Sequences review - exam questions (TL) [149 marks]

1. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1+u_3}{2}$ **(M1)**

$$5u_1 - 8 = \frac{u_1+(3u_1+8)}{2}$$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

[4 marks]

2. [Maximum mark: 12] EXN.2.SL.TZ0.7

Helen and Jane both commence new jobs each starting on an annual salary of $\$70,000$. At the start of each new year, Helen receives an annual salary increase of $\$2400$.

Let $\$H_n$ represent Helen's annual salary at the start of her nth year of employment.

(a) Show that $H_n = 2400n + 67\,600$. [2]

Markscheme

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uses $H_n = H_1 + (n-1)d$ with $H_1 = 70\,000$ and $d = 2400$ **(M1)**

$H_n = 70\,000 + 2400(n-1)$ **A1**

so $H_n = 2400n + 67\,600$ **AG**

[2 marks]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her nth year of employment is given by $J_n = 70\,000(1.03)^{n-1}$.

(b) Given that J_n follows a geometric sequence, state the value of the common ratio, r. [1]

Markscheme

$r = 1.03$ **A1**

[1 mark]

At the start of year N, Jane's annual salary exceeds Helen's annual salary for the first time.

(c.i) Find the value of N. [3]

Markscheme

evidence of use of an appropriate table or graph or GDC numerical solve feature to find the value of N such that $J_n > H_n$ **(M1)**

EITHER

for example, an excerpt from an appropriate table

N	H_n	J_n
11	94 000	94 074

(A1)

OR

for example, use of a GDC numerical solve feature to obtain $N = 10.800\ldots$ **(A1)**

Note: Award **A1** for an appropriate graph. Condone use of a continuous graph.

THEN

$N = 11$ **A1**

[3 marks]

(c.ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

Markscheme

$H_{11} = 94\,000\,(\$)$ **A1**

$J_{11} = 94\,074\,(\$)$ **A1**

Helen's annual salary is $\$94\,000$ and Jane's annual salary is $\$94\,074$

Note: Award **A1** for a correct H_{11} value and **A1** for a correct J_{11} value seen in part (c) (i).

[2 marks]

(d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

Markscheme

at the start of the 10th year, Jane will have worked for 9 years so the value of S_9 is required **R1**

Note: Award **R1** if S_9 is seen anywhere.

uses $S_n = \frac{J_1(r^n - 1)}{r-1}$ with $J_1 = 70\,000, r = 1.03$ and $n = 9$
(M1)

Note: Award **M1** if $n = 10$ is used.

$$S_9 = \frac{70\,000\left((1.03)^9 - 1\right)}{1.03 - 1} = 711\,137.42\ldots \quad \textbf{(A1)}$$

$$= 711\,137\,(\$)$$

Jane's total earnings are $\$711\,137$ (correct to the nearest dollar)

[4 marks]

3. [Maximum mark: 14] 23M.1.AHL.TZ1.10

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

Markscheme

recognition that $n = 5$ ***(M1)***

$S_5 = 45$ ***A1***

[2 marks]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

Markscheme

METHOD 1

recognition that $S_5 + u_6 = S_6$ ***(M1)***

$u_6 = 15$ ***A1***

METHOD 2

recognition that $60 = \frac{6}{2}\left(S_1 + u_6\right)$ ***(M1)***

$60 = 3(5 + u_6)$

$u_6 = 15$ ***A1***

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ ***(M1)***

$u_6 = 15$ ***A1***

[2 marks]

(b) Find u_1. [2]

Markscheme

recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)***

OR equations for S_5 and S_6 in terms of u_1 and d

$1 + 4$ OR $60 = \frac{6}{2}(U_1 + 15)$

$u_1 = 5$ ***A1***

[2 marks]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Markscheme

EITHER

valid attempt to find d (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

OR

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - \left(n^2 - 2n + 1 + 4n - 4\right)$ ***(A1)***

OR

equating $n^2 + 4n = \frac{n}{2}\left(5 + u_n\right)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

THEN

$u_n = 5 + 2(n - 1)$ OR $u_n = 2n + 3$ ***A1***

[3 marks]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

Markscheme

recognition that $v_2 r^2 = v_4$ OR $\left(v_3\right)^2 = v_2 \times v_4$ ***(M1)***

$r^2 = 3$ OR $v_3 = \left(\pm\right)5\sqrt{3}$ ***(A1)***

$r = \pm\sqrt{3}$ ***A1***

Note: If no working shown, award ***M1A1A0*** for $\sqrt{3}$.

[3 marks]

(e) Given that $v_{99} < 0$, find v_5. [2]

Markscheme

recognition that r is negative ***(M1)***

$v_5 = -15\sqrt{3}\ \left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

[2 marks]

4. [Maximum mark: 5] 22N.2.SL.TZ0.4

geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$. [5]

Markscheme

$86.4 = 50r^3$ ***(A1)***

$r = 1.2\left(= \sqrt[3]{\frac{86.4}{50}}\right)$ seen anywhere ***(A1)***

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ ***(A1)***

attempt to solve their geometric S_n inequality or equation ***(M1)***

sketch OR $n > 26.9045,\ n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic manipulation involving logarithms

$n = 27$ accept $n \geq 27$ ***A1***

[5 marks]

5. [Maximum mark: 18] 22M.1.AHL.TZ1.10

Consider the series $\ln x + p\ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R},\ x > 1$ and $p \in \mathbb{R},\ p \neq 0$.

Consider the case where the series is geometric.

(a.i) Show that $p = \pm\frac{1}{\sqrt{3}}$. [2]

Markscheme

EITHER

attempt to use a ratio from consecutive terms ***M1***

$\frac{p\ln x}{\ln x} = \frac{\frac{1}{3}\ln x}{p\ln x}$ OR $\frac{1}{3}\ln x = (\ln x)r^2$ OR $p\ln x = \ln x\left(\frac{1}{3p}\right)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in geometric sequence

Award ***M1*** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR

$r = p$ and $r^2 = \frac{1}{3}$ ***M1***

THEN

$p^2 = \frac{1}{3}$ OR $r = \pm\frac{1}{\sqrt{3}}$ ***A1***

$p = \pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

[2 marks]

(a.ii) Hence or otherwise, show that the series is convergent. [1]

Markscheme

EITHER

since, $|p| = \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}} < 1$ ***R1***

OR

since, $|p| = \frac{1}{\sqrt{3}}$ and $-1 < p < 1$ ***R1***

THEN

$\Rightarrow$ the geometric series converges. ***AG***

Note: Accept r instead of p.
Award ***R0*** if both values of p not considered.

[1 mark]

(a.iii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [3]

Markscheme

$\frac{\ln x}{1-\frac{1}{\sqrt{3}}}$ $\left(= 3 + \sqrt{3}\right)$ ***(A1)***

$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR
$\ln x = 3 - \sqrt{3} + \sqrt{3} - 1 \ (\Rightarrow \ln x = 2)$ ***A1***

$x = \mathrm{e}^2$ ***A1***

[3 marks]

Now consider the case where the series is arithmetic with common difference d.

(b.i) Show that $p = \frac{2}{3}$. [3]

Markscheme

METHOD 1

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p \ln x - \ln x = \frac{1}{3}\ln x - p \ln x$ OR
$\frac{1}{3}\ln x = \ln x + 2(p \ln x - \ln x)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in arithmetic sequence.

Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$

$2p \ln x = \frac{4}{3}\ln x \ \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p \ln x = \frac{\ln x + \frac{1}{3} \ln x}{2}$ ***A1***

$2p \ln x = \frac{4}{3} \ln x \ \left(\Rightarrow 2p = \frac{4}{3} \right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 3

attempt to find difference using u_3 ***M1***

$\frac{1}{3} \ln x = \ln x + 2d \ \left(\Rightarrow d = -\frac{1}{3} \ln x \right)$

$u_2 = \ln x + \frac{1}{2} \left(\frac{1}{3} \ln x - \ln x \right)$ OR $p \ln x - \ln x = -\frac{1}{3} \ln x$
A1

$p \ln x = \frac{2}{3} \ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

[3 marks]

(b.ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$. [1]

Markscheme

$d = -\frac{1}{3}\ln x$ ***A1***

[1 mark]

(b.iii) The sum of the first n terms of the series is $\ln\left(\frac{1}{x^3}\right)$.

Find the value of n. [8]

Markscheme

METHOD 1

$S_n = \frac{n}{2}\left\lfloor 2\ln x + (n-1) \times \left(-\frac{1}{3}\ln x\right)\right\rfloor$

attempt to substitute into S_n and equate to $\ln\left(\frac{1}{x^3}\right)$ ***(M1)***

$\frac{n}{2}\left\lfloor 2\ln x + (n-1) \times \left(-\frac{1}{3}\ln x\right)\right\rfloor = \ln\left(\frac{1}{x^3}\right)$

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3\ln x$ ***(A1)***

correct working with S_n (seen anywhere) ***(A1)***

$\frac{n}{2}\left\lfloor 2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right\rfloor$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR $\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ or equivalent

Note: Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$n^2 - 7n - 18 = 0$

attempt to solve their quadratic ***(M1)***

$(n - 9)(n + 2) = 0$

$n = 9$ ***A1***

METHOD 2

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3 \ln x$ ***(A1)***

listing the first 7 terms of the sequence ***(A1)***

$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \ldots$

recognizing first 7 terms sum to 0 ***M1***

8^{th} term is $-\frac{4}{3}\ln x$ ***(A1)***

9^{th} term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8^{th} and 9^{th} term $= -3 \ln x$ ***(A1)***

$n = 9$ ***A1***

[8 marks]

6. [Maximum mark: 9] 21N.2.SL.TZ0.6

The sum of the first n terms of a geometric sequence is given by

$S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

Markscheme

$u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ ***(M1)***

$= \frac{14}{24} \left(= \frac{7}{12} = 0.583333\ldots\right)$ ***A1***

[2 marks]

(b) Find S_∞. [3]

Markscheme

$r = \frac{7}{8} (= 0.875)$ ***(A1)***

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ ***(M1)***

$= \frac{14}{3} (= 4.66666\ldots)$ ***A1***

[3 marks]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

Markscheme

attempt to substitute their values into the inequality or formula for S_n ***(M1)***

$\frac{14}{3} - \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r < 0.001$ OR $S_n = \frac{\frac{7}{12}\left(1-\left(\frac{7}{8}\right)^n\right)}{\left(1-\frac{7}{8}\right)}$

attempt to solve their inequality using a table, graph or logarithms

(must be exponential) ***(M1)***

Note: Award ***(M0)*** if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value ***(A1)***

$63.2675\ldots$ OR $S_\infty - S_{63} = 0.001036\ldots$ OR
$S_\infty - S_{64} = 0.000906\ldots$

OR $S_\infty - S_{63} - 0.001 = 0.0000363683\ldots$ OR
$S_\infty - S_{64} - 0.001 = 0.0000931777\ldots$

least value is $n = 64$ ***A1***

[4 marks]

7. [Maximum mark: 5] 21M.1.SL.TZ1.3

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

Markscheme

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6,\ d = 2$ ***A1A1***

[5 marks]

8. [Maximum mark: 5] 21M.2.SL.TZ2.3

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Markscheme

attempt to use $u_1 + (n - 1)d = 0$ ***(M1)***

$60 - 2.5(k - 1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]

Markscheme

METHOD 1

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0),\ S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]

9. [Maximum mark: 6] 20N.1.SL.TZ0.T_15

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of $30\,\text{cm}^3$. The fifth smallest slice has a volume of $240\,\text{cm}^3$.

(a) Find the common ratio of the sequence. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$u_1r = 30$ **and** $u_1r^4 = 240,$ ***(M1)***

Note: Award ***(M1)*** for both the given terms expressed in the formula for u_n.

OR

$30r^3 = 240 \quad (r^3 = 8)$ ***(M1)***

Note: Award ***(M1)*** for a correct equation seen.

$(r =)\ 2$ ***(A1) (C2)***

[2 marks]

(b) Find the volume of the smallest slice of pie. [2]

Markscheme

$u_1 \times 2 = 30$ **OR** $u_1 \times 2^4 = 240$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in geometric sequence formula.

$(u_1 =)\ 15$ ***(A1)*****(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

(c) The apple pie has a volume of $61\,425\,\text{cm}^3$.

Find the total number of slices Mia can cut from this pie. [2]

Markscheme

$\frac{15(2^n-1)}{2-1} = 61425$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted geometric series formula equated to 61425.

$(n =)\ 12$ (slices) ***(A1)*****(ft)** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

10. [Maximum mark: 5] 20N.1.AHL.TZ0.H_5

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$. [5]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$u_5 = 4 + 4d = \log_2 625$ ***(A1)***

$4d = \log_2 625 - 4$

attempt to write an integer (*eg* 4 or 1) in terms of $\log_2$ ***M1***

$4d = \log_2 625 - \log_2 16$

attempt to combine two logs into one ***M1***

$4d = \log_2 \left(\frac{625}{16}\right)$

$d = \frac{1}{4}\log_2 \left(\frac{625}{16}\right)$

attempt to use power rule for logs ***M1***

$d = \log_2 \left(\frac{625}{16}\right)^{\frac{1}{4}}$

$d = \log_2 \left(\frac{5}{2}\right)$ ***A1***

[5 marks]

Note: Award method marks in any order.

11. [Maximum mark: 8] 20N.2.SL.TZ0.S_6

An infinite geometric series has first term $u_1 = a$ and second term $u_2 = \frac{1}{4}a^2 - 3a$, where $a > 0$.

(a) Find the common ratio in terms of a. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of dividing terms (in any order) ***(M1)***

eg $\frac{u_1}{u_2}$, $\frac{\frac{1}{4}a^2-3a}{a}$

$r = \frac{1}{4}a - 3$ ***A1 N2***

[2 marks]

(b) Find the values of a for which the sum to infinity of the series exists. [3]

Markscheme

recognizing $|r| < 1$ (must be in terms of a) ***(M1)***

eg

$\left|\frac{1}{4}a - 3\right| < 1, \quad -1 \leq \frac{1}{4}a - 3 \leq 1, \quad -4 < a - 12 < 4$

$8 < a < 16$ ***A2 N3***

[3 marks]

(c) Find the value of a when $S_\infty = 76$. [3]

Markscheme

correct equation ***(A1)***

eg $\frac{a}{1-\left(\frac{1}{4}a-3\right)} = 76,\ a = 76\left(4 - \frac{1}{4}a\right)$

$a = \frac{76}{5}\ (= 15.2)$ (exact) ***A2 N3***

[3 marks]

12. [Maximum mark: 6] 19N.1.SL.TZ0.S_1

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

Markscheme

valid approach ***(M1)***

eg $11 - 5, 11 = 5 + d$

$d = 6$ ***A1 N2***

[2 marks]

(b) Find the first term. [2]

Markscheme

valid approach ***(M1)***

eg $u_2 - d$, $5 - 6$, $u_1 + (3 - 1)(6) = 11$

$u_1 = -1$ ***A1 N2***

[2 marks]

(c) Find the sum of the first 20 terms. [2]

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6))$, $\frac{20}{2}(-1 + 113)$ ***(A1)***

$S_{20} = 1120$ ***A1 N2***

[2 marks]

13. [Maximum mark: 7] 19N.2.SL.TZ0.S_5

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

Markscheme

valid approach *(M1)*

eg $\frac{u_1}{u_2}$, $\frac{2.226}{2.1}$, $2.226 = 2.1r$

$r = 1.06$ (exact) ***A1 N2***

[2 marks]

(b) Find the value of u_{10}. [2]

Markscheme

correct substitution *(A1)*

eg 2.1×1.06^9

3.54790 ***A1 N2***

$u_{10} = 3.55$

[2 marks]

(c) Find the least value of n such that $S_n > 5543$. [3]

Markscheme

correct substitution into S_n formula *(A1)*

eg $\frac{2.1(1.06^n-1)}{1.06-1}$, $\frac{2.1(1.06^n-1)}{1.06-1} > 5543$,
$2.1\left(1.06^n - 1\right) = 332.58$, sketch of S_n and $y = 5543$

correct inequality for n **or** crossover values ***A1***

eg $n > 87.0316$, $S_{87} = 5532.73$ **and** $S_{88} = 5866.79$

$n = 88$ ***A1 N2***

[3 marks]

14. [Maximum mark: 5] 19N.2.AHL.TZ0.H_1

A geometric sequence has $u_4 = -70$ and $u_7 = 8.75$. Find the second term of the sequence. [5]

Markscheme

$u_1 r^3 = -70$, $u_1 r^6 = 8.75$ ***(M1)***

$r^3 = \frac{8.75}{-70} = -0.125$ ***(A1)***

$\Rightarrow r = -0.5$ ***(A1)***

valid attempt to find u_2 ***(M1)***

for example: $u_1 = \frac{-70}{-0.125} = 560$

$u_2 = 560 \times -0.5$

$= -280$ ***A1***

[5 marks]

15. [Maximum mark: 7] 19M.2.SL.TZ1.S_7

The first terms of an infinite geometric sequence, u_n, are 2, 6, 18, 54, …

The first terms of a second infinite geometric sequence, v_n, are 2, −6, 18, −54, …

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(a) Write down the first three **non-zero** terms of w_n. [3]

Markscheme

attempt to add corresponding terms ***(M1)***

eg $2+2,\ 6+(-6),\ 2(3)^{n-1} +\ 2(-3)^{n-1}$

correct value for w_5 ***(A1)***

eg 324

4, 36, 324 (accept 4 + 36 + 324) ***A1 N3***

[3 marks]

(b.i) Find the value of r. [2]

Markscheme

valid approach ***(M1)***

eg $4 \times r^1 = 36$, $4 \times 9^{n-1}$

$r = 9$ (accept $\sum_{k=0}^{m} 4 \times 9^k$; m may be incorrect) ***A1 N2***

[2 marks]

(b.ii) Find the value of m. [2]

Markscheme

recognition that 225 terms of w_n consists of 113 non-zero terms ***(M1)***

eg $\sum_{1}^{113}$, $\sum_{0}^{112}$, 113

$m = 112$ (accept $\sum_{k=0}^{1} 124 \times r^k$; r may be incorrect) ***A1 N2***

[2 marks]

16. [Maximum mark: 12] 19M.2.SL.TZ2.S_10

In an arithmetic sequence, $u_1 = 1.3$, $u_2 = 1.4$ and $u_k = 31.2$.

(b) Find the exact value of S_k. [2]

Markscheme

correct substitution *(A1)*

eg $\frac{300}{2}(1.3 + 31.2)$, $\frac{300}{2}[2(1.3) + (300 - 1)(0.1)]$, $\frac{300}{2}[2.6 + 299(0.1)]$

$S_k = 4875$ ***A1 N2***

[2 marks]

Consider the terms, u_n, of this sequence such that $n \leq k$.

Let F be the sum of the terms for which n is not a multiple of 3.

(c) Show that $F = 3240$. [5]

Markscheme

recognizing need to find the sequence of multiples of 3 (seen anywhere) *(M1)*

eg first term is u_3 (= 1.5) (accept notation $u_1 = 1.5$),

$d = 0.1 \times 3$ (= 0.3), 100 terms (accept $n = 100$), last term is 31.2

(accept notation $u_{100} = 31.2$), $u_3 + u_6 + u_9 + \ldots$ (accept $F = u_3 + u_6 + u_9 + \ldots$)

correct working for sum of sequence where n is a multiple of 3 ***A2***

$\frac{100}{2}(1.5 + 31.2)$, $50(2 \times 1.5 + 99 \times 0.3)$, 1635 valid approach (seen anywhere) ***(M1)***

eg $S_k - (u_3 + u_6 + \ldots)$, $S_k - \frac{100}{2}(1.5 + 31.2)$, S_k- (their sum for $(u_3 + u_6 + \ldots)$)

correct working (seen anywhere) ***A1***

eg $S_k - 1635$, 4875 – 1635

$F = 3240$ ***AG NO***

[5 marks]

(d) An infinite geometric series is given as $S_\infty = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \ldots, a \in \mathbb{Z}^+$.

Find the largest value of a such that $S_\infty < F$. [5]

Markscheme

attempt to find r ***(M1)***

eg dividing consecutive terms

correct value of r (seen anywhere, including in formula)

eg $\frac{1}{\sqrt{2}}$, 0.707106… , $\frac{a}{0.293\ldots}$

correct working (accept equation) ***(A1)***

eg $\frac{a}{1-\frac{1}{\sqrt{2}}} < 3240$

correct working ***A1***

METHOD 1 (analytical)

eg $3240 \times \left(1 - \frac{1}{\sqrt{2}}\right)$, $a < 948.974$, 948.974

METHOD 2 (using table, must find both S_∞ values)

eg when $a = 948$, $S_\infty = 3236.67\ldots$ **AND** when $a = 949$, $S_\infty = 3240.08\ldots$

$a = 948$ ***A1 N2***

[5 marks]

17. [Maximum mark: 6] 18M.1.SL.TZ1.T_7

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

(a.i) For that day find how much weight was added after each lift. [2]

Markscheme

$5d = 46 - 21$ **OR** $u_1 + 2d = 21$ and $u_1 + 7d = 46$ ***(M1)***

Note: Award ***(M1)*** for a correct equation in d or for two correct equations in u_1 and d.

$(d =)$ 5 (kg) ***(A1) (C2)***

[2 marks]

(a.ii) For that day find the weight of Sergei's first lift. [2]

Markscheme

$u_1 + 2 \times 5 = 21$ ***(M1)***

OR

$u_1 + 7 \times 5 = 46$ ***(M1)***

Note: Award ***(M1)*** for substitution of their d into either of the two equations.

$(u_1 =)$ 11 (kg) ***(A1)*(ft) *(C2)***

Note: Follow through from part (a)(i).

[2 marks]

(b) On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts. [2]

Markscheme

$\frac{12}{2}(2 \times 11 + (12 - 1) \times 5)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into arithmetic series formula.

= 462 (kg) ***(A1)*(ft) *(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

18. [Maximum mark: 6] 18M.2.SL.TZ2.T_4

A new café opened and during the first week their profit was $60.

The café's profit increases by $10 every week.

(b) Calculate the café's **total** profit for the first 12 weeks. [3]

Markscheme

$\frac{12}{2}(2 \times 60 + 11 \times 10)$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituting the arithmetic series formula, ***(A1)*****(ft)** for correct substitution. Follow through from their first term and common difference in part (a).

= ($) 1380 ***(A1)*****(ft)*****(G2)***

[3 marks]

A new tea-shop opened at the same time as the café. During the first week their profit was also $60.

The tea-shop's profit increases by 10 % every week.

(d) Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

Markscheme

$\frac{60(1.1^{12}-1)}{1.1-1}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituting the geometric series formula, ***(A1)*****(ft)** for correct substitution. Follow through from part (c) for their first term and common ratio.

$= (\$)1280 \ (1283.05\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 marks]

19. [Maximum mark: 7] 18M.2.SL.TZ2.S_4

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

correct substitution into infinite sum ***(A1)***

eg $200 = \frac{4}{1-r}$

$r = 0.98$ (exact) ***A1 N2***

[2 marks]

(b) Find the sum of the first 8 terms. [2]

Markscheme

correct substitution ***(A1)***

$\frac{4\left(1-0.98^8\right)}{1-0.98}$

29.8473

29.8 ***A1 N2***

[2 marks]

(c) Find the least value of n for which $S_n > 163$. [3]

Markscheme

attempt to set up inequality (accept equation) ***(M1)***

eg $\frac{4(1-0.98^n)}{1-0.98} > 163,\ \frac{4(1-0.98^n)}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values ***(A1)***

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ ***A1 N1***

[3 marks]