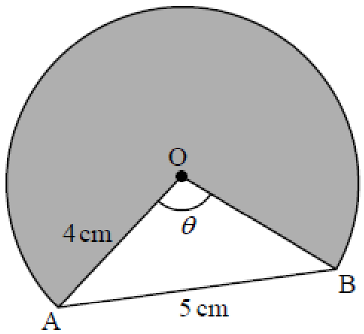
Trigonometry review (TL) [130 marks]

1. [Maximum mark: 6] SPM.2.SL.TZ0.2

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and $A\hat{O}B = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

Markscheme

METHOD 1

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2+4^2-5^2}{2\times4\times4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) Find the area of the shaded region. [3]

Markscheme

attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35\ldots)$ ***A1***

$= 39.5\ (\text{cm}^2)$ ***A1***

[3 marks]

2. [Maximum mark: 16] SPM.2.SL.TZ0.7

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

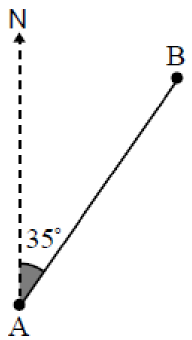
(a) Find the distance from point A to point B. [2]

Markscheme

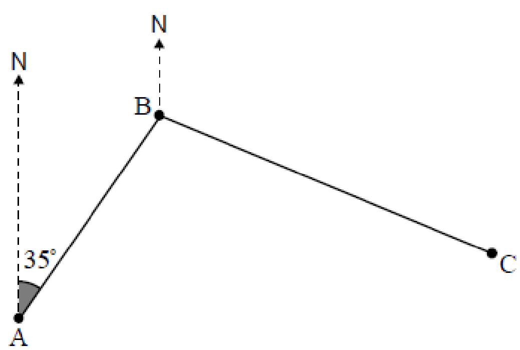
$\frac{4.2}{60} \times 45$ ***A1***

AB = 3.15 (km) ***A1***

[2 marks]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



(b.i) Show that $\hat{ABC}$ is 101°. [2]

Markscheme

66° or (180 – 114) ***A1***

35 + 66 ***A1***

$\hat{ABC}$ = 101° ***AG***

[2 marks]

(b.ii) Find the distance from the camp to point C. [3]

Markscheme

attempt to use cosine rule ***(M1)***

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101°$ (or equivalent) ***A1***

$AC = 6.05$ (km) ***A1***

[3 marks]

(c) Find $\mathrm{B\hat{C}A}$. [3]

Markscheme

valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(\mathrm{B\hat{C}A}\right)}{3.15} = \frac{\sin 101}{6.0507\ldots}$

$\mathrm{B\hat{C}A} = 30.7°$ ***A1***

[3 marks]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C .

(d) Find the bearing that Jacob must take to point C. [3]

Markscheme

$\mathrm{B\hat{A}C} = 48.267$ (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg $48.267 + 35$

bearing = 83.3° (accept 083°) ***A1***

[3 marks]

(e) Jacob hikes at an average speed of 3.9 km/h.

Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Markscheme

attempt to use $\text{time} = \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

t = 93 (minutes) ***A1***

[3 marks]

3. [Maximum mark: 13] SPM.2.SL.TZ0.9

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

Markscheme

correct approach ***A1***

eg $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

period = 12 ***A1***

[2 marks]

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

(b.i) Find the value of b. [2]

Markscheme

valid approach ***(M1)***

eg $\frac{\max+\min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8+10.2}{2}$, or equivalent

b = 16 ***A1***

[2 marks]

(b.ii) Hence, find the value of f(6). [2]

Markscheme

attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right) + 16$

f(6) = 13.1 ***A1***

[2 marks]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x-3.75)\right)+q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

(c) Find the value of p and the value of q. [5]

Markscheme

valid attempt to set up a system of equations ***(M1)***

two correct equations ***A1***

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q = 2.5,\ p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q = 15.1$

valid attempt to solve system ***(M1)***

$p = 8.4$; $q = 6.7$ ***A1A1***

[5 marks]

(d) Find the value of x for which the functions have the greatest difference. [2]

Markscheme

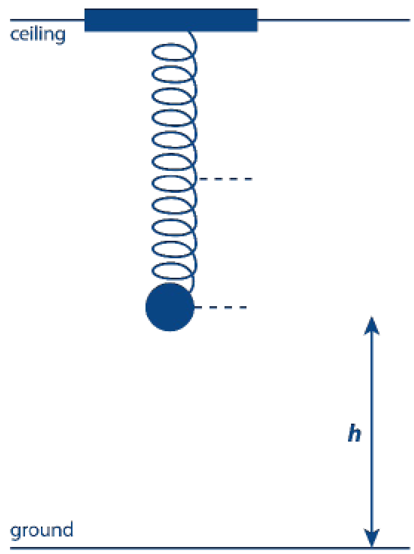
attempt to use $|f(x) - g(x)|$ to find maximum difference ***(M1)***

$x = 1.64$ ***A1***

[2 marks]

4. [Maximum mark: 11] EXN.1.SL.TZ0.9

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.



The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \geq 0$.

(a) Find the height of the ball above the ground when it is released. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts to find $h(0)$ **(M1)**

$h(0) = 0.4\cos(0) + 1.8 (= 2.2)$

2.2 (m) (above the ground) **A1**

[2 marks]

(b) Find the minimum height of the ball above the ground. [2]

Markscheme

EITHER

uses the minimum value of $\cos(\pi t)$ which is -1 **M1**

$0.4(-1) + 1.8$ (m)

OR

the amplitude of motion is 0.4 (m) and the mean position is 1.8 (m) **M1**

OR

finds $h\prime(t) = -0.4\pi \sin(\pi t)$, attempts to solve $h\prime(t) = 0$ for t and determines that the minimum height above the ground occurs at $t = 1, 3, \ldots$ **M1**

$0.4(-1) + 1.8$ (m)

THEN

1.4 (m) (above the ground) **A1**

[2 marks]

(c) Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. [2]

Markscheme

EITHER

the ball is released from its maximum height and returns there a period later **R1**

the period is $\frac{2\pi}{\pi} (= 2)\ (\mathrm{s})$ **A1**

OR

attempts to solve $h(t) = 2.2$ for t **M1**

$\cos(\pi t) = 1$

$t = 0, 2, \ldots$ **A1**

THEN

so it takes 2 seconds for the ball to return to its initial position for the first time **AG**

[2 marks]

(d) For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground. [5]

Markscheme

$0.4\cos(\pi t) + 1.8 = 1.8 + 0.2\sqrt{2}$ **(M1)**

$0.4\cos(\pi t) = 0.2\sqrt{2}$

$\cos(\pi t) = \frac{\sqrt{2}}{2}$ **A1**

$\pi t = \frac{\pi}{4}, \frac{7\pi}{4}$ **(A1)**

Note: Accept extra correct positive solutions for πt.

$t = \frac{1}{4}, \frac{7}{4}\ (0 \le t \le 2)$ **A1**

Note: Do not award **A1** if solutions outside $0 \le t \le 2$ are also stated.

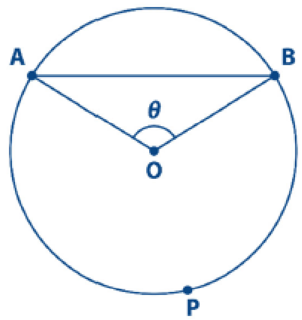
the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground for $\frac{7}{4} - \frac{1}{4}$ (s)

1.5 (s) **A1**

[5 marks]

5. [Maximum mark: 7] EXN.2.SL.TZ0.2

The following diagram shows a circle with centre O and radius 3.



Points A, P and B lie on the circumference of the circle.

Chord $[\mathrm{AB}]$ has length L and $\mathrm{A\hat{O}B} = \theta$ radians.

(a) Show that arc APB has length $6\pi - 3\theta$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

EITHER

uses the arc length formula **(M1)**

arc length is $3(2\pi - \theta)$ **A1**

OR

length of arc AB is 3θ **A1**

the sum of the lengths of arc AB and arc APB is 6π **A1**

THEN

so arc APB has length $6\pi - 3\theta$ **AG**

[2 marks]

(b) Show that $L = \sqrt{18 - 18\cos\theta}$. [2]

Markscheme

uses the cosine rule **(M1)**

$L^2 = 3^2 + 3^2 - 2(3)(3)\cos\theta$ **A1**

so $L = \sqrt{18 - 18\cos\theta}$ **AG**

[2 marks]

(c) Arc APB is twice the length of chord $[\text{AB}]$.

Find the value of θ. [3]

Markscheme

$6\pi - 3\theta = 2\sqrt{18 - 18\cos\theta}$ **A1**

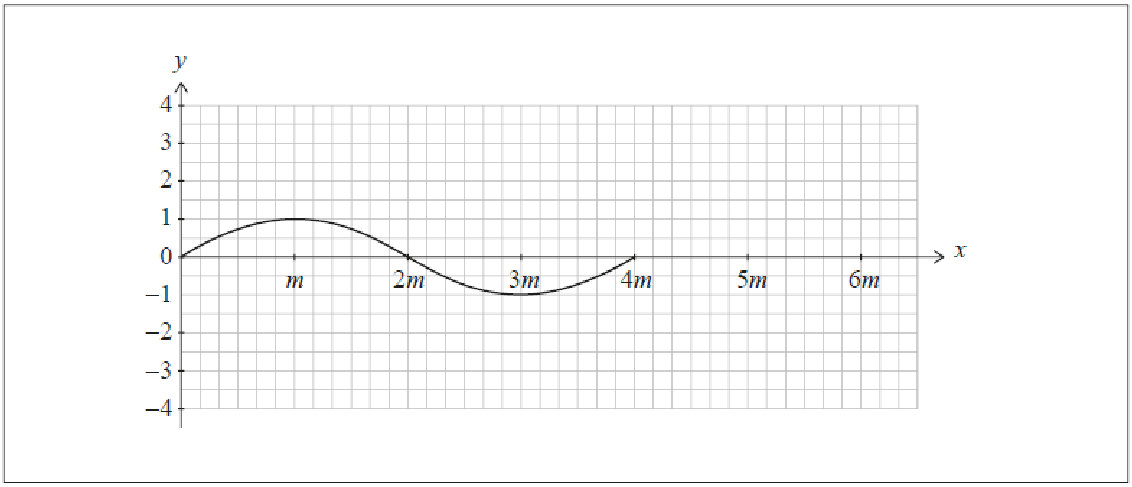
attempts to solve for θ **(M1)**

$\theta = 2.49$ **A1**

[3 marks]

6. [Maximum mark: 6] 23M.1.SL.TZ1.6

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \leq x \leq 4m$, where x is in radians. There are x-intercepts at $x = 0, 2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

Markscheme

recognition that period is $4m$ OR substitution of a point on f (except the origin) ***(M1)***

$4m = \frac{2\pi}{q}$ OR $1 = \sin qm$

$m = \frac{\pi}{2q}$ ***A1***

[2 marks]

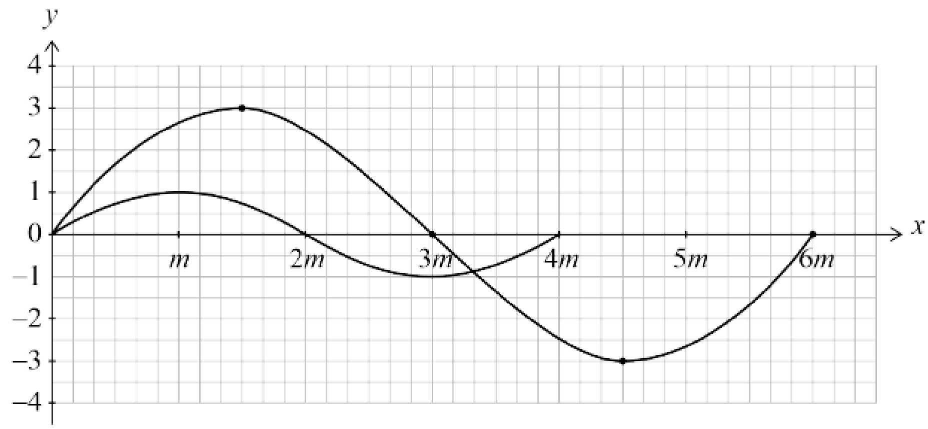
The function g is defined by $g\left(x\right) = 3 \sin \frac{2qx}{3}$, for $0 \leq x \leq 6m$.

(b) On the axes above, sketch the graph of g. [4]

Markscheme

horizontal scale factor is $\frac{3}{2}$ (seen anywhere) ***(A1)***

Note: This ***(A1)*** may be earned by seeing a period of $6m$, half period of $3m$ or the correct x-coordinate of the maximum/minimum point.



A1A1A1

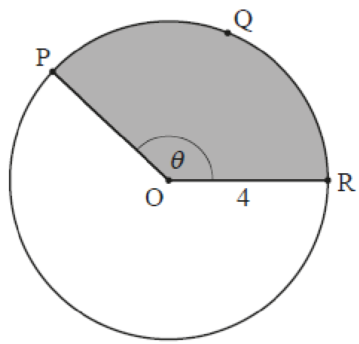
Note:
Curve must be an approximate sinusoidal shape (sine or cosine).
Only in this case, award the following:
A1 for correct amplitude.
A1 for correct domain.
A1 for correct max and min points **and** correct x-intercepts.

[4 marks]

7. [Maximum mark: 6] 23M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius $4\ \mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\ \mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

Markscheme

attempts to find perimeter ***(M1)***

$\mathrm{arc} + 2 \times \mathrm{radius}$ OR $10 + 4 + 4$

$= 18\,(\mathrm{cm})$ ***A1***

[2 marks]

(b) Find θ. [2]

Markscheme

$10 = 4\theta$ ***(A1)***

$\theta = \frac{10}{4}\left(= \frac{5}{2}, 2.5\right)$ ***A1***

[2 marks]

(c) Find the area of the shaded sector. [2]

Markscheme

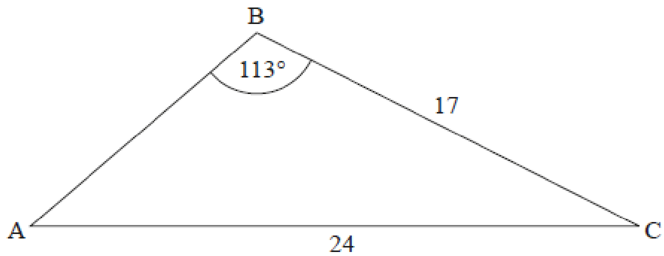
area $= \frac{1}{2}\left(\frac{10}{4}\right)\left(4^2\right)\ \left(= 1.25 \times 16\right)$ ***(A1)***

$= 20\left(\mathrm{cm}^2\right)$ ***A1***

[2 marks]

8. [Maximum mark: 6] 22N.2.SL.TZ0.2

The following diagram shows triangle ABC, with $\mathrm{AC}=24, \mathrm{BC}=17$, and $\mathrm{A\widehat{B}C}=113^\circ$.



(a) Find $\mathrm{B\widehat{A}C}$. [3]

Markscheme

attempt to use sine rule ***(M1)***

$\frac{24}{\sin 113^\circ}=\frac{17}{\sin \mathrm{B\widehat{A}C}}$ OR $\left(\sin \mathrm{B\widehat{A}C}=\right)0.652024\ldots$ ***(A1)***

$40.6943\ldots$

$\mathrm{B\widehat{A}C}=40.7^\circ$ ***A1***

[3 marks]

(b) Find AB. [3]

Markscheme

METHOD 1 (cosine rule with $\mathrm{A\widehat{B}C}$ or $\mathrm{B\widehat{A}C}$)

attempt to use the cosine rule ***(M1)***

$24^2=\mathrm{AB}^2+17^2-2\cdot 17\cdot \mathrm{AB}\cdot\cos 113^\circ \quad \left(\mathrm{AB}^2+13.2848\ldots\mathrm{AB}-287=0\right)$ OR

$17^2=\mathrm{AB}^2+24^2-2\cdot 24\cdot \mathrm{AB}\cdot\cos 40.6943\ldots^\circ \quad \left(\mathrm{AB}^2-36.3935\ldots\mathrm{AB}+287=0\right)$ ***(A1)***

$11.5543\ldots$

$\mathrm{AB}=11.6$ ***A1***

METHOD 2 (cosine rule with $\mathrm{B\widehat{C}A}$)

attempt to use cosine rule *(M1)*

correct substitution *(A1)*

$\mathrm{AB}^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056\ldots^\circ$ OR $\mathrm{AB}^2 = 133.502\ldots$

$11.5543\ldots$

$\mathrm{AB} = 11.6$ ***A1***

METHOD 3 (sine rule)

attempt to use sine rule *(M1)*

correct substitution *(A1)*

$\frac{\mathrm{AB}}{\sin 26.3056\ldots^\circ} = \frac{24}{\sin 113^\circ} = \frac{17}{\sin 40.6943\ldots^\circ}$ OR $\mathrm{AB} = \frac{24 \cdot \sin(180^\circ - 113^\circ - 40.6943\ldots^\circ)}{\sin 113^\circ}$

$11.5543\ldots$

$\mathrm{AB} = 11.6$ ***A1***

[3 marks]

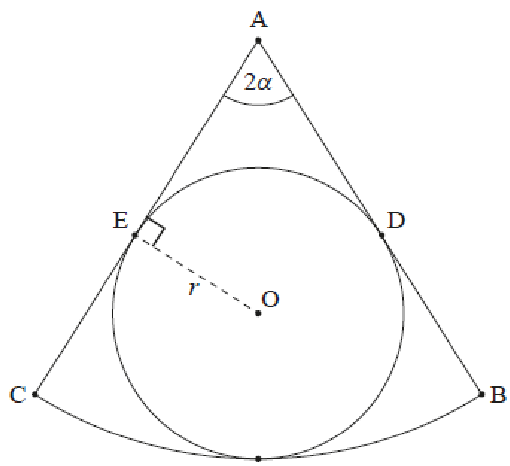
9. [Maximum mark: 13] 22N.2.SL.TZ0.8

The following diagram shows a sector ABC of a circle with centre A. The angle $\text{B}\hat{\text{A}}\text{C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\text{O}\hat{\text{E}}\text{A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\frac{r^2}{\tan\alpha}$. [4]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

attempt to use right angled trigonometry or sine rule to find AE in terms of r and α ***(M1)***

$\tan\alpha = \frac{r}{\text{AE}}$ OR $\frac{\text{AE}}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{r}{\sin\alpha}$

$\text{AE} = \frac{r}{\tan\alpha}$ OR $\text{AE} = \frac{r\sin\left(\frac{\pi}{2}-\alpha\right)}{\sin\alpha}$ OR $\text{AE} = \frac{r\cos\alpha}{\sin\alpha}$ ***A1***

valid approach to find the area of ADOE ***(M1)***

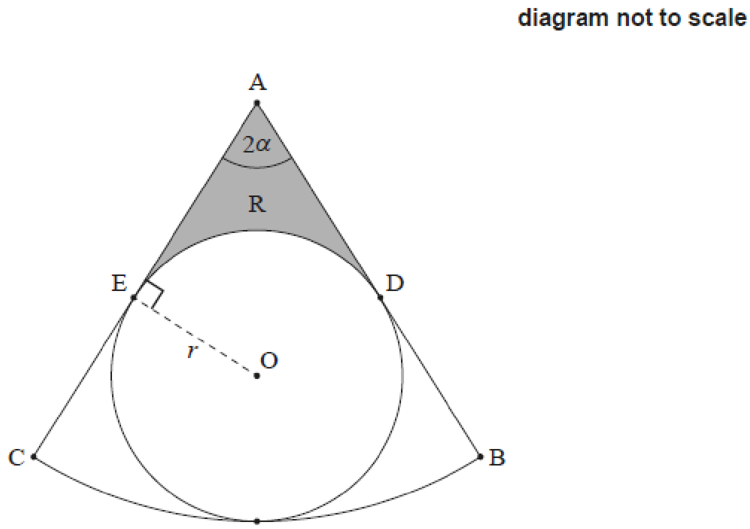
$2\times$ area of triangle AOE OR area of triangle AED + area of triangle OED OR $\text{OE}\times\text{AE}$

Area $\text{ADOE} = 2\left(\frac{1}{2}\cdot\frac{r}{\tan\alpha}\cdot r\right)$ OR $r\times\text{AE}$ ***A1***

Area $\text{ADOE} = \frac{r^2}{\tan\alpha}$ ***AG***

[4 marks]

R represents the shaded region shown in the following diagram.



(b.i) Find $\mathrm{D\hat{O}E}$ in terms of α. [2]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

recognizing that the sum of the angles of a kite is 2π ***(M1)***

$\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 2\pi$ OR $2\alpha + 2 \cdot \frac{\pi}{2} + \mathrm{D\hat{O}E} = 2\pi$

$\mathrm{D\hat{O}E} = \pi - 2\alpha$ ***A1***

Note: Award ***M1A0*** if candidate uses degrees (i.e. $\mathrm{D\hat{O}E} + \mathrm{O\hat{E}A} + \mathrm{E\hat{A}D} + \mathrm{A\hat{D}O} = 360^\circ$ or $2\alpha + 2 \cdot \frac{\pi}{2} + \mathrm{D\hat{O}E} = 360^\circ$) and obtains $\mathrm{D\hat{O}E} = 180^\circ - 2\alpha$.

[2 marks]

(b.ii) Hence or otherwise, find an expression for the area of R. [3]

Markscheme

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

valid approach to find the area of R *(M1)*

area of kite – area of sector OR 2(area of triangle $\mathrm{AOE} - 0.5$ area of sector OED)

Area of sector $= \frac{1}{2}r^2.\,\mathrm{D\widehat{O}E}\left(= \frac{1}{2}r^2(\pi - 2\alpha)\right)$ seen anywhere *(A1)*

Area of $\mathrm{R} = \frac{r^2}{\tan \alpha} - \frac{1}{2}r^2(\pi - 2\alpha)$ *A1*

Note: Accept $\frac{r^2}{\tan \alpha} - \frac{1}{2}r^2.\,\mathrm{D\widehat{O}E}$.

[3 marks]

(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

Markscheme

equating their area formula to πr^2 *(M1)*

$\frac{r^2}{\tan \alpha} - \frac{1}{2}r^2(\pi - 2\alpha) = \pi r^2$

correct equation in terms of α *A1*

$\frac{1}{\tan \alpha} - \frac{1}{2}(\pi - 2\alpha) = \pi$

valid approach to solve the equation *(M1)*

$\alpha = 0.218979\ldots$

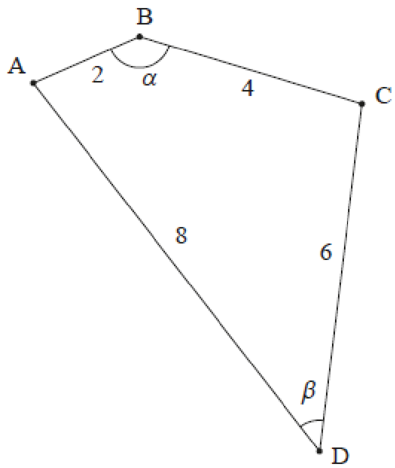
$\alpha = 0.219$ *A1*

[4 marks]

10. [Maximum mark: 8] 22N.2.AHL.TZ0.9

Consider a quadrilateral ABCD such that $\mathrm{AB} = 2,\ \mathrm{BC} = 4,\ \mathrm{CD} = 6$ and $\mathrm{DA} = 8$, as shown in the following diagram. Let $\alpha = \mathrm{A\hat{B}C}$ and $\beta = \mathrm{A\hat{D}C}$.

diagram not to scale



(a.i) Find AC in terms of α. [2]

Markscheme

attempt to use the cosine rule *(M1)*

$$\mathrm{AC} = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha}\left(= \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha}\right) \quad \textit{A1}$$

[2 marks]

(a.ii) Find AC in terms of β. [1]

Markscheme

$$\mathrm{AC} = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta}\left(= \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta}\right) \quad \textit{A1}$$

[1 mark]

(a.iii) Hence or otherwise, find an expression for α in terms of β. [1]

Markscheme

$$5 - 4\cos\alpha = 25 - 24\cos\beta$$

$\alpha = \arccos(6 \cos \beta - 5)$ ***A1***

[1 mark]

(b) Find the maximum area of the quadrilateral ABCD. [4]

Markscheme

attempt to find the sum of two triangle areas using $A = \frac{1}{2}ab \sin C$ ***(M1)***

Note: Do not award this ***M1*** if the triangle is assumed to be right angled.

$\text{Area} = \frac{1}{2}(8) \sin \alpha + \frac{1}{2}(48) \sin \beta$ ***(A1)***

attempt to express the area in terms of one variable only ***(M1)***

$= 4\sqrt{1 - (6 \cos \beta - 5)^2} + 24 \sin \beta$ or $4 \sin(\arccos(6 \cos \beta - 5)) + 24 \sin \beta$ OR

$4 \sin \alpha + 24\sqrt{1 - \left(\frac{5+\cos \alpha}{6}\right)^2}$ or $4 \sin \alpha + 24 \sin\left(\arccos\left(\frac{5+\cos \alpha}{6}\right)\right)$

Max area $= 19.5959\ldots$

$= 19.6$ ***A1***

[4 marks]

11. [Maximum mark: 7] 22M.1.SL.TZ1.6

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

Markscheme

translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r. [5]

Markscheme

METHOD 1

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4 + 2.5 + q \geq 7$

$q \geq 8.5$ (accept $q = 8.5$) ***A1***

substituting $x = 0$ and their $q\ (= 8.5)$ to find r ***(M1)***

$(r =)\ 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + 8.5$

$4 + 2.5 + 8.5$ ***(A1)***

smallest value of r is 15 ***A1***

METHOD 2

substituting $x = 0$ to find an expression (for r) in terms of q ***(M1)***

$(g(0) = r =)\ 4\ \sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$

$(r =)\ 6.5 + q$ ***A1***

minimum of $4\ \sin\left(x - \frac{3\pi}{2}\right)$ is -4 ***(M1)***

$-4 + 2.5 + q \geq 7$

$-4 + 2.5 + (r - 6.5) \geq 7$ (accept $=$) ***(A1)***

smallest value of r is 15 ***A1***

METHOD 3

$4\ \sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\ \cos\ x + 2.5 + q$ ***A1***

y-intercept of $4\ \cos\ x + 2.5 + q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$r = 2 \times 4 + 7$

smallest value of r is 15 ***A1***

[5 marks]

12. [Maximum mark: 5] 22M.1.SL.TZ2.5

Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$. [5]

Markscheme

determines $\frac{\pi}{4}$ (or 45°) as the first quadrant (reference) angle ***(A1)***

attempts to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}$ ***(M1)***

Note: Award ***M1*** for attempting to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}, \frac{7\pi}{4}(, \ldots)$

$\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4} \Rightarrow x<0$ and so $\frac{\pi}{4}$ is rejected ***(R1)***

$\frac{x}{2}+\frac{\pi}{3}=2\pi-\frac{\pi}{4}\left(=\frac{7\pi}{4}\right)$ ***A1***

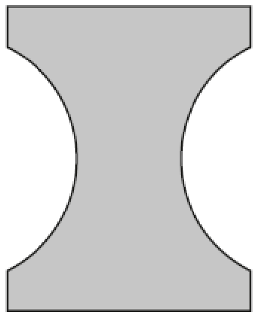
$x=\frac{17\pi}{6}$ (must be in radians) ***A1***

[5 marks]

13. [Maximum mark: 6] 22M.2.SL.TZ1.3

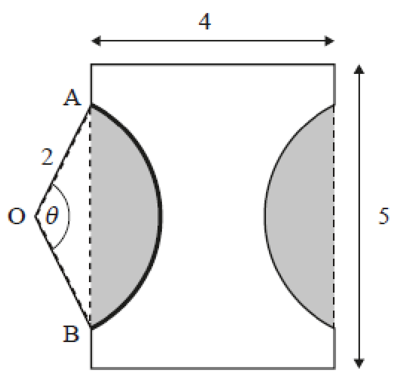
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

Markscheme

valid approach to find area of segment by finding area of sector – area of triangle ***(M1)***

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2 \sin\theta$ ***(A1)***

area $= 2\theta - 2\sin\theta$ ***A1***

[3 marks]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

Markscheme

EITHER

area of logo = area of rectangle – area of segments ***(M1)***

$5 \times 4 - 2 \times (2\theta - 2\sin\theta) = 13.4$ ***(A1)***

OR

area of one segment $= \frac{20-13.4}{2}$ $(= 3.3)$ ***(M1)***

$2\theta - 2\sin\theta = 3.3$ ***(A1)***

THEN

$\theta = 2.35672\ldots$

$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

Note: Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

14. [Maximum mark: 6] 22M.2.SL.TZ2.1

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $\mathrm{A\hat{O}B} = 1.9$ radians.

(a) Find the length of the chord $[\mathrm{AB}]$. [3]

Markscheme

EITHER

uses the cosine rule ***(M1)***

$\mathrm{AB}^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 1.9$ ***(A1)***

OR

uses right-angled trigonometry ***(M1)***

$\frac{\frac{\mathrm{AB}}{2}}{5} \sin 0.95$ ***(A1)***

OR

uses the sine rule ***(M1)***

$\alpha = \frac{1}{2}(\pi - 1.9)(= 0.6207\ldots)$

$\frac{\mathrm{AB}}{\sin 1.9} = \frac{5}{\sin 0.6207\ldots}$ ***(A1)***

THEN

$\mathrm{AB} = 8.13415\ldots$

$\mathrm{AB} = 8.13\,(\mathrm{m})$ ***A1***

[3 marks]

(b) Find the area of the shaded sector. [3]

Markscheme

let the shaded area be A

METHOD 1

attempt at finding reflex angle *(M1)*

$\mathrm{A\hat{O}B} = 2\pi - 1.9\ (= 4.3831\ldots)$

substitution into area formula *(A1)*

$A = \frac{1}{2} \times 5^2 \times 4.3831\ldots$ OR $\left(\frac{2\pi - 1.9}{2\pi}\right) \times \pi\left(5^2\right)$

$= 54.7898\ldots$

$= 54.8\ \left(\mathrm{m}^2\right)$ *A1*

METHOD 2

let the area of the circle be A_C and the area of the unshaded sector be A_U

$A = A_C - A_U$ *(M1)*

$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9\ \ (= 78.5398\ldots - 23.75)$ *(A1)*

$= 54.7898\ldots$

$= 54.8\ \left(\mathrm{m}^2\right)$ *A1*

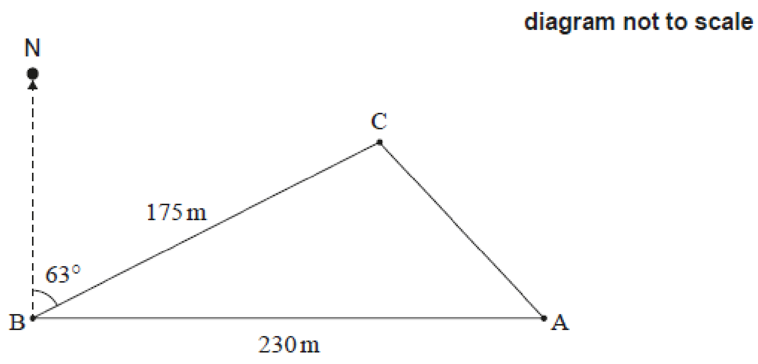
[3 marks]

15. [Maximum mark: 14] 22M.2.SL.TZ2.7

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.



(a) Find the distance from point A to point C. [4]

Markscheme

$\mathrm{A\hat{B}C} = 27^{\circ}$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$175^2 + 230^2 - 2(175)(230)\cos 27^{\circ}$ ***(A1)***

$108.62308\ldots$

$\mathrm{AC} = 109\,(\mathrm{m})$ ***A1***

[4 marks]

(b) Find the area of this piece of land. [2]

Markscheme

correct substitution into area formula ***(A1)***

$\frac{1}{2} \times 175 \times 230 \times \sin 27^{\circ}$

$9136.55\ldots$

area $= 9140\left(\mathrm{m}^2\right)$ ***A1***

[2 marks]

(c) Find $\hat{CAB}$. [3]

Markscheme

attempt to substitute into sine rule or cosine rule ***(M1)***

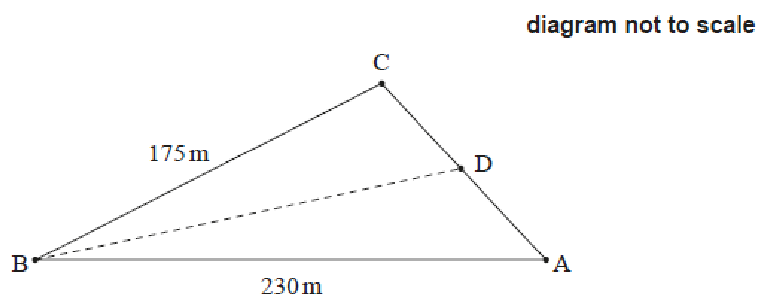
$\frac{\sin 27^\circ}{108.623\ldots} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623\ldots)^2 + 230^2 - 175^2}{2 \times 108.623\ldots \times 230}$ ***(A1)***

$47.0049\ldots$

$\hat{CAB} = 47.0^\circ$ ***A1***

[3 marks]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.



(d) Find the distance from point B to point D. [5]

Markscheme

METHOD 1

recognizing that for areas to be equal, $AD = DC$ ***(M1)***

$AD = \frac{1}{2}AC = 54.3115\ldots$ ***A1***

attempt to substitute into cosine rule to find BD ***(M1)***

correct substitution into cosine rule ***(A1)***

$BD^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049^\circ$

$BD = 197.009\ldots$

$\mathrm{BD} = 197\,(\mathrm{m})$ ***A1***

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin x^\circ$ and $\frac{1}{2} \times \mathrm{BD} \times 175 \times \sin\,(27 - x)^\circ$ OR

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin\,(27 - x)^\circ$ and $\frac{1}{2} \times \mathrm{BD} \times 175 \times \sin x^\circ$

correct equation in terms of x ***(A1)***

$175 \sin(27 - x) = 230 \sin x$ or $175 \sin x = 230 \sin(27 - x)$

$x = 11.6326\ldots$ or $x = 15.3673\ldots$ ***(A1)***

substituting their value of x into equation to solve for BD ***(M1)***

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin 11.6326\ldots = \frac{1}{2} \times \mathrm{BD} \times 175 \times \sin 15.3673\ldots$ or

$\frac{1}{2} \times \mathrm{BD} \times 230 \times \sin 11.6326\ldots = \frac{1}{2} \times 9136.55\ldots$

$\mathrm{BD} = 197\,(\mathrm{m})$ ***A1***

[5 marks]