

## Sequences review - exam questions (TL) [149 marks]

1. [Maximum mark: 6]

EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are  $u_1$ ,  $5u_1 - 8$  and  $3u_1 + 8$ .

(a) Show that  $u_1 = 4$ . [2]

(b) Prove that the sum of the first  $n$  terms of this arithmetic sequence is a square number. [4]

2. [Maximum mark: 12]

EXN.2.SL.TZ0.7

Helen and Jane both commence new jobs each starting on an annual salary of \$70,000. At the start of each new year, Helen receives an annual salary increase of \$2400.

Let  $\$H_n$  represent Helen's annual salary at the start of her  $n$ th year of employment.

(a) Show that  $H_n = 2400n + 67600$ . [2]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary,  $\$J_n$ , at the start of her  $n$ th year of employment is given by  $J_n = 70000(1.03)^{n-1}$ .

(b) Given that  $J_n$  follows a geometric sequence, state the value of the common ratio,  $r$ . [1]

At the start of year  $N$ , Jane's annual salary exceeds Helen's annual salary for the first time.

(c.i) Find the value of  $N$ . [3]

(c.ii) For the value of  $N$  found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

(d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

3. [Maximum mark: 14]

23M.1.AHL.TZ1.10

Consider the arithmetic sequence  $u_1, u_2, u_3, \dots$ .

The sum of the first  $n$  terms of this sequence is given by  $S_n = n^2 + 4n$ .

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that  $S_6 = 60$ , find  $u_6$ . [2]

(b) Find  $u_1$ . [2]

(c) Hence or otherwise, write an expression for  $u_n$  in terms of  $n$ . [3]

Consider a geometric sequence,  $v_n$ , where  $v_2 = u_1$  and  $v_4 = u_6$ .

(d) Find the possible values of the common ratio,  $r$ . [3]

(e) Given that  $v_{99} < 0$ , find  $v_5$ . [2]

4. [Maximum mark: 5]

22N.2.SL.TZ0.4

geometric sequence has a first term of 50 and a fourth term of 86. 4.

The sum of the first  $n$  terms of the sequence is  $S_n$ .

Find the smallest value of  $n$  such that  $S_n > 33500$ . [5]

5. [Maximum mark: 18]

22M.1.AHL.TZ1.10

Consider the series  $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$ , where  $x \in \mathbb{R}$ ,  $x > 1$  and  $p \in \mathbb{R}$ ,  $p \neq 0$ .

Consider the case where the series is geometric.

(a.i) Show that  $p = \pm \frac{1}{\sqrt{3}}$ . [2]

(a.ii) Hence or otherwise, show that the series is convergent. [1]

(a.iii) Given that  $p > 0$  and  $S_\infty = 3 + \sqrt{3}$ , find the value of  $x$ . [3]

Now consider the case where the series is arithmetic with common difference  $d$ .

(b.i) Show that  $p = \frac{2}{3}$ . [3]

(b.ii) Write down  $d$  in the form  $k \ln x$ , where  $k \in \mathbb{Q}$ . [1]

(b.iii) The sum of the first  $n$  terms of the series is  $\ln\left(\frac{1}{x^3}\right)$ .

Find the value of  $n$ . [8]

6. [Maximum mark: 9]

21N.2.SL.TZ0.6

The sum of the first  $n$  terms of a geometric sequence is given by

$$S_n = \sum_{r=1}^n \frac{2}{3} \left(\frac{7}{8}\right)^r.$$

(a) Find the first term of the sequence,  $u_1$ . [2]

(b) Find  $S_\infty$ . [3]

(c) Find the least value of  $n$  such that  $S_\infty - S_n < 0.001$ . [4]

7. [Maximum mark: 5] 21M.1.SL.TZ1.3  
Consider an arithmetic sequence where  $u_8 = S_8 = 8$ . Find the value of the first term,  $u_1$ , and the value of the common difference,  $d$ . [5]

8. [Maximum mark: 5] 21M.2.SL.TZ2.3  
An arithmetic sequence has first term 60 and common difference  $-2.5$ .

(a) Given that the  $k$ th term of the sequence is zero, find the value of  $k$ . [2]

(b) Let  $S_n$  denote the sum of the first  $n$  terms of the sequence.  
Find the maximum value of  $S_n$ . [3]

9. [Maximum mark: 6] 20N.1.SL.TZ0.T\_15  
Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of  $30 \text{ cm}^3$ . The fifth smallest slice has a volume of  $240 \text{ cm}^3$ .

(a) Find the common ratio of the sequence. [2]

(b) Find the volume of the smallest slice of pie. [2]

(c) The apple pie has a volume of  $61\,425 \text{ cm}^3$ .  
Find the total number of slices Mia can cut from this pie. [2]

- 10.** [Maximum mark: 5] 20N.1.AHL.TZ0.H\_5  
The first term in an arithmetic sequence is 4 and the fifth term is  $\log_2 625$ .

Find the common difference of the sequence, expressing your answer in the form  $\log_2 p$ , where  $p \in \mathbb{Q}$ . [5]

- 11.** [Maximum mark: 8] 20N.2.SL.TZ0.S\_6  
An infinite geometric series has first term  $u_1 = a$  and second term  $u_2 = \frac{1}{4}a^2 - 3a$ , where  $a > 0$ .

(a) Find the common ratio in terms of  $a$ . [2]

(b) Find the values of  $a$  for which the sum to infinity of the series exists. [3]

(c) Find the value of  $a$  when  $S_\infty = 76$ . [3]

- 12.** [Maximum mark: 6] 19N.1.SL.TZ0.S\_1  
In an arithmetic sequence,  $u_2 = 5$  and  $u_3 = 11$ .

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]

13. [Maximum mark: 7] 19N.2.SL.TZ0.S\_5  
The first two terms of a geometric sequence are  $u_1 = 2.1$  and  $u_2 = 2.226$ .

- (a) Find the value of  $r$ . [2]
- (b) Find the value of  $u_{10}$ . [2]
- (c) Find the least value of  $n$  such that  $S_n > 5543$ . [3]

14. [Maximum mark: 5] 19N.2.AHL.TZ0.H\_1  
A geometric sequence has  $u_4 = -70$  and  $u_7 = 8.75$ . Find the second term of the sequence. [5]

15. [Maximum mark: 7] 19M.2.SL.TZ1.S\_7  
The first terms of an infinite geometric sequence,  $u_n$ , are 2, 6, 18, 54, ...  
The first terms of a second infinite geometric sequence,  $v_n$ , are 2, -6, 18, -54, ...  
The terms of a third sequence,  $w_n$ , are defined as  $w_n = u_n + v_n$ .

The finite series,  $\sum_{k=1}^{225} w_k$ , can also be written in the form  $\sum_{k=0}^m 4r^k$ .

- (a) Write down the first three **non-zero** terms of  $w_n$ . [3]
- (b.i) Find the value of  $r$ . [2]
- (b.ii) Find the value of  $m$ . [2]

16. [Maximum mark: 12]

19M.2.SL.TZ2.S\_10

In an arithmetic sequence,  $u_1 = 1.3$ ,  $u_2 = 1.4$  and  $u_k = 31.2$ .

(b) Find the exact value of  $S_k$ . [2]

Consider the terms,  $u_n$ , of this sequence such that  $n \leq k$ .

Let  $F$  be the sum of the terms for which  $n$  is not a multiple of 3.

(c) Show that  $F = 3240$ . [5]

(d) An infinite geometric series is given as  
$$S_\infty = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \dots, a \in \mathbb{Z}^+.$$

Find the largest value of  $a$  such that  $S_\infty < F$ . [5]

17. [Maximum mark: 6]

18M.1.SL.TZ1.T\_7

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei's lifts form an arithmetic sequence.

Sergei's friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei's lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

(a.i) For that day find how much weight was added after each lift. [2]

(a.ii) For that day find the weight of Sergei's first lift. [2]

(b) On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts. [2]

18. [Maximum mark: 6] 18M.2.SL.TZ2.T\_4

A new café opened and during the first week their profit was \$60.

The café's profit increases by \$10 every week.

(b) Calculate the café's **total** profit for the first 12 weeks. [3]

A new tea-shop opened at the same time as the café. During the first week their profit was also \$60.

The tea-shop's profit increases by 10 % every week.

(d) Calculate the tea-shop's **total** profit for the first 12 weeks. [3]

19. [Maximum mark: 7] 18M.2.SL.TZ2.S\_4

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

(b) Find the sum of the first 8 terms. [2]

(c) Find the least value of  $n$  for which  $S_n > 163$ . [3]