Sequences (AI) + Finances - review (TL) [188 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.2

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

2. [Maximum mark: 18] SPM.2.AHL.TZ0.3

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i) Find the repayment made each quarter. [3]

(a.ii) Find the total amount paid for the car. [2]

(a.iii) Find the interest paid on the loan. [2]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b.i) Find the amount to be borrowed for this option. [2]

(b.ii) Find the annual interest rate, r. [3]

(c) State which option Bryan should choose. Justify your answer. [2]

(d) Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

If they invest it in an account paying 0.4 % interest per month and inflation is 0.1 % per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

3. [Maximum mark: 7] EXN.1.SL.TZ0.9

Sophia pays $\$200$ into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

(a) Find the value of her investment after a period of 5 years. [3]

The average rate of inflation per year over the 5 years was 2%.

(b) Find an approximation for the real interest rate for the money invested in the account. [2]

(c) Hence find the real value of Sophia's investment at the end of 5 years. [2]

4. [Maximum mark: 7] EXN.1.SL.TZ0.12

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.

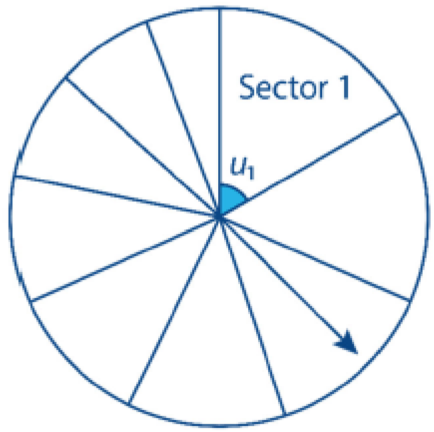


Diagram not to scale

(a) Write down the value of $\sum_{i=1}^{9} u_i$. [1]

It is given that $u_9 = \frac{1}{3}u_1$.

(b) Find the value of u_1. [4]

(c) A game is played in which the arrow attached to the centre of the disc is spun and the sector in which the arrow stops is noted. If the arrow stops in sector 1 the player wins 10 points, otherwise they lose 2 points.

Let X be the number of points won

Find $\mathrm{E}(X)$. [2]

5. [Maximum mark: 5] EXM.1.SL.TZ0.1

Give your answers to this question correct to two decimal places.

Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

(a) Calculate the value of her savings after 10 years. [2]

(b) The rate of inflation during this 10 year period is 1.5% per year.

Calculate the real value of her savings after 10 years. [3]

6. [Maximum mark: 6] EXM.1.SL.TZ0.6

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of \$4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

(a) Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. [3]

(b) Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund.

Calculate the amount Yejin needs to save each month, to meet her retirement goal. [3]

7. [Maximum mark: 15] EXM.2.SL.TZ0.2

Sophie is planning to buy a house. She needs to take out a mortgage for \$120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay \$1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

(a.i) Calculate the monthly repayment using option 1. [2]

(a.ii) Calculate the total amount Sophie would pay, using option 1. [2]

(b.i) Calculate the number of months it will take to repay the mortgage using option 2. [3]

(b.ii) Calculate the total amount Sophie would pay, using option 2. [2]

Give a reason why Sophie might choose

(c.i) option 1. [1]

(c.ii) option 2. [1]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

(d.i) Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years. [2]

(d.ii) Hence calculate her monthly repayment for the final 10 years. [2]

8. [Maximum mark: 15] 23M.2.SL.TZ2.2

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12 th day. [3]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

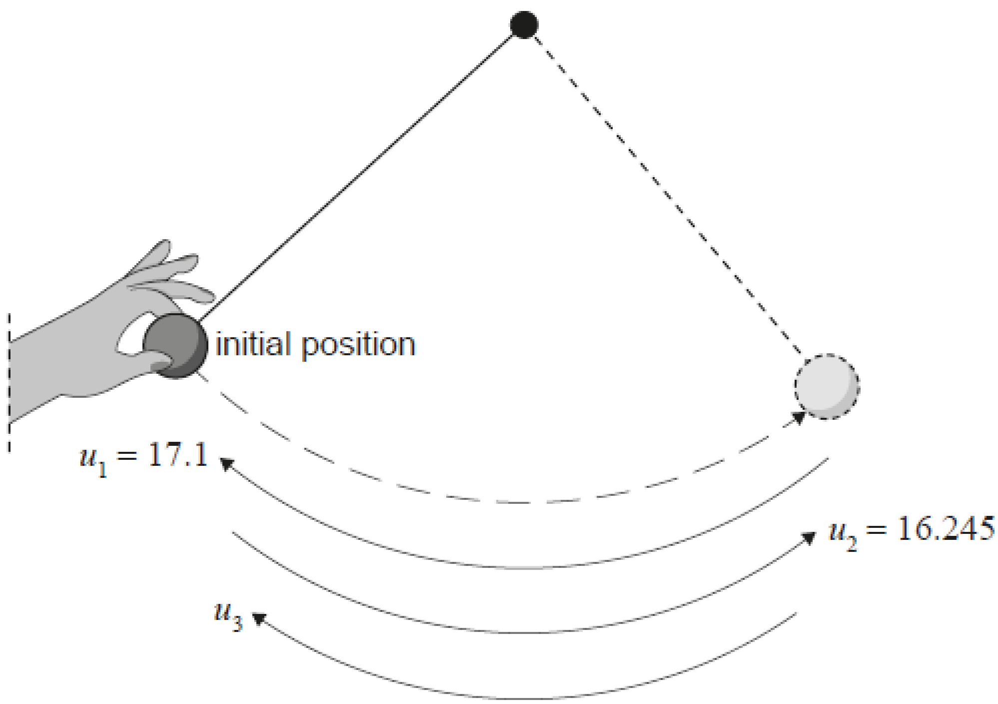
(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

(d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

9. [Maximum mark: 7] 22N.1.SL.TZ0.2

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

(a) Find the number of trees to be planted in the 15th month. [3]

(b) Find the total number of trees to be planted in the first 15 months. [2]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

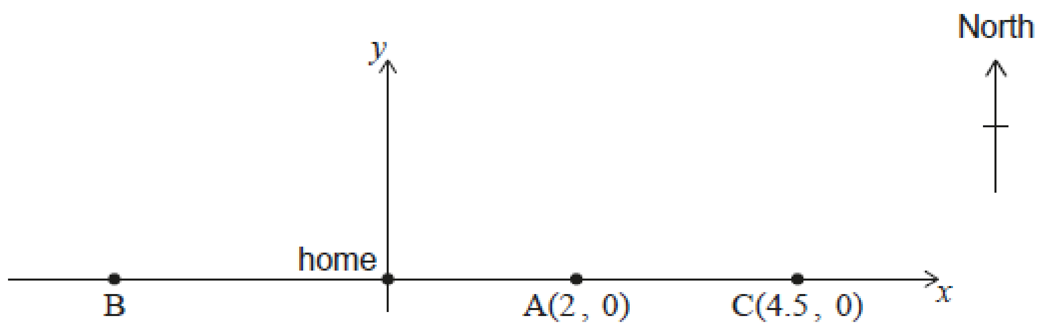
10. [Maximum mark: 7] 22N.1.SL.TZ0.11

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

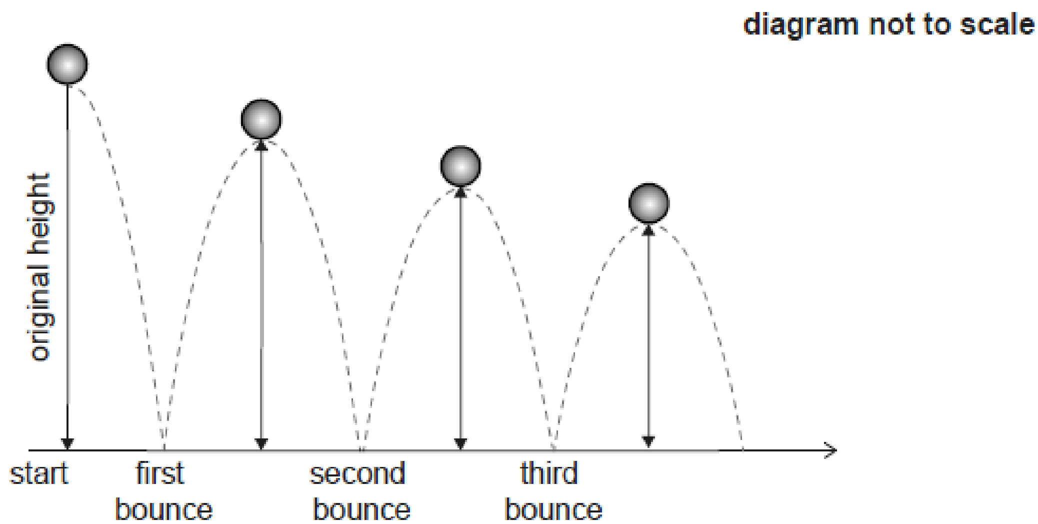
On the $6\mathrm{th}$ day, Kristi runs to point F.

(b) Find the location of point F. [2]

(c) Find the total distance Kristi runs during the first 7 days of training. [3]

11. [Maximum mark: 7] 22M.1.SL.TZ1.13

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

12. [Maximum mark: 5] 22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

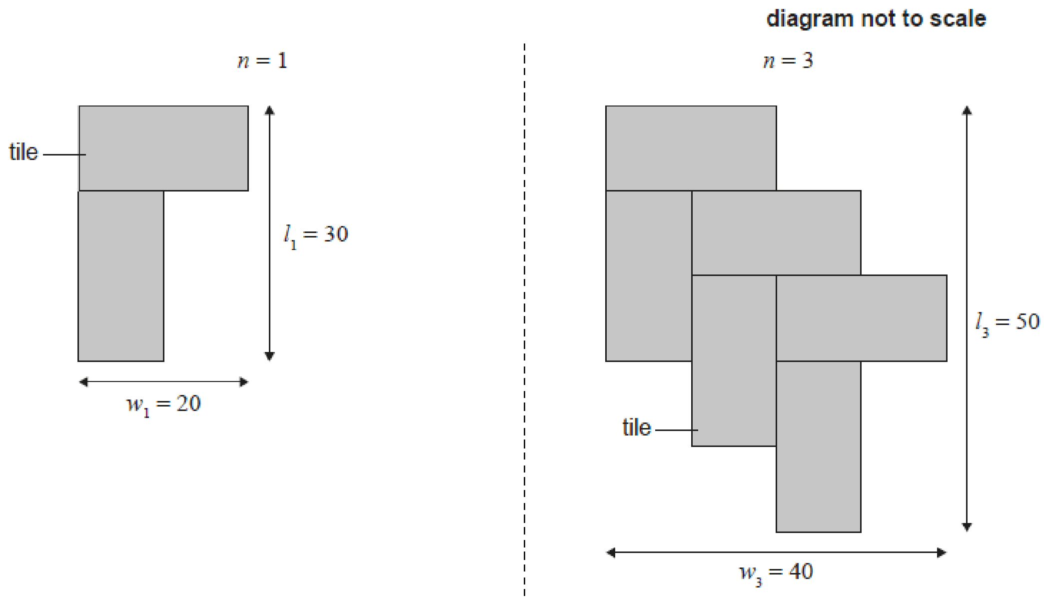
Find the third term. Justify your answer. [5]

13. [Maximum mark: 19] 22M.2.SL.TZ1.2

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\,\text{cm}$ wide and $20\,\text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

(a.i) a. [1]

(a.ii) b. [1]

Write down an expression in terms of n for

(b.i) w_n. [2]

(b.ii) l_n. [1]

Eddie's lawn has a length $740\,\text{cm}$.

(c.i) Show that Eddie needs 144 tiles. [2]

(c.ii) Find the value of w_n for this path. [1]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer. [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

14. [Maximum mark: 13] 22M.2.SL.TZ2.2

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

(a.i) fed to the dog per day. [3]

(a.ii) remaining in the bag at the end of the first day. [1]

(b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

(c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

(d.i) Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$. [1]

(d.ii) Describe what the value in part (d)(i) represents in this context. [2]

(e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

15. [Maximum mark: 5] 21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

16. [Maximum mark: 16] 21N.2.SL.TZ0.2

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b.i) Write down the common ratio of the sequence. [1]

(b.ii) Find an expression for u_n. [1]

(b.iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [2]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n . [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

17. [Maximum mark: 8] 21M.1.SL.TZ1.8

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\mathrm{m}$. On each subsequent day, Charlie ran $100\,\mathrm{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

Calculate how far

(a.i) Charlie ran on day 20 of his fitness programme. [2]

(a.ii) Daniella ran on day 20 of her fitness programme. [3]

(b) On day n of the fitness programmes Daniella runs more than Charlie for the first time.

Find the value of n. [3]

18. [Maximum mark: 6] 21M.1.AHL.TZ2.7

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

19. [Maximum mark: 16] 21M.2.SL.TZ2.3

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

Find:

(a.i) the number of seats in the last row. [3]

(a.ii) the total number of seats in the concert hall. [2]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020. [2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. [5]

(d) It is assumed that the concert hall will host 50 concerts each year.

Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025. [4]