]*



The sector *OAB* has area 15 cm^2.
The perimeter of the sector is 4 times the length of the arc *AB*.
Find the radius, *r*. *[4]*

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2 *[Maximum mark: 5]*

The scores out of 10 in a mathematics test are summarized in the table:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	1	0	1	1	2	2	3	2	1	1

a Find the median score. *[1]*

b Find the interquartile range. *[1]*

c Determine, with clear calculations, whether any of the scores are outliers. *[3]*

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3 *[Maximum mark: 6]*

Laura is training for a marathon. The number of miles she runs in each training session forms an arithmetic sequence.

In her fifth session she runs 8 miles. The total distance she has run in the first eight sessions is 58 miles.

a Find the distance she ran in her first training session and the increase in distance between consecutive sessions. *[4]*

b Hence find the number of the training session in which she first runs a full marathon distance of 26 miles. *[2]*

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4 *[Maximum mark: 6]*

The voltage, V, current, I, and resistance, R, in an electrical circuit are related by the formula $V = IR$.

Suzi measures the voltage to be 219 to three significant figures and the current to be 18.4 to three significant figures.

a Find

i the upper bound on the resistance

ii the lower bound on the resistance. *[4]*

b Hence state the value of the resistance to an appropriate degree of accuracy, justifying your choice. *[2]*

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5 *[Maximum mark: 6]*

a Find an expression for the gradient of the chord joining the point (1, 0) to the point (x, ln x^2). *[2]*

b Use technology to find the limit of the gradient of this chord as x tends towards 1. *[2]*

c Explain the significance of your answer to part **b**. *[2]*

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6 *[Maximum mark: 8]*

Sven has recently moved from Gamla Stan to Nya Stan. He suspects that the average temperature in Nya Stan is warmer than in Gamla Stan at this time of year and has gathered the following temperature readings from each town.

Gamla Stan (°C)	12.3	14.1	16.3	15.9	10.6	13.2		
Nya Stan (°C)	14.8	11.7	16.5	17.6	19.3	12.1	16.9	15.4

He uses these samples to carry out a test at the 10% level to investigate his suspicion.

a State the null and alternative hypotheses. *[2]*

b Calculate the p-value for this test. *[2]*

c State the conclusion of the test. *[2]*

d State two assumptions needed to conduct this test. *[2]*

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7 *[Maximum mark: 6]*

The fourth term of a geometric sequence is 13.5 and the sum of the first three terms is 74.

Find the first term and common ratio of the sequence. *[6]*

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8 *[Maximum mark: 6]*

Erica pays £6000 each year into a pension scheme that guarantees a 2.5% per annum interest rate. She plans to retire in 30 years.

a Find the value of her pension on retirement. *[3]*

She will use the sum saved in the scheme to buy an annuity also at a 2.5% per annum interest rate. The annuity will pay out £750 per month for life.

b How long after retiring must Erica live before she starts receiving money that was not saved in her pension? *[3]*

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9 *[Maximum mark: 8]*

A manufacturer produces x hundred items of a particular product each week and makes a profit $P(x)$ in thousands of US dollars.

He knows that the rate of change of profit with respect to the number of items produced is given by $-3x^2 + 5x + 2$.

a Find the number of items he should produce each week to maximize profit. *[3]*

He makes a profit of $2000 when producing 100 items.

b Find $P(x)$. *[5]*

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10 *[Maximum mark: 9]*

A surgery manager claims that patient waiting times for pre-booked appointments at his surgery are normally distributed with a mean of 14 minutes and a variance of 36 minutes.

A sample of the waiting times for 80 patients is taken:

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Observed frequency	3	8	23	30	16

This sample is used to conduct a χ^2 goodness of fit test to investigate the manager's claim. The test is conducted at the 5% level.

a State the null and alternative hypotheses. *[2]*

b Copy and complete the following table. *[2]*

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Expected frequency	5.34				

c Find the p-value for the test. *[3]*

d State the conclusion of the test. *[2]*

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11 *[Maximum mark: 8]*

The London Eye is an observation wheel with a diameter of 120 m that rotates once every 30 minutes. The pods that carry customers are arranged around the rim of the wheel.

A particular pod starts at the lowest point of the circle 2 m above ground level. The height, h metres, of that pod at time t minutes can be modelled by the function $h = a\cos(bt) + c$.

a Find the values of a, b and c. *[5]*

b Find the length of time for which the pod is higher than 50 m above ground level in any one revolution of the wheel. *[3]*

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12 *[Maximum mark: 8]*

The table below is the probability distribution for the score obtained in a game of chance:

x	0	1	2	3	4
$P(X=x)$	0.1	a	b	0.2	0.15

A player is charged $2 to play and they win, in dollars, the score they achieve in the game.

a Show that $a + b = 0.55$. *[2]*

b Given that the game is fair, find a and b. *[3]*

Zhuo plays the game twice.

c Find the probability that he makes a loss overall. *[3]*

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1 *[Maximum mark: 15]*
Fola is researching the possible relationship between height and weight of staff at his school.
He initially decides to select his sample by taking an alphabetic list of all staff and selecting every 10th person on the list.

a **i** Name this sampling technique.
ii Explain why this does not produce a simple random sample. *[2]*

He then decides to change his sampling technique by taking a stratified sample of men and women.
He wants a sample size of 12 and knows there are 46 men and 63 women at the school.

b **i** Find the number of men he should include in his sample.
ii State the sampling method he then needs to employ to select the particular men and women.
iii State how cluster sampling differs from stratified sampling. *[4]*

He collects the following data:

Height/cm	153	158	161	162	164	165	167	172	175	179	184	190
Weight/kg	52.4	54.6	59.7	57.1	58.5	74.2	62.8	73.1	82.3	60.2	74.3	86.6

c Find Pearson's product moment correlation coefficient and interpret this value in context. *[2]*

d Use an appropriate regression line to estimate the weight of a person with height
i 140 cm
ii 170 cm. *[3]*

e Comment on the reliability of the predictions in parts **di** and **dii**. *[2]*

f Suggest two ways Fola could improve the reliability of any predictions made from linear regression for this population. *[2]*

2 *[Maximum mark: 18]*
A pleasure boat runs trips around the local bay.
It leaves its mooring and manoeuvres onto a straight line path that keeps it equidistant from the end of the harbour walls located at the points with coordinates (1, 8) and (5, 2).

a Find the equation of its path in the form $ax + by + c = 0$. *[4]*

As the boat passes between the harbour walls, the captain observes that the angle of elevation to the top of one of the walls is 12°. The harbour master is 50 m closer to that wall and observes that the angle of elevation is 55°.

b Find the height of the harbour wall. *[5]*

Once clear of the harbour, the boat reaches a buoy at *A* and from there moves on a bearing of 310° for 20 km until it reaches point *B*.
It then moves on a bearing of 055° for 30 km to point *C*.

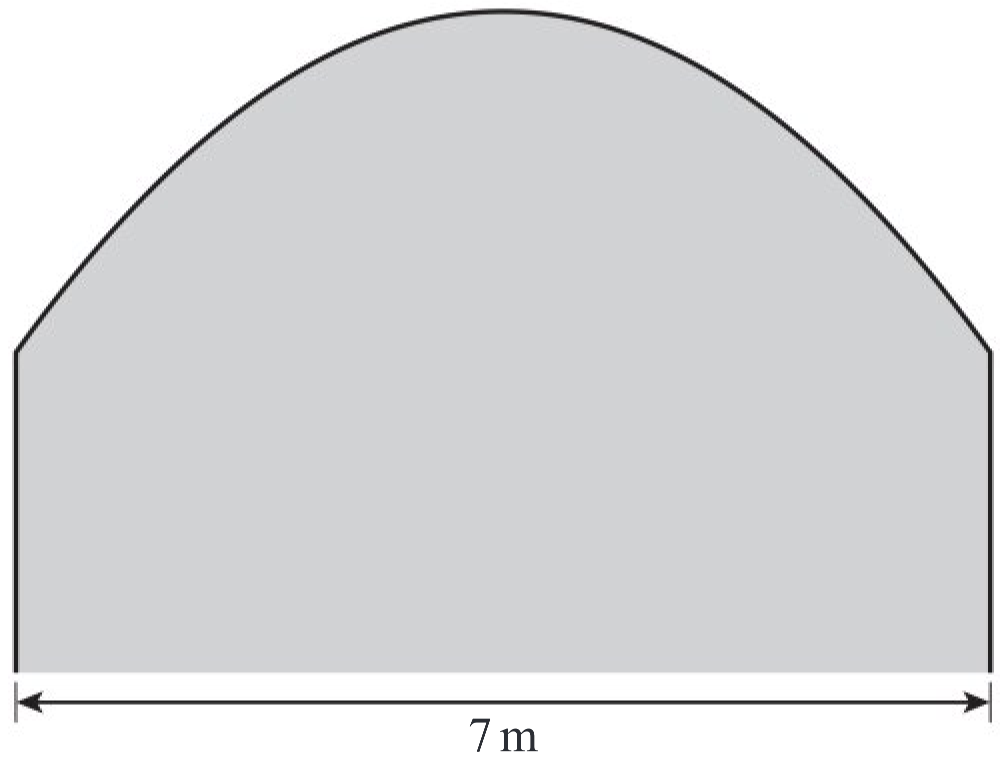
c Find the angle $A\hat{B}C$. *[2]*

d Find the shortest distance from *C* back to the buoy at *A*. *[3]*

e Find the bearing the boat must travel on to cover the shortest distance from *C* back to *A*. *[4]*

3 *[Maximum mark: 16]*

The entrance to a railway tunnel is shaped as shown below:



John measures the height, h, at various distances, x, from one side.

x/m	0	1	2	3	4	5	6	7
h/m	2.3	3.5	4.3	4.7	4.7	4.3	3.5	2.3

a Use the trapezoidal rule with 5 strips to estimate the cross-sectional area of the tunnel. *[3]*

b Explain whether your answer in part **a** is an underestimate or overestimate of the true cross-sectional area. *[2]*

In fact the curve of the entrance is a parabola, $h = ax^2 + bx + \text{c}$.

c Find a, b and c. *[4]*

d Find the maximum height of the tunnel. *[2]*

e Find the exact value of the actual cross-sectional area. *[2]*

f Find the percentage error in the estimate from part **a**. *[2]*

g How could the accuracy of the estimate in part **a** be improved? *[1]*

4 *[Maximum mark: 16]*

A telesales worker has constant probability of 0.04 of a call resulting in a sale.

a Find the probability of achieving exactly two sales in the first 10 calls made. *[2]*

b Find the probability of achieving at least two sales in the first 10 calls made. *[2]*

c **i** Find the number of calls he needs to make in a day to average two sales per day.

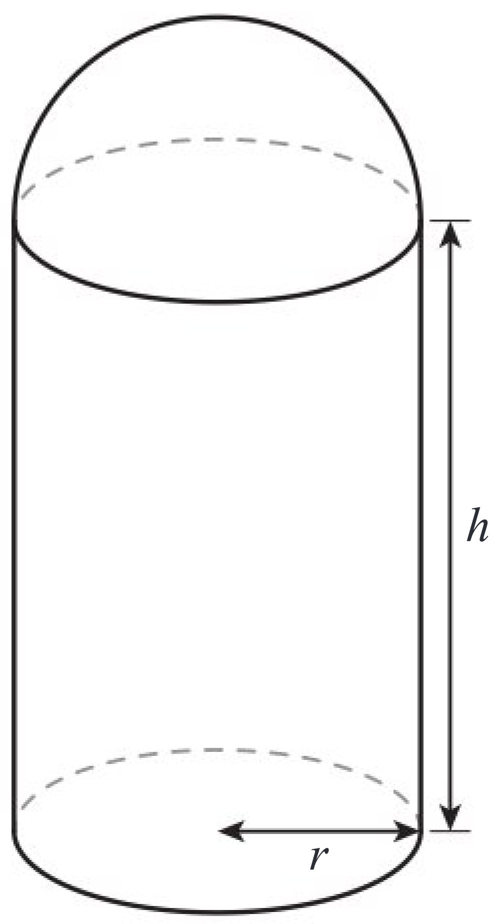
ii In this case, find the variance of the number of sales achieved. *[4]*

d In a 5-day week, find the probability that he achieves at least two sales in the first 10 calls made on more than one day. *[4]*

e Find the least number of calls he needs to make in order that the probability of making at least one sale is greater than 95%. *[4]*

5 *[Maximum mark: 15]*

A wooden salt shaker is formed from a hemisphere of radius r on top of a cylinder of height h as shown.



The volume of the salt shaker is $300\,\text{cm}^3$.

The manufacturer wants to use the least amount of wood possible in the production process.

a Show that $h = \dfrac{900 - 2\pi r^3}{3\pi r^2}$. *[4]*

b Hence find an expression for the surface area, A, of the salt shaker in the form $A = ar^b + cr^d$, where a, b, c and d are constants to be found. *[5]*

c Find

i the minimum amount of wood needed

ii the radius to achieve this minimum

iii the height to achieve this minimum. *[5]*

d State one reason why the manufacturer might not wish to use the dimensions found in parts **cii** and **ciii**. *[1]*

Mathematics: applications and interpretation
Standard level
Paper 1
Practice Set C

Candidate session number

1 hour 30 minutes

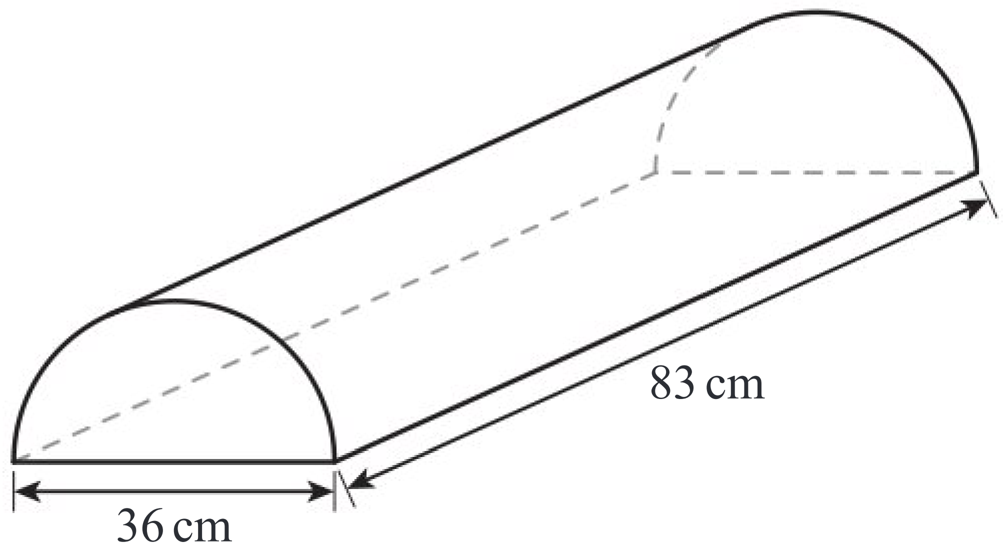
Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions. Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1 *[Maximum mark: 6]*

A metal bar is in the shape of a prism with a semicircular cross-section. The dimensions are shown in the diagram.



a Find the volume of the bar. Give your answer in cm^3, in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. *[3]*

The bar is melted down and all the metal used to make a sphere.

b Find the radius of the sphere. *[3]*

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2 *[Maximum mark: 5]*

A triangle has sides $AB = 6.8\,\text{cm}$, $BC = 4.7\,\text{cm}$ and $CA = 9.1\,\text{cm}$.

a Find, in degrees, the size of the angle $A\hat{B}C$. *[3]*

b Find the area of the triangle. *[2]*

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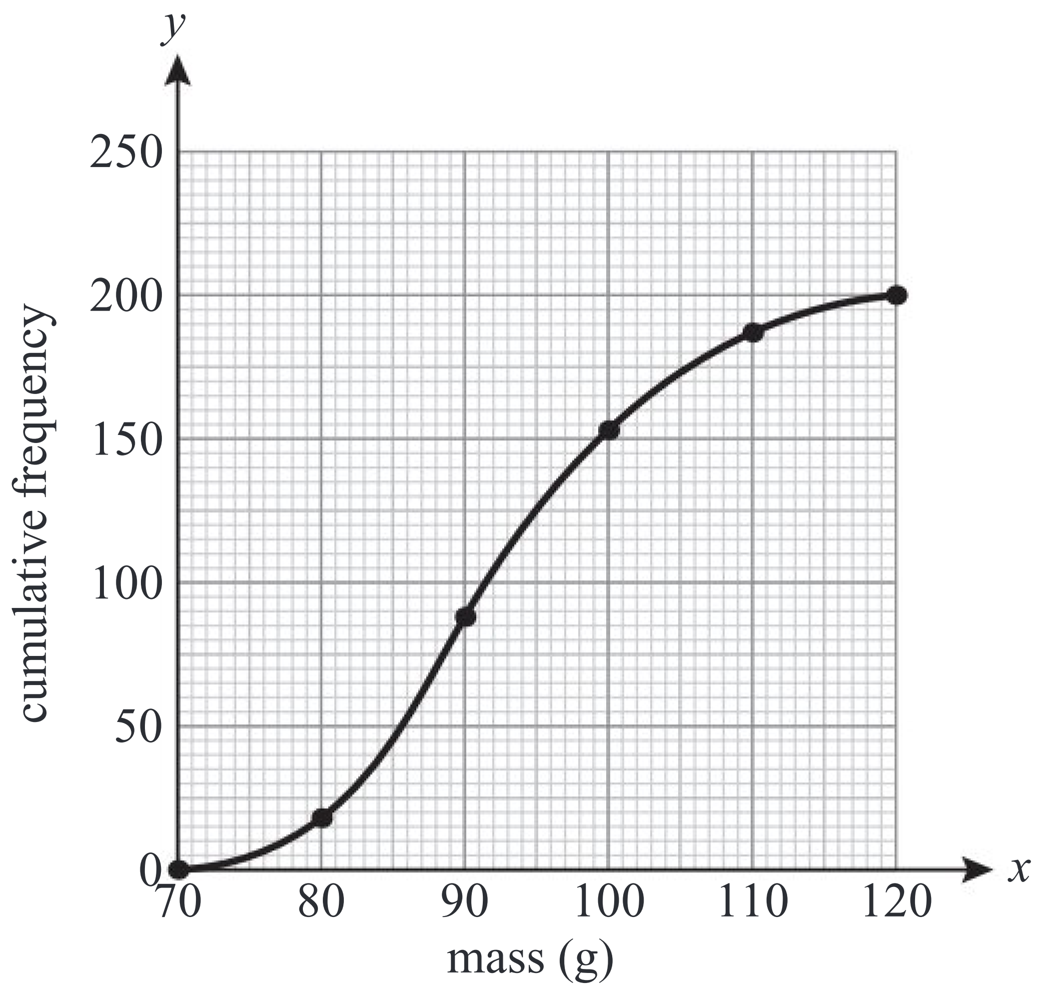
3 *[Maximum mark: 5]*

Roshni collected 200 apples from her orchard. The cumulative frequency graph below shows their mass in grams.



a Estimate how many apples weigh more than 90 g. *[2]*

b The heaviest 15% of the apples are going to be sent to the local restaurant. Estimate the least weight of an apple sent to the restaurant. *[3]*

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4 *[Maximum mark: 4]*

An arithmetic sequence has first term 7 and the sum of the first 20 terms is 640. Find

a the 20th term of the sequence *[2]*

b the 39th term of the sequence. *[2]*

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5 *[Maximum mark: 5]*

A sector of a circle has radius 10 cm. The angle at the centre of the sector is $\theta°$. The area of the sector is 75 cm^2.

a Find the value of θ. *[2]*

b Find the perimeter of the sector. *[3]*

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6 *[Maximum mark: 6]*

The heights of 30 flowers, measured in cm, are summarized in the table.

Height/cm	8–12	12–15	15–20	20–25	25–28
Frequency	5	6	8	7	4

a Estimate the mean and the standard deviation of the 30 flowers. *[3]*

The measurements are converted into inches, where 1 inch = 2.54 cm.

b Find the mean and variance of the heights in inches. *[3]*

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7 *[Maximum mark: 7]*

Notebooks are delivered to schools in boxes of different sizes. A teacher thinks that the volume, V cm^3, of a box of height x cm can be modelled by the equation

$V = ax^3 + bx^2 + cx$.

He measures heights and volumes of three boxes and obtains the following results:

x	8	10	15
V	1890	1690	703

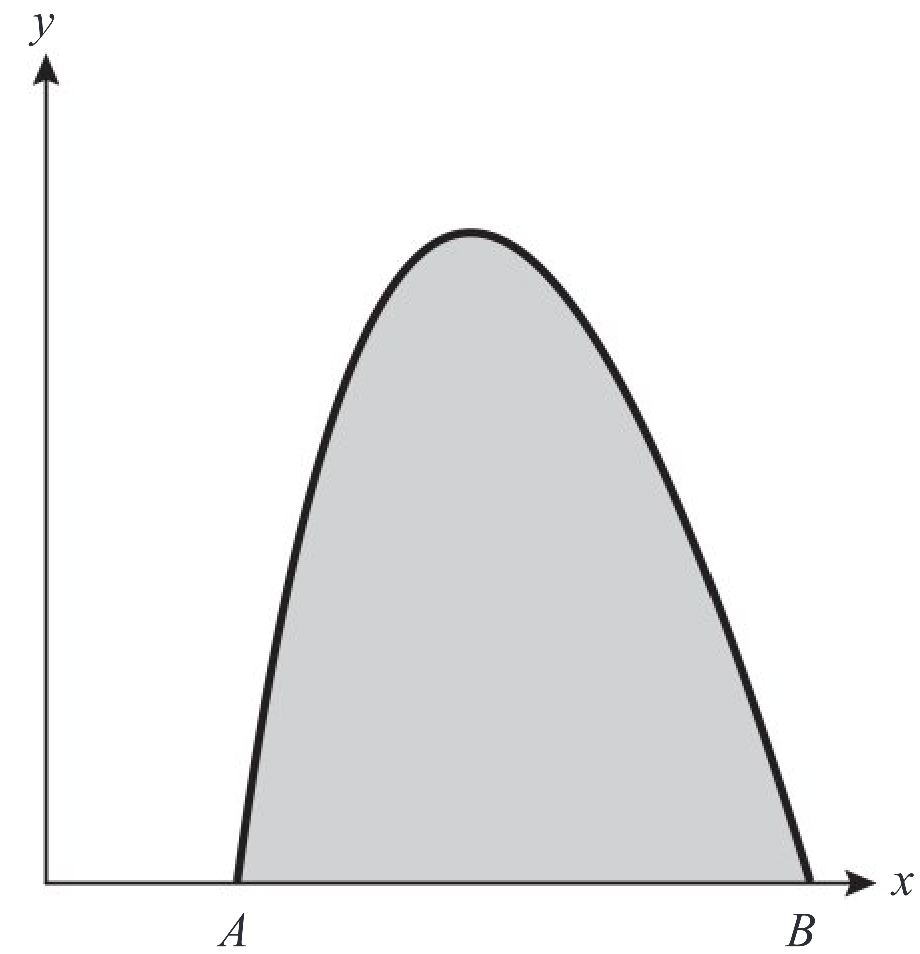
a Find the values of a, b and c. *[3]*

b According to this model, what should the height of a box with volume 1720 cm^3 be?
Give your answer correct to one decimal place. *[2]*

c Would this be a good model for a box of height 20 cm? Explain your answer. *[2]*

8 *[Maximum mark: 6]*

The diagram shows a cave entrance, whose outline can be modelled by the equation $y = 0.8(4 - x)\ln x$, where x and y are measured in metres. Points A and B are on the ground.



a Find the coordinates of A and B. *[2]*

b Find the height of the cave entrance at its highest point. *[2]*

c Find the area of the opening. *[2]*

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9 *[Maximum mark: 7]*

A small business needs to take out a loan of $50 000. A bank offers a loan with an annual interest rate of 2.4%, compounded monthly.

a If the business pays back $1000 per month, how long will it take to pay off the loan? *[2]*

b Find the monthly payment if the loan needs to be repaid in four years. *[2]*

c Compare the total amounts the business will pay for the loan in cases **a** and **b**. *[3]*

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10 *[Maximum mark: 6]*

The graph of $y = 2x^3 - ax^2 + x + 2b$ has a local minimum point at $(2, -6)$.

Find the values of a and b. *[6]*

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11 *[Maximum mark: 6]*

Rin has started a new martial arts club. The number of members during the first six months is shown in the table:

Month	1	2	3	4	5	6
Number of members	26	34	44	51	59	66

Rin thinks that the number of members can be modelled by a sequence that is approximately arithmetic.

a Find an average increase in the number of members over the first six months. *[2]*

b Use this model, with the first term equal to 26, to predict the number of members at the end of the year (in month 12). *[2]*

Gabor thinks that a better way to predict the number of members in future is to use a regression line.

c Use the data from the table to find the equation of the regression line and use it to predict the number of members at the end of the year. *[2]*

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12 *[Maximum mark: 5]*

The fuel consumption of a car depends on the speed at which the car is travelling. When the speed of the car is $v\,\text{km h}^{-1}$, the fuel consumption, in litres per 100 km, is modelled by the function $F(v)$. The table shows the rate of change of $F(v)$ for some values of v.

v	20	30	35	40	50
$F'(v)$	−0.0773	−0.0023	0.0195	0.0307	0.0217

a For which values of v in the table does the fuel consumption decrease as the speed increases? *[1]*

b Suggest a whole number speed, v, with $20 \leqslant v \leqslant 50$, at which the car should travel to minimize its fuel consumption. Justify your choice. *[2]*

c The minimum fuel consumption is 4.2 litres per 100 km. When the car is travelling at $20\,\text{km h}^{-1}$, the fuel consumption is 4.6 litres per 100 km. Use the information in the table to sketch the graph of F against v for $20 \leqslant v \leqslant 50$. *[2]*

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13 *[Maximum mark: 6]*

A running coach investigates whether different running shoes make a difference to athletes' running times. He times eight of his runners over the same distance, wearing two different pairs of shoes. The times, in seconds, are recorded in the table.

	Athlete							
Shoes	A	B	C	D	E	F	G	H
Pair 1	26.2	31.5	28.2	22.7	33.8	25.2	29.7	30.3
Pair 2	27.3	30.8	29.3	25.7	31.8	26.5	31.2	33.1

The coach decides to conduct a hypothesis test, using a 5% significance level, to test whether the mean times are different for the two pairs of shoes.

a State suitable hypotheses for the test. *[1]*

b Complete the table showing the difference in times for each athlete. *[2]*

Athlete	A	B	C	D	E	F	G	H
Time difference	–1.1	0.7						

c Hence carry out the test and state your conclusion. *[3]*

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14 *[Maximum mark: 6]*

Events A and B are such that:

$P(B) = \frac{1}{6}$, $P(A \cup B) = \frac{1}{5}$ and $P(A \mid B) = 4P(A)$.

Find $P(A \cap B)$. *[6]*

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Mathematics: applications and interpretation
Standard level
Paper 2
Practice Set C

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer **all** questions in an answer booklet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the mathematics: applications and interpretation formula booklet is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer **all** questions.

Do **not** write solutions on this page.

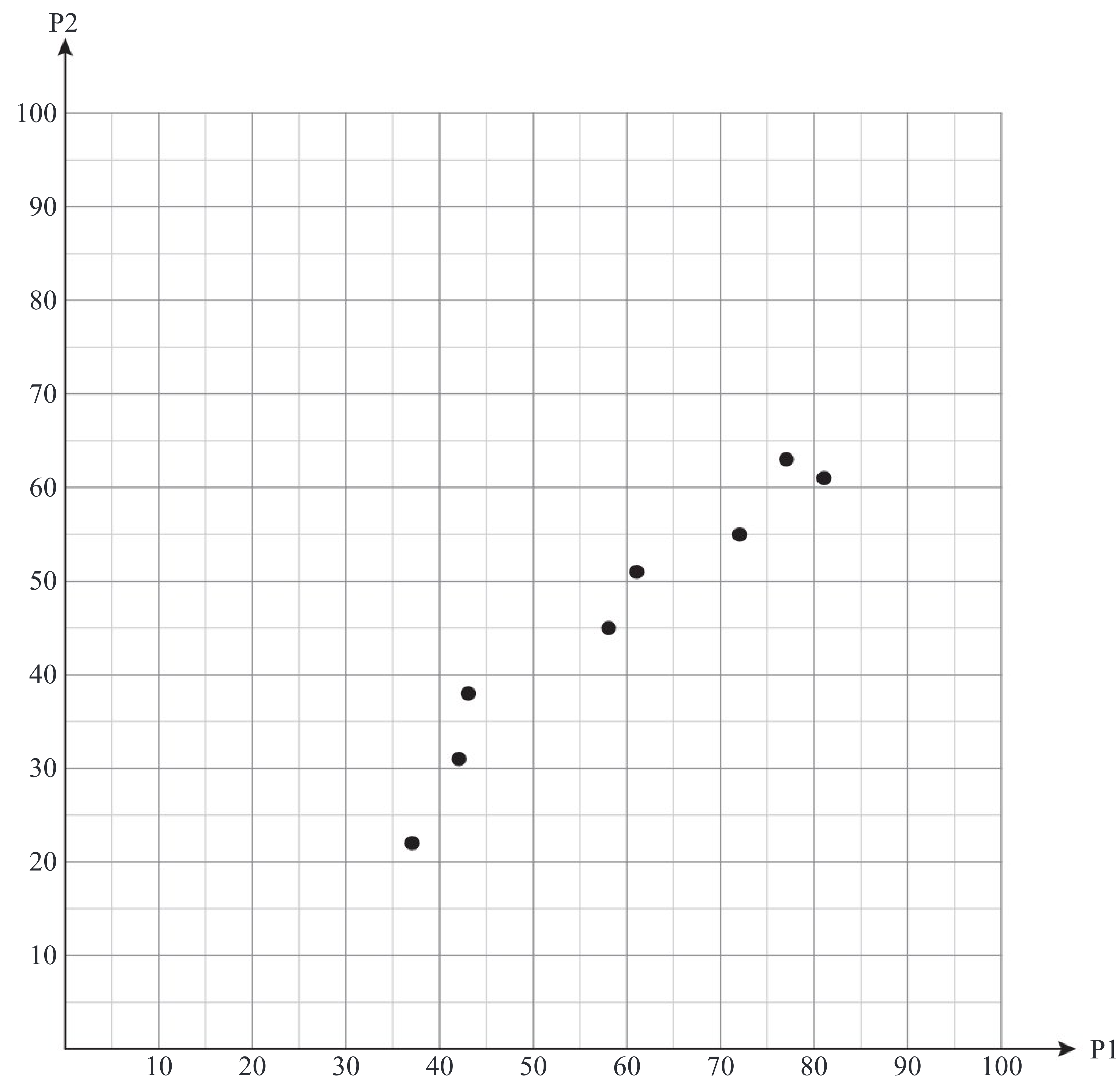
Answer all questions in an answer booklet. Please start each question on a new page.

1 *[Maximum mark: 17]*

The table shows the marks eight students obtained on two different exam papers.

	Student							
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Paper 1	37	61	81	43	42	72	58	77
Paper 2	22	51	61	38	31	55	45	63

The data are also shown on the scatter diagram.



a Find the mean mark for each paper and add the corresponding point to the scatter graph. *[2]*

b Draw a line of best fit. *[1]*

c Calculate Pearson's product moment correlation coefficient for the data. *[1]*

Two students did not take the second paper and a teacher wants to estimate what mark they would have got in it.

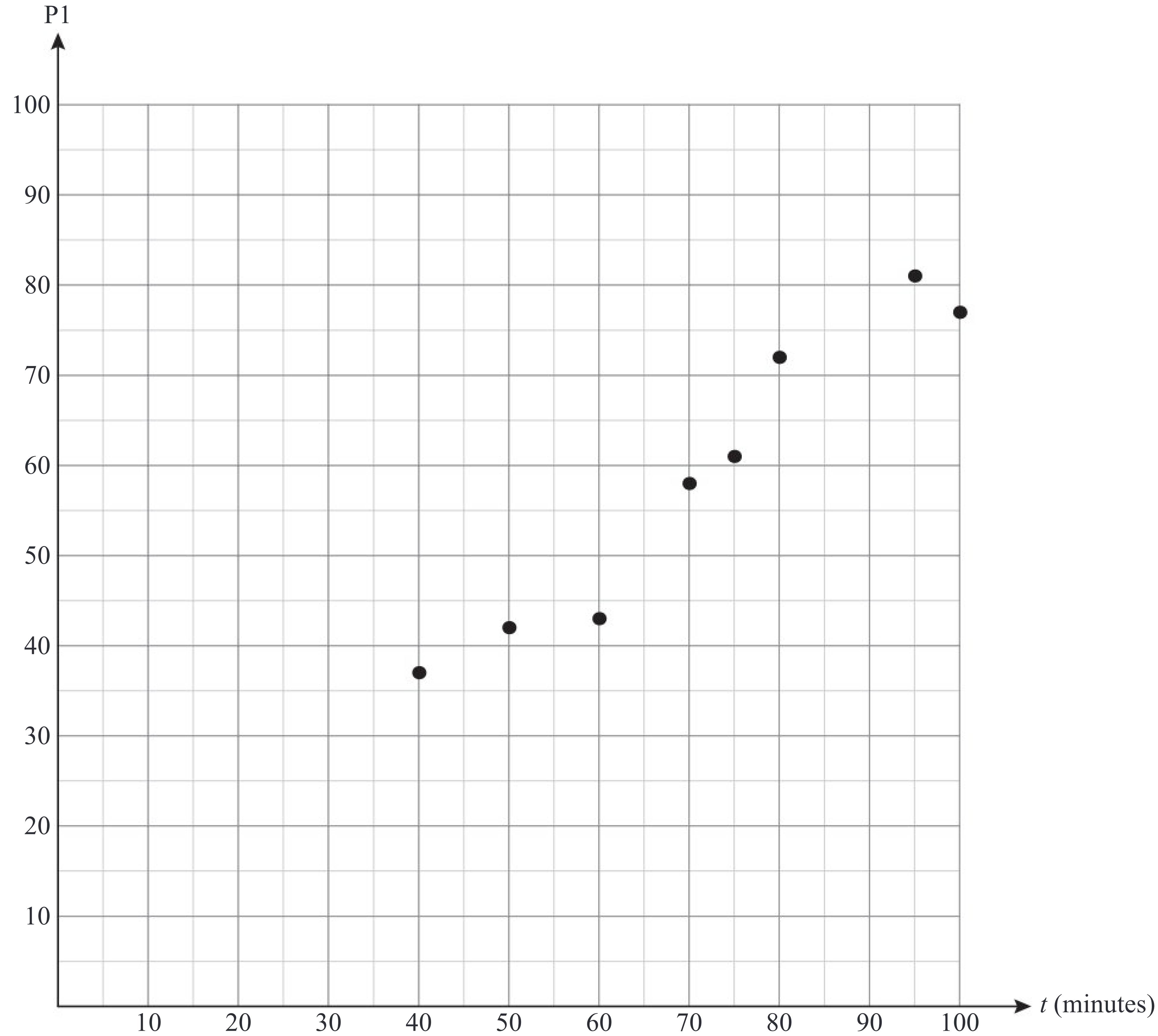
d Find the equation of the appropriate regression line that the teacher should use. *[2]*

e In Paper 1, Student J got 57 marks and Student K got 23 marks.

i Use your regression line to estimate how many marks each student would have got in Paper 2.

ii For each student, comment on the reliability of the estimate, giving reasons for your answers. *[5]*

Students A to H recorded how long they spent revising for Paper 1. The graph shows the time and the Paper 1 mark for each student. The teacher wants to determine whether there is any evidence of positive correlation between the time spent revising and the mark on Paper 1.



f By referring to the graph, explain why Pearson's product moment correlation is not an appropriate measure of correlation. *[1]*

g **i** Complete the table of ranks below.

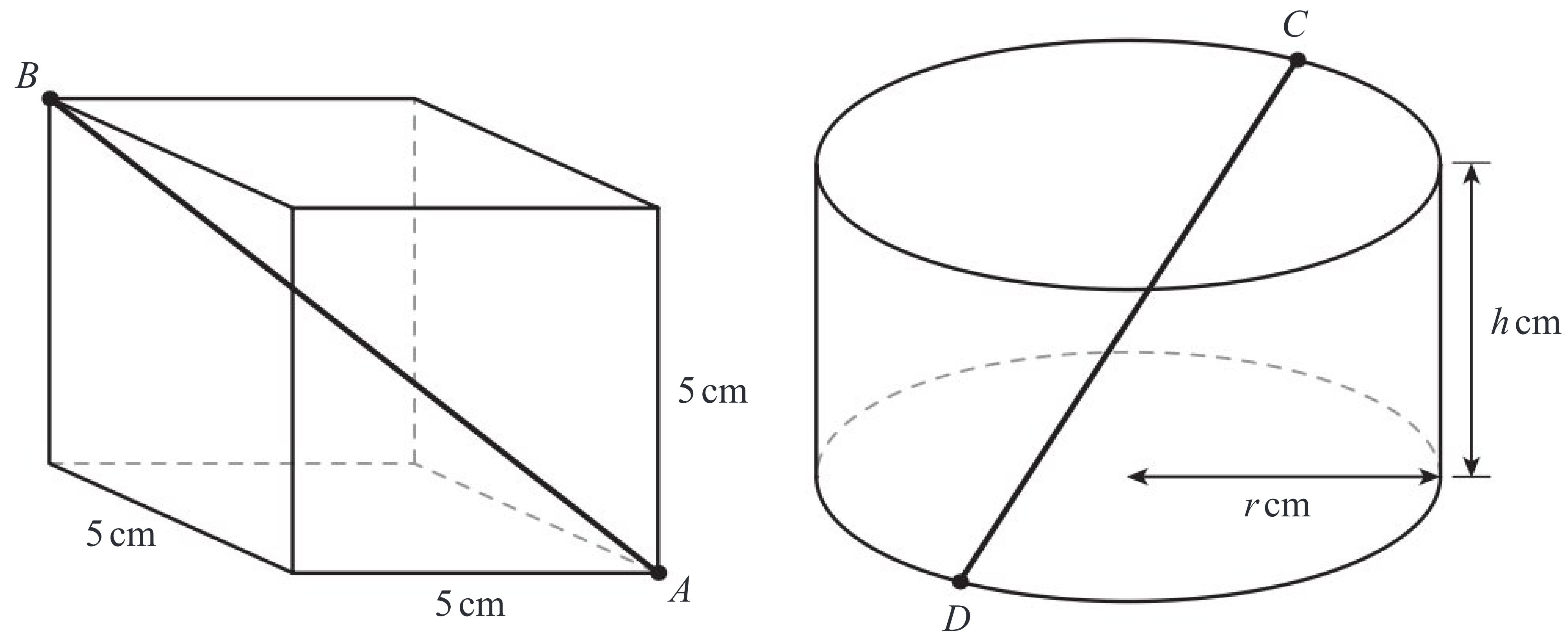
Student	A	B	C	D	E	F	G	H
Revision time rank	1	5		3			4	8
Paper 1 rank	1			3	2		4	7

ii Calculate Spearman's rank correlation coefficient.

iii The critical value of the correlation coefficient for the 5% significance level is 0.643. Stating your hypotheses and conclusion clearly test, at the 5% significance level, whether there is evidence of positive correlation between the time spent revising and the mark on Paper 1. *[5]*

2 *[Maximum mark: 14]*

The diagram shows a cube with side 5 cm and a cylinder with base radius r cm and height h cm.



a Find the length of AB. *[2]*

b Find the angle that the line AB makes with the horizontal base of the cube. *[2]*

The cylinder and the cube have the same volume.

c Show that the surface area of the cylinder is given by $\frac{250}{r} + 2\pi r^2$. *[4]*

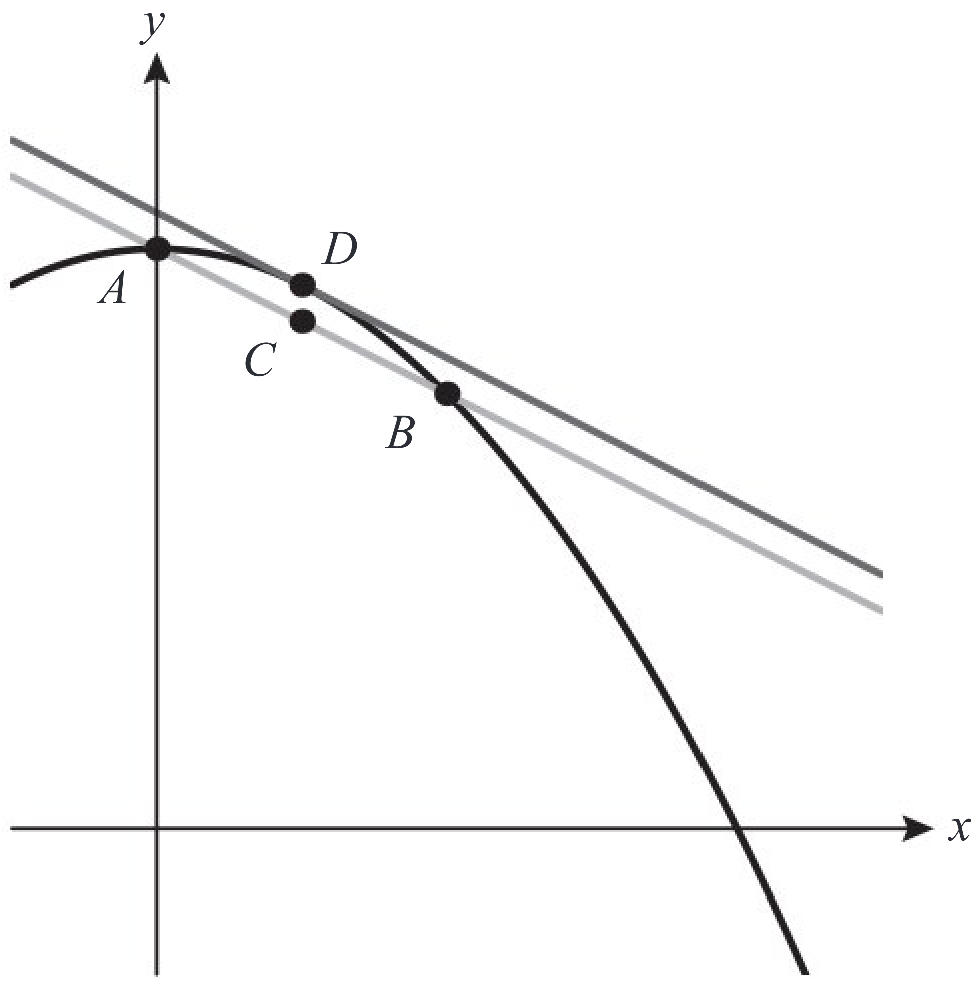
d Compare the minimum possible surface area of the cylinder to the surface area of the cube. *[3]*

e Assume the cylinder has the minimum possible surface area found in part **d**. The line CD is the longest line that can be drawn between the bottom base and the top base of the cylinder.

Find the angle that this line makes with the base of the cylinder. *[3]*

3 *[Maximum mark: 11]*

The diagram shows the curve with equation $y = 4 - x^2$. The line $y = 4 - x$ intersects the curve at the points A and B. The point C is the midpoint of AB. The line $y = k - x$ is tangent to the curve at point D.



a Find the coordinates of C. *[3]*

b Find the x-coordinate of D. *[3]*

c Find the value of k. *[3]*

d Find the distance CD. *[2]*

4 *[Maximum mark: 11]*

The times taken by children to complete a race can be modelled by a normal distribution with mean 5.56 minutes and standard deviation 2.5 minutes.

a Find the probability that a randomly selected child completes the race in less than 9.2 minutes. *[1]*

b Given that a randomly selected child completes the race in less than 9.2 minutes, find the probability that they complete the race in less than 8.3 minutes. *[2]*

Twenty randomly selected children run the race.

c Find the expected number of children who complete the race in less than 9.2 minutes. *[2]*

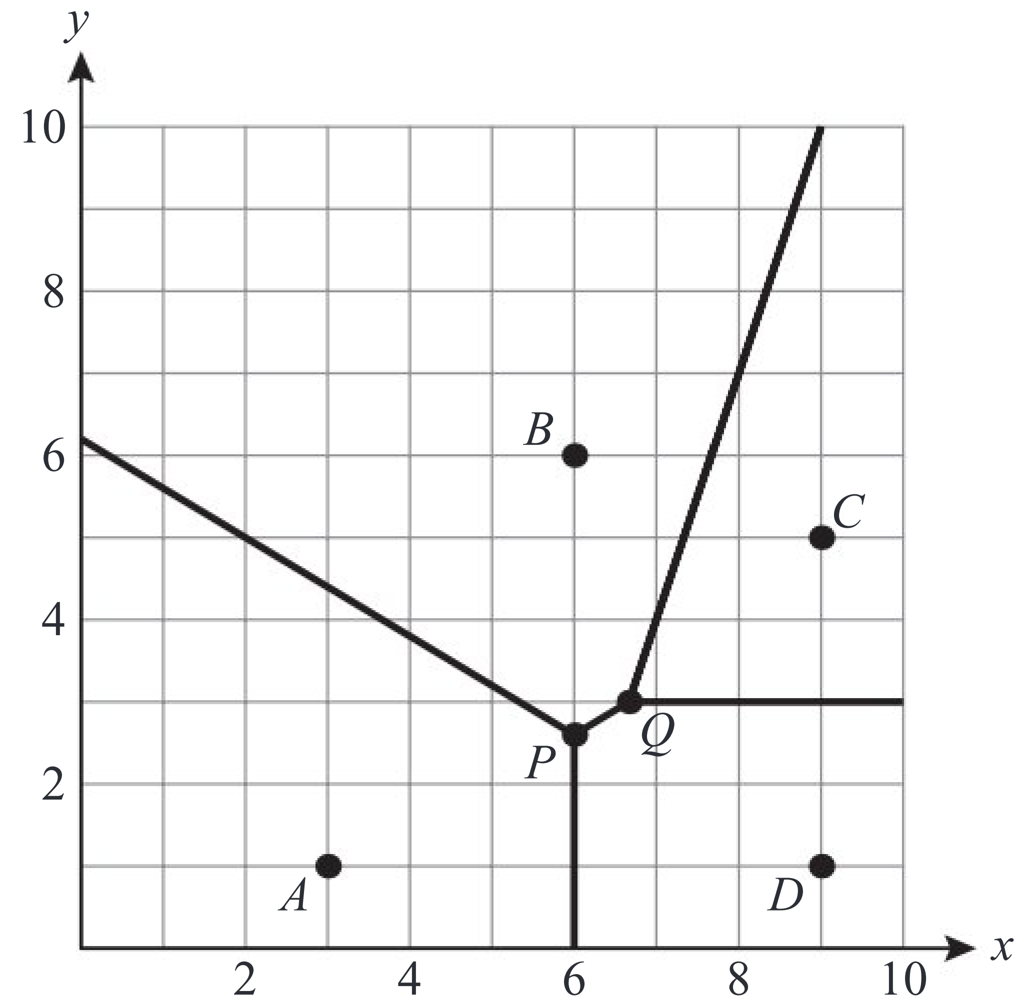
d Find the probability that at least 18 of the 20 children complete the race in less than 9.2 minutes. *[3]*

Two separate groups of 20 children run the race.

e Find the probability that in exactly one of the groups, at least 18 children complete the race in less than 9.2 minutes. *[3]*

5 *[Maximum mark: 15]*

In the Voronoi diagram below, post offices are located at sites $A(3, 1)$, $B(6, 6)$, $C(9, 5)$ and $D(9, 1)$.



a A shop is located at the point with coordinates (5, 4). The manager wants to go to the nearest post office. Which post office should she go to? *[1]*

b Write down the equations of the perpendicular bisectors of AD and CD. *[2]*

c Find the equation of the perpendicular bisector of BD, writing your answer in the form $ax + by = c$ where a, b and c are integers. *[4]*

d Find the coordinates of the vertices P and Q. *[3]*

e A new post office is to be opened at one of P or Q. Which of the two locations should be chosen if the new post office is to be as far as possible from the existing post offices? Show your method clearly. *[5]*

6 *[Maximum mark: 12]*

Newton's law of cooling states that the difference between the temperature of a cooling object and the background temperature decreases exponentially with time. This model can be represented by the equation $T = B + A \times 10^{-kt}$, where T is the temperature of the object in °C, B is the background temperature, t is the time in minutes, and A and k are constants.

A hot cake is placed in a room whose temperature can be assumed to be constant. The difference between the temperature of the cake and the room temperature halves every 3 minutes. The initial temperature of the cake is 93 °C.

a Show that the temperature of the cake after 9 minutes is given by $T = \dfrac{93 + 7B}{8}$ *[4]*

b Show that $10^{3k} = 2$. *[3]*

It is found that the temperature of the cake after 9 minutes is 30 °C.

c How much longer will it take for the cake to cool down to 24 °C? *[5]*

Practice Set A: Paper 1 Mark scheme

1 **a** 6.45×10^6 (m) A1

[1 mark]

b $4 \times \pi \times (6.45 \times 10^6)^2$ (M1)

$= 5.23 \times 10^{14}$ A1

[2 marks]

c $\frac{5.23 \times 10^{14} - 5.10 \times 10^{14}}{5.10 \times 10^{14}} \times 100$ (M1)

$= 2.51\%$ A1

[2 marks]

Total [5 marks]

2 **a** $V = \frac{1}{3} \times 6 \times 4^2$ (M1)

32 (cm^3) A1

[2 marks]

b Diagonal of square $= \sqrt{4^2 + 4^2}$ (= 5.66) (M1)

Length from corner to centre of square = 2.83 (A1)

Angle is $\tan^{-1}\left(\frac{6}{2.83}\right)$ (M1)

= 64.8° (1.13 radians) A1

[4 marks]

Total [6 marks]

3 **a** $\frac{54}{54 + 156 + 12 + 34}$ (M1)

$\frac{54}{256}\left(= \frac{27}{128}\right)$ A1

[2 marks]

b $\frac{156 + 34}{54 + 156 + 12 + 34}$ (M1)

$\frac{190}{256}\left(= \frac{95}{128}\right)$ A1

[2 marks]

c $\frac{12}{12 + 34}$ (M1)

$\frac{12}{46}\left(= \frac{6}{23}\right)$ A1

[2 marks]

Total [6 marks]

4 **a** 1.10 A1

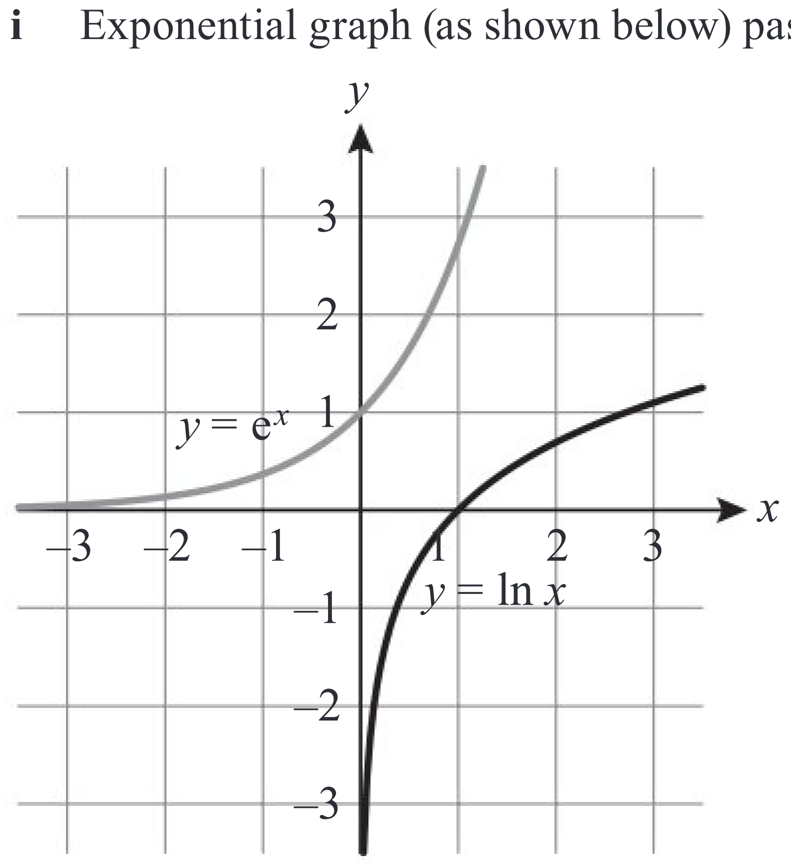
[1 mark]

b Logarithmic graph with y axis as asymptote A1

Passing through (1,0) and roughly (3,1.10) A1

[2 marks]

c **i** Exponential graph (as shown below) passing through (0, 1) A1



ii $y = e^x$ A1

[2 marks]

Total [5 marks]

5 a Ranks are

P	8	7	6	4.5	4.5	3	2	1
D	8	6	7	3	4	5	2	1

M1A1

(Or ranks could be reversed)

So from GDC, $r_s = 0.898$ A1

[3 marks]

b $0.898 > 0.643$, therefore there is evidence of a positive association A1

[1 mark]

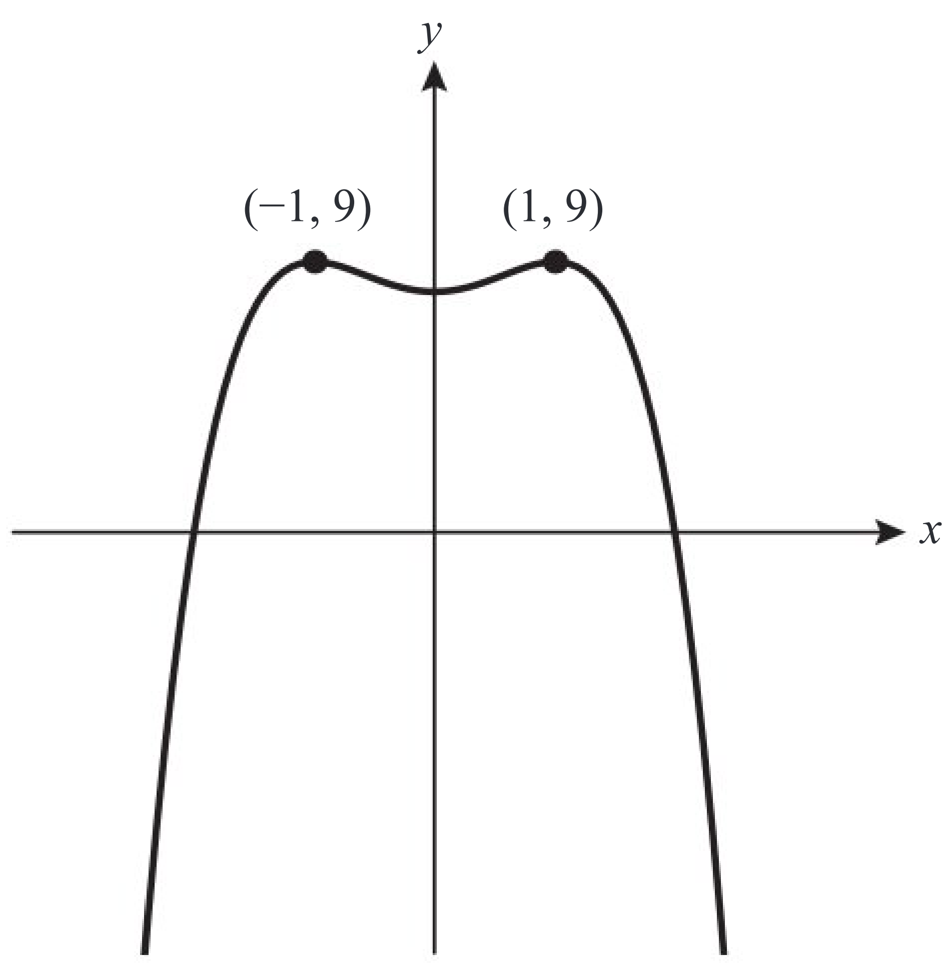
c Value would not change A1

Since an increase in the largest value of D would not change its rank R1

[2 marks]

Total [6 marks]

6 a Sketch of graph (M1)



From GDC, max value is 9 A1

[2 marks]

b Solve $8 + 2x^2 - x^4 = 0$ (M1)

$x = \pm 2$ (A1)

Area $= \int_{-2}^{2} 8 + 2x^2 - x^4 \, dx$ (M1)

$= \frac{448}{15} \approx 29.9$ A1

[4 marks]

Total [6 marks]

7 Expected frequencies are

HH	Hh	hh
37.5	75	37.5

M1A1

2 degrees of freedom A1

$\chi^2 = 1.72$ (M1)

p-value = 0.423 A1

p-value > 0.05, therefore there is no evidence that hypothesis is incorrect R1

Total [6 marks]

8 a $y = \int 3x^2 + 2\, dx$ (M1)

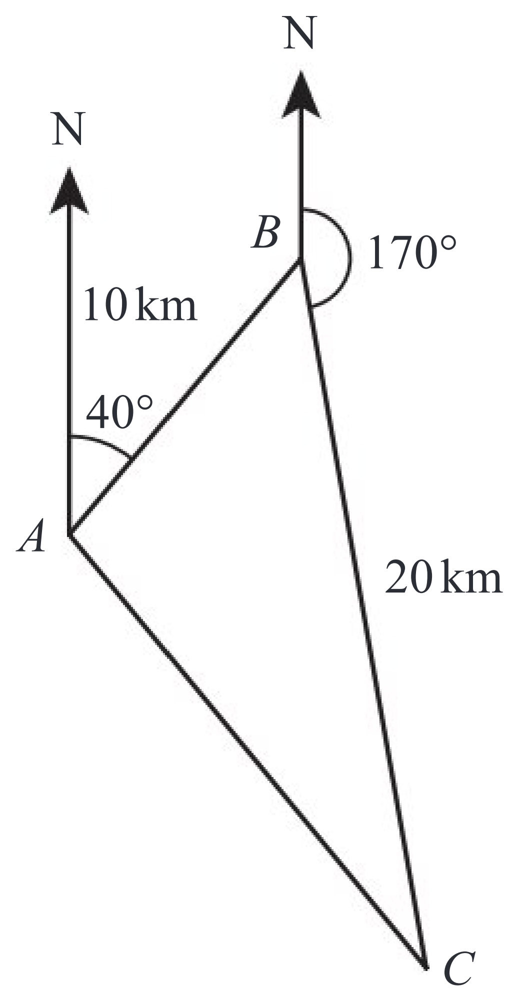
$= x^3 + 2x + c$ (A1)

When $x = 0$, $y = 1$ so $y = x^3 + 2x + 1$ (M1) A1

[4 marks]

	b	When $x = 0$, gradient of tangent is 2	(M1)
		So gradient of normal is $-\frac{1}{2}$	(A1)
		$y = -\frac{1}{2}x + 1$	(M1)
		$2y + x = 2$	A1
			[4 marks]
			Total [8 marks]
9	EITHER		
	$S_1 = 7$		(A1)
	$S_2 = 10$		(A1)
	$u_1 = 7$		A1
	Note: Must be made clear that this is the first term		
	$u_2 = 3$		(M1)
	$d = -4$		A1
			[5 marks]
	OR		
	$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$		(M1)(A1)
	Comparing coefficients:		
	$\frac{d}{2} = -2$ and $a - \frac{d}{2} = 9$		(M1)
	$d = -4$		A1
	$a = 7$		A1
			[5 marks]
			Total [5 marks]
10	**a**	Gradient of AB $= \frac{8-4}{2-0}$	(M1)
		$= 2$	(A1)
		So gradient of perpendicular line is $-\frac{1}{2}$	(A1)
		So equation is $y - 3 = -\frac{1}{2}(x - 1)$	M1A1
		$(y = -0.5x + 3.5)$	*[5 marks]*
	b	Equation of AB is $y = 2x + 4$	(A1)
		Solve simultaneously to find point of intersection	(M1)
		$(-0.2, 3.6)$	A1
			[3 marks]
			Total [8 marks]

11 a



Angle between trajectories is 50° (A1)

Using cosine rule:

$b^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 50$ M1

(= 242.88 …)

$c = 15.6$ km A1

[3 marks]

b Using sine rule

$\frac{\sin C}{10} = \frac{\sin 50}{15.6}$ (M1)

$C = \sin^{-1}\left(\frac{10 \sin 50}{15.6}\right) = 29.4°$ A1

Note – could also be found using the cosine rule

Bearing is 360 – 10 – 29.4 = 321 A1

[3 marks]

Total [6 marks]

12 a $C = 8 + 0.05x$ A1

[1 mark]

b $C = \begin{cases} 10 & 0 < x \leqslant 100 \\ 0.2x - 10 & x > 10 \end{cases}$ A1A1

[2 marks]

c Intersects first branch at $x = 40$ A1

Intersects second branch at $x = 120$ A1

So cheaper for $40 < x < 120$ A1

[3 marks]

Total [6 marks]

13 The midpoint of AB is (2, 3) (A1)

The gradient of AB is 1 (A1)

Therefore, the equation of the perpendicular bisector is

$y = 5 - x$ A1

Then EITHER perpendicular bisector of BC is $y = \frac{1}{2}x$

OR perpendicular bisector of AC is $y = 2x - 5$ A2

Intersecting any two perpendicular bisectors M1

$\left(\frac{10}{3}, \frac{5}{3}\right) \approx (3.33, 1.67)$ A1

Total [7 marks]

Practice Set A: Paper 2 Mark scheme

1 **a** 4000 A1

[1 mark]

b $3x^2 - 12x + 9$ (M1)A1

Note: Award M1 for at least one correct term.

[2 marks]

c Rate of change of number of subscribers A1

[1 mark]

d From GDC: $x = 3$ or $x = 1$ A1A1

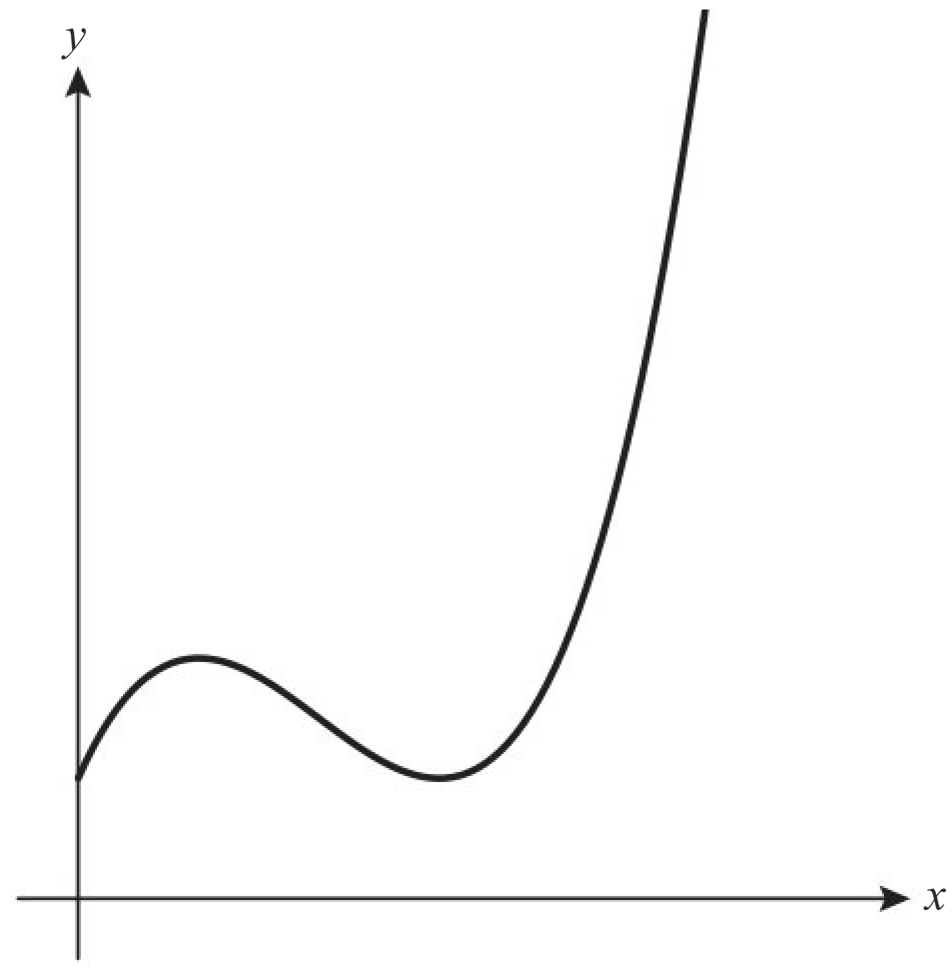
[2 marks]

e $f'(x) > 0$ (M1)

$x > 3$ or $x < 1$ A1

[2 marks]

f



Correct shape, with no negative x values A1

Intercept labelled at $y = 4$ A1

Max labelled at (1, 8), min at (3, 4) A1

[3 marks]

g Solving $x^3 - 6x^2 + 9x + 4$ graphically or using polynomial solver M1

4.20 (weeks) A1

[2 marks]

Total [13 marks]

2 **a** Using a lattice diagram or other systematic list (M1)

s	2	3	4	5	6	7	8
P(*S* = *s*)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\mathbf{\frac{3}{16}}$	$\mathbf{\frac{2}{16}}$	$\mathbf{\frac{1}{16}}$

A1

[2 marks]

b $E(S) = 2 \times \frac{1}{16} + 3 \times \frac{2}{16} \cdots$ (M1)

$= 5$ A1

[2 marks]

c $P(X > 4) = \frac{10}{16}$; $P(X > 6) = \frac{3}{16}$ A1

$P(X > 6 \mid X > 4) = \frac{3}{16} \div \frac{10}{16} = \frac{3}{10}$ M1A1

[3 marks]

d If W is the winnings then:

w	–1	k
P($W = w$)	$\frac{6}{16}$	$\frac{10}{16}$

M1A1

$\mathrm{E}(W) = -\frac{6}{16} + \frac{10k}{16}$ A1

For the game to be fair, E(W) = 0 so M1

$\frac{10k}{16} = \frac{6}{16}$

$k = 0.6$ A1

[5 marks]

Total [12 marks]

3 a i 16.1 A1

ii 1.73 A1

iii −0.901 A1

Strong negative correlation A1

[4 marks]

b i 8 A1

ii 6 A1

[2 marks]

c i $y = \begin{cases} -0.361x + 18.8 & x < 8 \\ -0.0686x + 15.1 & x \geq 10 \end{cases}$ M1A1A1

ii Both are negative so there is a trend of improving in both parts of the season A1

The modulus of the first coefficient is larger, so there is greater improvement each week in the pre-competition training A1

[5 marks]

d $-0.361 \times 7 + 18.8 \approx 16.6$ (minutes) M1A1

[2 marks]

e This is an example of extrapolation, which is not generally valid A1

[1 mark]

Total [14 marks]

4 a i $1000 + 5 \times 50 = 1250$ (M1)A1

ii $1000 \times 1.04^5 = 1216.65$ (M1)A1

Note: May be done using TVM so no working shown.

[4 marks]

b 4.56% (M1)A1

Note: award M1 for any evidence of using TVM package, eg stating principal value of 1000 and final value of 1250.

[2 marks]

c Solving $1000 + 50n = 1000 \times 1.04^n$ (M1)

Evidence of graphical, tabular or trial and error approach (M1)

$n \geq 12$ A1

Note: do not accept non-integer values.

[3 marks]

d Effective interest rate = 1.5% (A1)

Evidence of TVM or $1400 = 1000 \times 1.015^n$ (M1)

22.599 years (A1)

So needs 23 years A1

[4 marks]

Total [13 marks]

5 a From GDC, 0.0478 A1

[1 mark]

b Using inverse normal distribution M1

138(.4465) A1

[2 marks]

c If X = "number of people with score $\geqslant$ 150 out of 5"
$X \sim B(5, 0.0478)$ (M1)(A1)
$P(X \geqslant 2) = 1 - P(X \leqslant 1)$ (M1)
$= 0.0207$ A1
[4 marks]

d We need one success in the first four, then a success (M1)
If Y = "number of people with score $\geqslant$ 150 out of 4"
$Y \sim B(4, 0.0478)$
Required probability is $P(Y = 1) \times 0.0478$ (M1)
$= 0.00789$ A1
[3 marks]

e If A is the score of a member then we require
$P(A > 170 | A > 150)$ (M1)
$= \frac{P(A > 170 \cap A > 150)}{P(A > 150)}$
$= \frac{P(A > 170)}{P(A > 150)}$ (OR use a Venn diagram) (M1)
$= \frac{0.0107}{0.05}$ (M1)
Note: Award M1 for evidence of using GDC to calculate any probability from a N(100, "their value") distribution, even outside of context of conditional probability.
$= 0.214$ A1
[4 marks]

f That the membership of the high intelligence society is representative of the whole population R1
[1 mark]
Total [15 marks]

6 **a** 1 diameter of 2 cm and 2 radii each of 1 cm R1
[1 mark]

b Total area of metal in each repeating unit $= 2 \times \pi \times 1^2 = 2\pi$ M1A1
If side of the square is x then $x^2 + x^2 = 16$ M1
So proportion of box filled is $\frac{2\pi}{8} = \frac{\pi}{4}$ A1
[4 marks]

c for example, that the extra space at the edge of the box is negligible R1
Note: Accept any reasonable criticism of the model.
[1 mark]

d Diagonal is 4 and width is 2 A1
So height is $\sqrt{4^2 - 2^2} = \sqrt{12}$ M1A1
Ratio is $\frac{2 \times \pi \times 1^2}{2\sqrt{12}} = \frac{\pi}{\sqrt{12}}$ M1A1
[5 marks]

e $\sqrt{12} < \sqrt{16} = 4$ R1
Therefore method 2 can pack more rods A1
Note: Do not award R0A1
[2 marks]
Total [13 marks]

Practice Set B: Paper 1 Mark scheme

1 $\frac{1}{2}r^2\theta = 15$ (A1)

$2r + r\theta = 4r\theta$

$\theta = \frac{2}{3}$ A1

Substituting their θ into their area equation: (M1)

$\frac{1}{2}r^2\left(\frac{2}{3}\right) = 15$

$r = \sqrt{45} = 6.71$ cm A1

Total [4 marks]

2 a Median = 6.5 A1

[1 mark]

b $IQR = 8 - 5 = 3$ A1

[1 mark]

c x is an outlier if $x < Q_1 - 1.5(Q_3 - Q_1)$, so if $x < 5 - 1.5 \times 3 = 0.5$ A1

OR if $x > Q_3 + 1.5(Q_3 - Q_1)$, so if $x > 8 + 1.5 \times 3 = 12.5$ A1

No data values smaller than 0.5 or larger than 12.5 so no outliers R1

[3 marks]

Total [5 marks]

3 a $a + (5 - 1)\,d = 8$

$a + 4d = 8$ A1

$\frac{8}{2}(2a + (8 - 1)d) = 58$

$4a + 14d = 29$ A1

$a = 2,\ d = \frac{3}{2}$ A1A1

Note: If a and d both incorrect then award M1A0 for attempt to solve simultaneous equations.

[4 marks]

b $2 + (n - 1)\frac{3}{2} = 26$ (M1)

$3(n - 1) = 48$

$n = 17$ A1

[2 marks]

Total [6 marks]

4 a i Upper bound $= \frac{219.5}{18.35} = 11.961\,85$ M1A1

ii Lower bound $= \frac{218.5}{18.45} = 11.842\,81$ M1A1

Note: Award M1 each time for $\frac{219.5 \text{ or } 218.5}{18.35 \text{ or } 18.45}$

[4 marks]

b Agreement between upper and lower bound to two significant figures R1

so, $R = 12$ (2 s.f.) A1

[2 marks]

Total [6 marks]

5 a $m = \frac{\ln x^2 - 0}{x - 1}$ (M1)

$= \frac{\ln x^2}{x - 1}$ A1

[2 marks]

b Finds sequence of values of m for values of x that approach 1 M1

m tends towards 2 A1

[2 marks]

c The gradient of the function $f(x) = \ln x^2$ A1

at $x = 1$ A1

[2 marks]

Total [6 marks]

6	**a**	$H_0: \mu_G = \mu_N$	A1
		$H_1: \mu_G < \mu_N$	A1
			[2 marks]
	b	0.0986	A2
			[2 marks]
	c	$0.0986 < 0.1$	R1
		So reject H_0. There is sufficient evidence at the 10% level that Nya Stan is warmer	A1
		Note: Award R1 for correct comparison of their p-value. Must have conclusion in context for A1. Do not award R0A1.	
			[2 marks]
	d	The population temperatures are normally distributed	A1
		The population variances are equal	A1
			[2 marks]
			Total [8 marks]
7		$ar^3 = 13.5\ldots$ (1)	A1
		$a\left(\frac{1-r^3}{1-r}\right) = 74\ldots$ (2)	A1
		Dividing their (1) by (2) or substituting:	(M1)
		$\frac{r^3(1-r)}{1-r^3} = \frac{13.5}{74}$	
		$74r^3 - 74r^4 = 13.5 - 13.5r^3$	
		$74r^4 - 87.5r^3 + 13.5 = 0$	M1
		Note: Award M1 for rearranging to a quartic equation $p(r) = 0$	
		$r = \frac{3}{4}$ (reject $r = 1$)	A1
		$a = 32$	A1
			Total [6 marks]
8	**a**	$N = 30$	
		$I\% = 2.5$	
		$PV = 0$	
		$PMT = -6000$	
		$P/Y = C/Y = 1$	(M1)(A1)
		Note: Award M1 for attempt to use financial app; A1 for all values correct.	
		$FV = £309\,263\,416.22$	A1
			[3 marks]
	b	$I\% = \frac{2.5}{12}$	
		$PV = 309\,736.06$	
		$PMT = 750$	
		$FV = 0$	
		$P/Y = C/Y = 1$	(M1)(A1)
		Note: Award M1 for attempt to use financial app; A1 for all values correct.	
		$N = 263.8$	
		So, 264 months or 22 years	A1
			[3 marks]
			Total [6 marks]
9	**a**	Attempt to solve $-3x^2 + 5x + 2 = 0$	(M1)
		$x = 2\left(\text{reject } -\frac{1}{3}\right)$	(A1)
		So 200 items	A1
			[3 marks]

b $P(x) = \int -3x^2 + 5x + 2\, dx$ (M1)

Note: Award M1 for attempt at integration

$= -x^3 + 2.5x^2 + 2x + c$ A1A1

Note: Award A1 for any two correct terms in x; second A1 for all correct including constant of integration

$2 = -1^3 + 2.5 \times 1^2 + 2 \times 1 + c$ M1

$c = -1.5$

So, $P(x) = -x^3 + 2.5x^2 + 2x - 1.5$ A1

[5 marks]

Total [8 marks]

10 a H_0: Waiting times follow a $N(14, 36)$ distribution A1

H_1: Waiting times do not follow a $N(14, 36)$ distribution A1

[2 marks]

b

Waiting time/min	< 5	5–10	10–15	15–20	> 20
Expected frequency	5.34	14.85	25.10	22.01	12.69

A2

Note: Award A2 for all four correct; A1 for two or three correct; A0 otherwise.

[2 marks]

c $v = 4$ (A1)

p-value = 0.0871 A2

[3 marks]

d $0.0871 > 0.05$ R1

So do not reject H_0. There is insufficient evidence to reject the manager's claim A1

Note: Award R1 for correct comparison of their p-value. Must have conclusion in context for A1. Do not award R0A1

[2 marks]

Total [9 marks]

11 a $30 = \frac{2\pi}{b}$ so $b = \frac{\pi}{15}$ A1

When $t = 0$, $h = 2$ so $2 = a\cos 0 + c$ (M1)

$2 = a + c$

When $t = 15$, $h = 122$ so $122 = a\cos\left(\frac{\pi}{15} \times 15\right) + c$ (M1)

$122 = c - a$

Solving simultaneously, $a = -60$, $c = 62$ A1A1

[5 marks]

b $50 = -60\cos\left(\frac{\pi}{15}t\right) + 62$ (M1)

From GDC, $t = 6.54, 23.5$ (A1)

So time above 50 m is 16.9 minutes A1

[3 marks]

Total [8 marks]

12	**a**	$0.1 + a + b + 0.2 + 0.15 = 1$	M1A1
		$a + b = 0.55$	AG
			[2 marks]
	b	Game fair so $E(X) = 2$	
		$(0 \times 0.1) + a + 2b + (3 \times 0.2) + (4 \times 0.15) = 2$	(M1)
		$a + 2b = 0.8$	A1
		Solving simultaneously with $a + b = 0.55$,	
		$a = 0.3$, $b = 0.25$	A1
			[3 marks]
	c	Will make loss if $X_1 + X_2 < 4$	(M1)
		(0, 0), (1,1)	
		(0,1), (0,2), (0,3), (1,2) AND REVERSES	
		$P(X_1 + X_2 < 4) = 0.1^2 + 0.3^2$	
		$+ 2\,(0.1 \times 0.3 + 0.1 \times 0.25 + 0.1 \times 0.2 + 0.3 \times 0.25)$	M1
		$= 0.4$	A1
			[3 marks]
			Total [8 marks]

Practice Set B: Paper 2 Mark scheme

1 a i Systematic sampling — A1

ii Not all samples are possible, eg adjacent people on the list cannot be chosen — A1

[2 marks]

b i Men = $\frac{46}{46+63} \times 12 = 5.06$ — M1

So, 5 men — A1

ii Simple random sampling — A1

iii Uses opportunity sampling rather than simple random sampling to select the participants in each group — A1

[4 marks]

c $r = 0.787$ — A1

Reasonable positive correlation between height and weight; as one increases, so does the other — A1

[2 marks]

d i $w = 0.806h - 70.0$ — (M1)

$w = 0.806 \times 140 - 70.0 = 42.8$ kg — A1

ii $w = 0.806 \times 170 - 70.0 = 67.0$ kg — A1

[3 marks]

e 140 cm is significantly outside the range of the given data so extrapolation of the relationship makes the prediction unreliable — A1

170 cm is within the range of the data and reasonable positive correlation so prediction reasonably reliable — A1

[2 marks]

f for example, take a larger sample; create separate regression lines for men and women — A1A1

[2 marks]

Total [15 marks]

2 a Midpoint of (1, 8) and (5, 2) = $\left(\frac{1+5}{2}, \frac{8+2}{2}\right) = (3,5)$ — A1

Gradient of line segment from (1, 8) to (5, 2) = $\frac{2-8}{5-1} = -\frac{3}{2}$ — A1

So gradient of perpendicular bisector is $\frac{2}{3}$ — (M1)

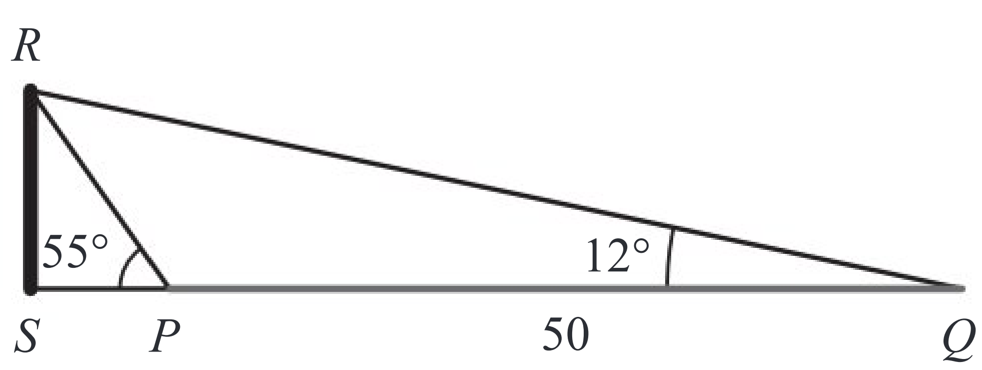
Note: Award M1 for gradient of perpendicular = $-\frac{1}{\text{their } m}$

Equation of perpendicular bisector: $y - 5 = \frac{2}{3}(x - 3)$

$2x - 3y = -9$ — A1

[4 marks]

b



$R\hat{P}Q = 125°$ so $P\hat{R}Q = 180 - 125 - 12 = 43°$ — A1

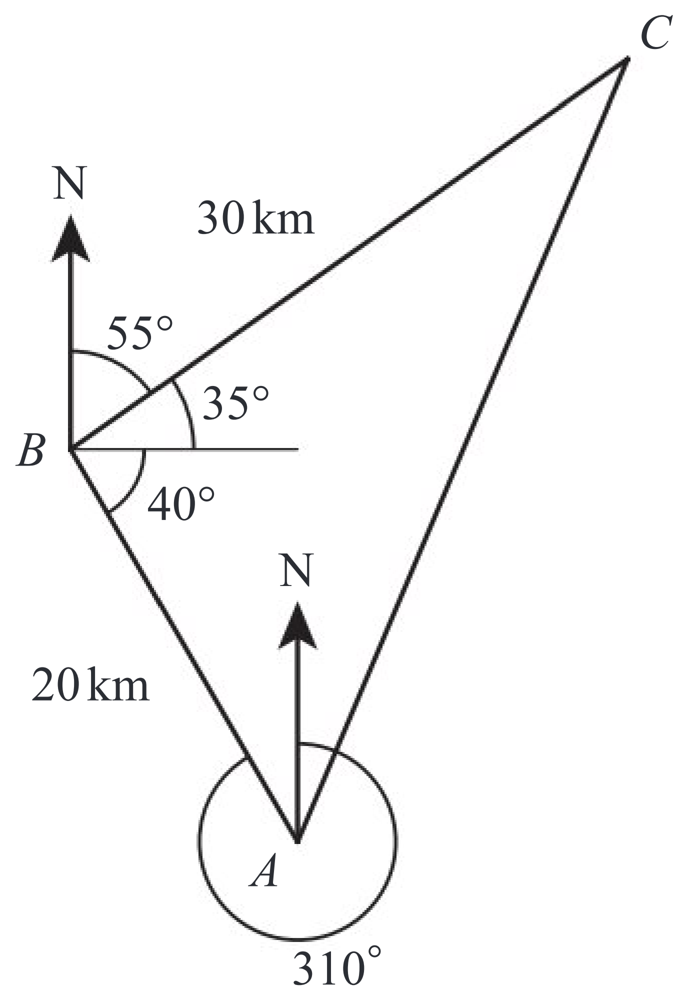
By sine rule $\frac{PR}{\sin 12} = \frac{50}{\sin 43}$ — (M1)

$PR = \frac{50 \sin 12}{\sin 43} = 15.24283\ \ldots$ — A1

$h = PR \sin 55 = 12.5$ m — (M1)A1

[5 marks]

c



$A\hat{B}C = 40 + 35$ M1

$= 75°$ A1

[2 marks]

d By cosine rule, $AC = \sqrt{20^2 + 30^2 - 2 \times 20 \times 30 \cos 75}$ (M1A1)

Note: Award M1 for attempt to use cosine rule

= 31.5 km A1

[3 marks]

e By sine rule, $\frac{\sin B\hat{C}A}{20} = \frac{\sin 75}{31.455}$ (M1)

Note: Award M1 for attempt to use sine rule

$B\hat{C}A = 37.9°$ A1

So bearing = 360 – 125 – their $B\hat{C}A$ (M1)

= 197° A1

[4 marks]

Total [18 marks]

3 a $A = \frac{1}{2}[2.3 + 2.3 + 2(3.5 + 4.3 + 4.7 + 4.7 + 4.3 + 3.5)]$ M1A1

$= 27.3\,\text{m}^2$ A1

[3 marks]

b Since the curve bows out, the trapezia are all under the curve... R1

... so this gives an underestimate A1

Note: Do not award R0A1

[2 marks]

c $h = ax^2 + bx + 2.3$ A1

Substitute in any other two pairs of data: M1

$3.5 = 1^2a + 1b + 2.3$

$4.3 = 2^2a + 2b + 2.3$

Solve simultaneously to give $a = -0.2$, $b = 1.4$ A1A1

[4 marks]

d Finds max point of their quadratic from GDC (M1)

Max height is $h = 4.75$ m A1

[2 marks]

e $A = \int_0^7 -0.2x^2 + 1.4x + 2.3\,dx$ (M1)

$= \frac{413}{15}$ A1

[2 marks]

f % error $= \frac{\frac{413}{15} - 27.3}{\frac{413}{15}} \times 100$ (M1)

= 0.847% A1

[2 marks]

g Take more strips A1

[1 mark]

Total [16 marks]

4 a $X \sim B(10, 0.04)$ (M1)
$P(X = 2) = 0.0519$ A1
[2 marks]

b $P(X \geq 2) = 1 - P(X \leq 1)$ (M1)
$= 0.0582$ A1
[2 marks]

c i $0.04n = 2$ (M1)
$n = 50$ A1
ii $\text{Var}(X) = 50 \times 0.04 \times 0.96$ (M1)
$= 1.92$ A1
[4 marks]

d $Y \sim B(5, 0.0582)$ (M1A1)
Note: Award M1 for use of binomial with $n = 5$
$P(Y > 1) = 1 - P(Y \leq 1)$ (M1)
$= 0.0301$ A1
[4 marks]

e $P(X \geq 1) > 0.95$
$1 - P(X = 0) > 0.95$ (M1)
$P(X = 0) < 0.05$
$0.96^n < 0.95$ M1
$n > 73$ A1
that is, smallest number of calls is $n = 74$ A1
[4 marks]
Total [16 marks]

5 a $\pi r^2 h + \frac{2}{3}\pi r^3 = 300$ M1A1
Note: Award M1 for correct volume of cylinder or hemisphere
$3\pi r^2 h + 2\pi r^3 = 900$ A1
$3\pi r^2 h = 900 - 2\pi r^3$ A1
$h = \frac{900 - 2\pi r^3}{3\pi r^2}$ AG
[4 marks]

b $A = 2\pi rh + \pi r^2 + 2\pi r^2$ (M1A1)
$= 2\pi r\left(\frac{900 - 2\pi r^3}{3\pi r^2}\right) + 3\pi r^2$ M1
$= \frac{600}{r} - \frac{4}{3}\pi r^2 + 3\pi r^2$
$= 600r^{-1} + \frac{5}{3}\pi r^2$ A1A1
Note: Award A1 for $ar^{-1} + cr^2$; second A1 for all correct
[5 marks]

c i Attempt to find minimum point of $y = 600x^{-1} + \frac{5}{3}\pi x^2$ from GDC or otherwise (M1)
$A = 233\text{ cm}^2$ A1
ii $r = 3.86$ cm A1
iii Substituting their r into $h = \frac{900 - 2\pi r^3}{3\pi r^2}$ M1
$h = 3.86$ cm A1
[5 marks]

d For example, may want taller and thinner design for aesthetic reasons, or for ergonomic reasons A1
[1 mark]
Total [15 marks]

Practice Set C: Paper 1 Mark scheme

1	a	$\frac{18^2 \pi}{2} \times 83$	(M1)
		$4.22 \times 10^4 \text{cm}^3$	A1A1
			[3 marks]
	b	$\frac{4}{3}\pi r^3 =$ their volume	M1
		$r^3 = \frac{\text{volume} \times 3}{4\pi}$	(M1)
		$r = 21.6\,\text{cm}$	A1
			[3 marks]
			Total [6 marks]
2	a	$9.1^2 = 6.8^2 + 4.7^2 - 2(6.8)(4.7)\cos B$	(M1)
		$\cos B = -0.227$	(A1)
		$B = 103°$	A1
			[3 marks]
	b	$\frac{1}{2}(6.8)(4.7)\sin(\text{their } B)$	M1
		$15.6\,\text{cm}^2$	A1
			[2 marks]
			Total [5 marks]
3	a	(90, 88)	(M1)
		200 – 88 = 112	A1
			[2 marks]
	b	15% of 200 = 30	(M1)
		Line at 170 on graph crosses at (104, 170)	A1
		104 g	A1
			[3 marks]
			Total [5 marks]
4	a	Use $640 = \frac{20}{2}(7 + u_{20})$ or $640 = \frac{20}{2}(14 + 19d)$	(M1)
		57	A1
			[2 marks]
	b	$19d = 50$ or $u_{39} - u_{20} = u_{20} - u_1$	(M1)
		107	A1
			[2 marks]
			Total [4 marks]
5	a	$\frac{\theta}{360} \times \pi \times 10^2 = 75$	M1
		$\theta = 85.9$	A1
			[2 marks]
	b	$\frac{\text{their } \theta}{360} \times 2\pi \times 10$	M1
		+20	(M1)
		35 cm	A1
			[3 marks]
			Total [5 marks]
6	a	midpoints: 10, 13.5, 17.5, 22.5, 26.5	(M1)
		mean = 17.8	A1
		SD = 5.40	A1
			[3 marks]
	b	"17.8" × 2.54	(M1)
		mean = 45.3	A1
		variance = 188	A1
			[3 marks]
			Total [6 marks]

7	**a**	Attempt to find three simultaneous equations	M1
		$\begin{cases} 512a + 64b + 8c = 1890 \\ 1000a + 100b + 10c = 1690 \\ 3375a + 225b + 15c = 703 \end{cases}$	
		All three equations correct	A1
		$a = 1.31$, $b = -57.3$, $c = 610$	A1
			[3 marks]
	b	Attempt to solve $ax^3 + bx^2 + cx = 1720$	M1
		4.6 cm, 9.7 cm or 29.4 cm	A1
			[2 marks]
	c	Find $V(20)$	M1
		$V = -240$; No, model predicts negative volume	A1
			[2 marks]
			Total [7 marks]
8	**a**	$A(1, 0)$, $B(4, 0)$	A1A1
			[2 marks]
	b	Maximum point marked on a sketch	(M1)
		1.14 m	A1
			[2 marks]
	c	$\int_1^4 0.8x(4 - x)\,dx$ (condone lack of limits)	M1
		$= 2.27\text{ m}^2$	A1
			[2 marks]
			Total [6 marks]
9	**a**	Using TVM solver: $PV = 50\,000$, $PMT = -1000$, $I = 2.4$, P/Y = C/Y = 12	
		[to get $N = 52.73$]	M1
		53 months (4 years and 5 months)	A1
			[2 marks]
	b	Change N to 48 and find PMT	M1
		$1093.51	A1
			[2 marks]
	c	In part **a**: Amount left after 52 payments of $1000	M1
		($FV = 731.56$)	
		Total paid = $52 731.56	A1
		In part **b**: Total paid = 48 × 1093.51 = $52 488.48, which is less	A1
			[3 marks]
			Total [7 marks]
10		$\frac{dy}{dx} = 6x^2 - 2ax + 1$	(M1)
		$24 - 4a + 1 = 0$	M1
		$a = \frac{25}{4}$	A1
		$-6 = 16 - 4\left(\text{their } \frac{25}{4}\right) + 2 + 2b$	M1A1
		$b = 0.5$	A1
			Total [6 marks]
11	**a**	increases: 8, 10, 7, 8, 7	(M1)
		average = 8	A1
			[2 marks]
	b	26 + 11 × "8"	M1
		114	A1
			[2 marks]
	c	$y = 8.06x + 18.5$	M1
		115	A1
			[2 marks]
			Total [6 marks]

12 a 20 and 30 — A1

[1 mark]

b Any one of 31, 32, 33, 34 — A1

The gradient is zero somewhere between 30 and 35 — R1

[2 marks]

c Minimum at (v, 4.2) where $v \in \{31, 32, 33, 34\}$ — A1

Decreases from 4.6 to 4.2, then increases — A1

[2 marks]

Total [5 marks]

13 a $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$ — A1

[1 mark]

b

A	B	C	D	E	F	G	H
–1.1	0.7	–1.1	–3.0	2.0	–1.3	–1.5	–2.8

M1A1

[2 marks]

c $\bar{x} = -1.01$, $t = -1.72$ (evidence of using t-test) — (M1)

$p = 0.130 > 0.05$ — A1

Insufficient evidence that the means are different — R1

[3 marks]

Total [6 marks]

14 $P(A \cap B) = P(A|B)P(B) \left[= \frac{2}{3}P(A) \right]$ — M1

Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — M1

$\frac{1}{5} = P(A) + \frac{1}{6} - \frac{2}{3}P(A)$ — A2

$P(A) = \frac{1}{10}$ — (A1)

$P(A \cap B) = \frac{1}{15}$ — A1

Total [6 marks]

Alternative:

Draw a Venn diagram with $\frac{1}{30}, x, \frac{1}{6} - x$ — M1A1

$\frac{x}{1/6} = 4\left(x + \frac{1}{30}\right)$ — M1A1

$x = \frac{1}{15}$ — M1A1

Practice Set C: Paper 2 Mark scheme

1 **a** Means 58.9 and 45.8 A1
Point added on the diagram A1
[2 marks]

b Line of best fit through the means A1
[1 mark]

c 0.969 A1
[1 mark]

d Attempt the correct line M1
$y = 0.833x - 3.28$ A1
[2 marks]

e **i** Attempt to use the line to find y from x M1
Student J: 44 marks A1
Student K: 16 marks A1
ii Student J: reliable as strong correlation R1
Student K: not reliable, as extrapolation R1
[5 marks]

f The correlation does not seem to be linear R1
[1 mark]

g **i** Time ranks correct A1
Paper 1 ranks correct A1

Student	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Revision time rank	1	5	7	3	2	6	4	8
Paper 1 rank	1	5	8	3	2	6	4	7

ii $r_S = 0.976$ A1
iii H_0: There is no correlation between the revision time and the marks
H_1: There is a positive correlation
[both correct] A1
0.976 > 0.643, so there is evidence of positive correlation between the revision time and the marks A1
[5 marks]
Total [17 marks]

2 **a** $5^2 + 5^2 + 5^2$ [=75] M1
8.66 cm A1
[2 marks]

b $\sin^{-1}\left(\frac{5}{8.66}\right)$ or $\tan^{-1}\left(\frac{5}{\sqrt{50}}\right)$ M1
35.3° A1
[2 marks]

c $\pi r^2 h = 125$ M1
$h = \frac{125}{\pi r^2}$ A1
SA $= 2\pi rh + 2\pi r^2$, replace h by $\frac{125}{\pi r^2}$ M1
Simplify $2\pi r \times \frac{125}{\pi r^2}$ to $\frac{250}{r}$ A1
[4 marks]

d Graph of $y = \frac{250}{x} + 2\pi x^2$ M1
Minimum value is 138 A1
The surface area of the cylinder is smaller (138 versus 150) A1
[3 marks]

e $r = 2.71$, $h = \frac{125}{\pi r^2} = 5.42$ A1
$\tan\theta = \frac{5.42}{2 \times 2.71}$ M1
$\theta = 45.0°$ A1
[3 marks]
Total [14 marks]

3	**a**	$[4 - x^2 = 4 - x]$ or use GDC	(M1)
		$A(0, 4), B(1, 3)$	A1
		$(0.5, 3.5)$	A1
			[3 marks]
	b	$\frac{dy}{dx} = -2x$	A1
		$= -1$	M1
		$x = \frac{1}{2}$	A1
			[3 marks]
	c	$y = 4 - \left(\frac{1}{2}\right)^2 = \frac{15}{4}$	M1
		$k - \frac{1}{2} = \frac{15}{4}$	(M1)
		$k = \frac{17}{4}$	A1
			[3 marks]
	d	(their y_D) – (their y_C)	M1
		$\frac{1}{4}$	A1
			[2 marks]
			Total [11 marks]
4	**a**	0.927	A1
			[1 mark]
	b	$\frac{P(X < 8.3)}{\text{answer } \mathbf{a}}$	M1
		0.931	A1
			[2 marks]
	c	20 × answer **a**	(M1)
		18.5	A1
			[2 marks]
	d	Using B(20, answer **a**)	M1
		$1 - P(X \leq 17)$	M1
		0.824	A1
			[3 marks]
	e	Using answer **d**	(M1)
		$2 \times 0.824 \times (1 - 0.824)$	M1
		0.290	A1
			[3 marks]
			Total [11 marks]
5	**a**	B	A1
			[1 mark]
	b	$x = 6, y = 3$	A1A1
			[2 marks]
	c	Gradient of $BD = -\frac{5}{3}$	A1
		Midpoint $= \left(\frac{15}{2}, \frac{7}{2}\right)$	A1
		Equation: $y - \frac{7}{2} = \frac{3}{5}\left(x - \frac{15}{2}\right)$	M1
		$3x - 5y = 5$	A1
			[4 marks]
	d	Intersect $3x - 5y = 5$ with $x = 6$ and with $y = 3$	M1
		$P\left(6, \frac{13}{5}\right), Q\left(\frac{20}{3}, 3\right)$	A1A1
			[3 marks]

	e	Attempt to find distances from P and Q to one of B or D.	M1
		$PB = 6 - \frac{13}{5} = 3.4$	A1
		$QB = \sqrt{\left(\frac{20}{3} - 6\right)^2 + (3-6)^2} = 4.01$	A1
		The post office should be built at Q	A1
		Because $QB > PB$	R1
			[5 marks]
			Total [15 marks]
6	**a**	Using $T - B$ halves every 3 minutes	
		When $t = 0$: $93 - B = A$	A1
		When $t = 9$: $T - B = \frac{1}{8}A$	M1
		$T - B = \frac{1}{8}(93 - B)$	M1
		Rearranges correctly to $T = \frac{93 + 7B}{8}$	A1AG
			[4 marks]
	b	Using $t = 9$ and $t = 0$: $\frac{1}{8}A = A \times 10^{-9k}$	M1
		$\frac{1}{8} = 10^{-9k}$	A1
		$10^{9k} = 8 \Rightarrow 10^{3k} = 2$	A1AG
			[3 marks]
	c	When $t = 9$: $30 = \frac{93 + 7B}{8}$	
		When $t = 0$: $93 = B + A$	M1
		$A = 72, B = 21$	A1
		$10^{3k} = 2$ so $k = \frac{1}{3}\log 2$ (=0.1003)	A1
		Attempt to solve $24 = 21 + 72 \times 10^{-kt}$ with $k = \frac{1}{3}\log 2$	M1
		$t = 13.75$, so another 4.75 minutes	A1
			[5 marks]
			Total [12 marks]