Quadratics - review (TL) [116 marks]

1. [Maximum mark: 5] EXN.2.SL.TZ0.5

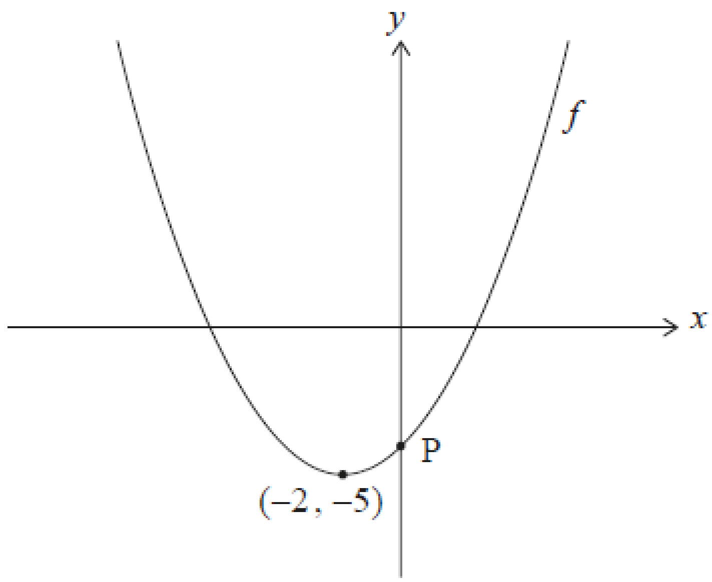
The quadratic equation $(k-1)x^2 + 2x + (2k-3) = 0$, where $k \in \mathbb{R}$, has real distinct roots.

Find the range of possible values for k. [5]

2. [Maximum mark: 16] 23M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2,\ -5)$ and the y-intercept is at point P.



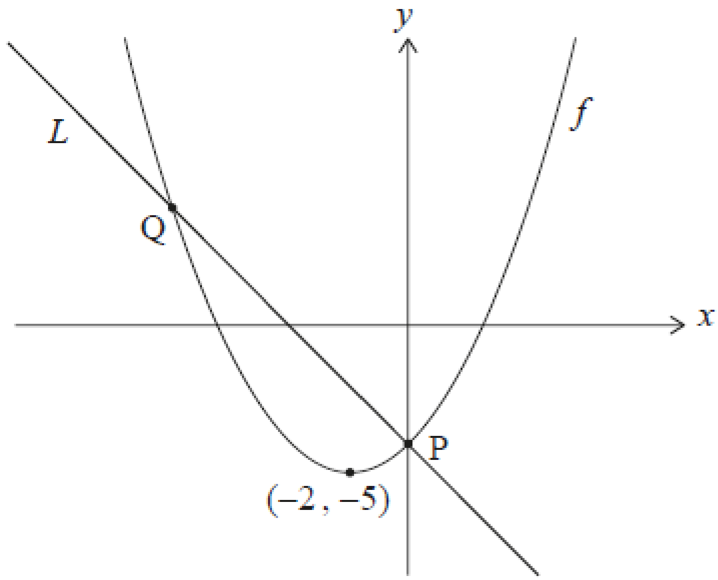
(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f\left(x\right) = \frac{1}{4}(x - h)^2 + k$, where h, $k \in \mathbb{Z}$.

(b) Write down the values of h and k. [2]

(c) Find the y-coordinate of P. [2]

In the following diagram, the line L is normal to the graph of f at point P.



(d) Find the equation of the line L, in the form $y = ax + b$. [3]

The line L intersects the graph of f at a second point, Q, as shown above.

(e) Calculate the distance between P and Q. [8]

3. [Maximum mark: 6] 23M.1.AHL.TZ1.4

Find the range of possible values of k such that $\mathrm{e}^{2x} + \ln k = 3\mathrm{e}^{x}$ has at least one real solution. [6]

4. [Maximum mark: 7] 23M.1.AHL.TZ2.5

The functions f and g are defined for $x \in \mathbb{R}$ by

$f(x) = ax + b$, where $a, b \in \mathbb{Z}$

$g(x) = x^2 + x + 3$.

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

[7]

5. [Maximum mark: 16] 22N.1.SL.TZ0.7

(a) The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f\left(x\right) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$.

In the case where $g(-3) = g(1) = 4$,

(b.i) find the value of p and the value of t. [4]

(b.ii) find the range of g. [3]

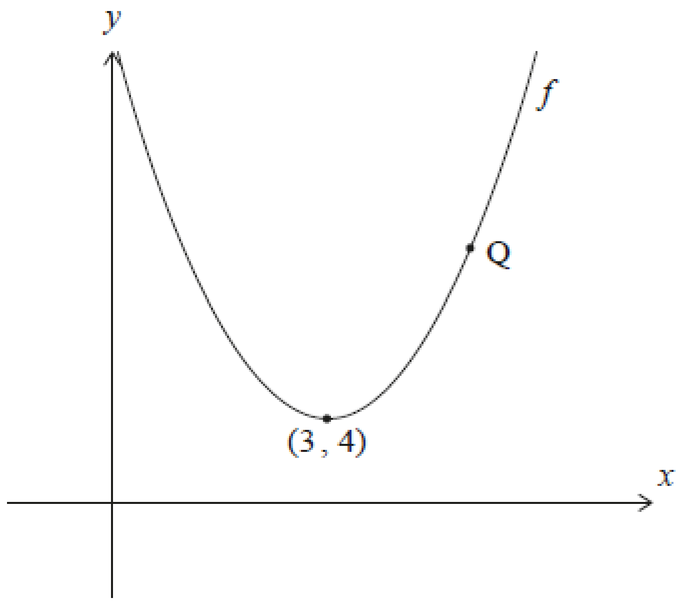
(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R},\ p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

6. [Maximum mark: 15] 22M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3,\ 4)$, and it passes through point Q as shown.



(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f\left(x\right) = a(x-h)^2 + k$.

(b.i) Write down the values of h and k. [2]

(b.ii) Point Q has coordinates $(5,\ 12)$. Find the value of a. [2]

The line L is tangent to the graph of f at Q.

(c) Find the equation of L. [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g\prime(x) = f(x) - d$, where $d \in \mathbb{R}$.

(d) Find the values of d for which g is an increasing function. [3]

(e) Find the values of x for which the graph of g is concave-up. [3]

7. [Maximum mark: 4] 22M.2.AHL.TZ1.8

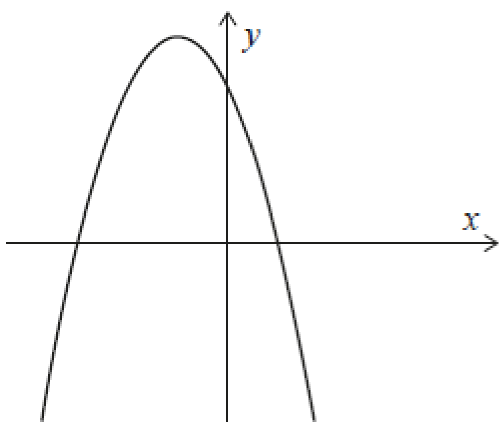
Consider the equation $kx^2 - (k+3)x + 2k + 9 = 0$, where $k \in \mathbb{R}$.

(a) Write down an expression for the product of the roots, in terms of k. [1]

(b) Hence or otherwise, determine the values of k such that the equation has one positive and one negative real root. [3]

8. [Maximum mark: 7] 21N.1.SL.TZ0.1

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



For the graph of f

(a.i) find the x-coordinates of the x-intercepts. [2]

(a.ii) find the coordinates of the vertex. [3]

(b) The function f can be written in the form $f(x) = -2(x-h)^2 + k$.

Write down the value of h and the value of k. [2]

9. [Maximum mark: 7] 21N.1.AHL.TZ0.7

The equation $3px^2 + 2px + 1 = p$ has two real, distinct roots.

(a) Find the possible values for p. [5]

(b) Consider the case when $p = 4$. The roots of the equation can be expressed in the form $x = \frac{a \pm \sqrt{13}}{6}$, where $a \in \mathbb{Z}$. Find the value of a. [2]

10. [Maximum mark: 14] 21M.1.SL.TZ1.7

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p,\ q \in \mathbb{R}$.

(b) Find the value of p and the value of q. [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h,\ k \in \mathbb{R}$.

(c) Find the value of h and the value of k. [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

11. [Maximum mark: 6] 21M.2.SL.TZ2.5

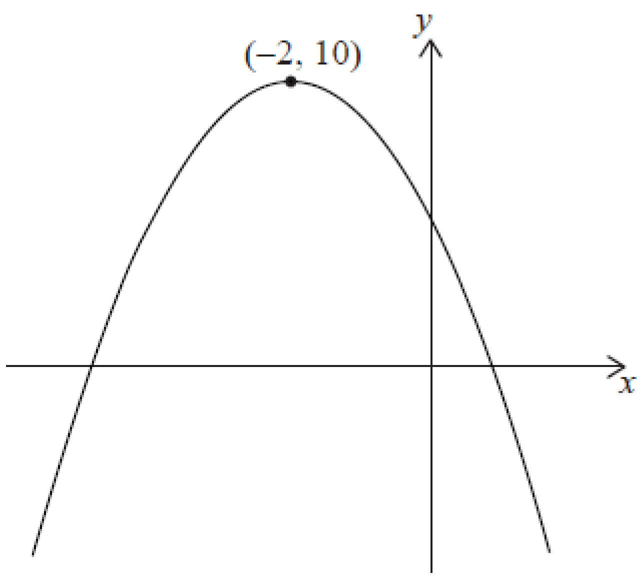
The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f. [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. [4]

12. [Maximum mark: 6] 20N.1.SL.TZ0.T_11

The diagram shows the graph of the quadratic function $f(x) = ax^2 + bx + c$, with vertex $(-2,\ 10)$.



The equation $f(x) = k$ has two solutions. One of these solutions is $x = 2$.

(a) Write down the other solution of $f(x) = k$. [2]

(b) Complete the table below placing a tick (✔) to show whether the unknown parameters a and b are positive, zero or negative. The row for c has been completed as an example.

	positive	zero	negative
a			
b			
c	✓		

[2]

(c) State the values of x for which $f(x)$ is decreasing. [2]

13. [Maximum mark: 7] 19N.1.SL.TZ0.S_3

Let $g\left(x\right)=x^2+bx+11$. The point $\left(-1,\ 8\right)$ lies on the graph of g.

(a) Find the value of b. [3]

(b) The graph of $f\left(x\right)=x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]