Trigonometry review (TL) [130 marks]
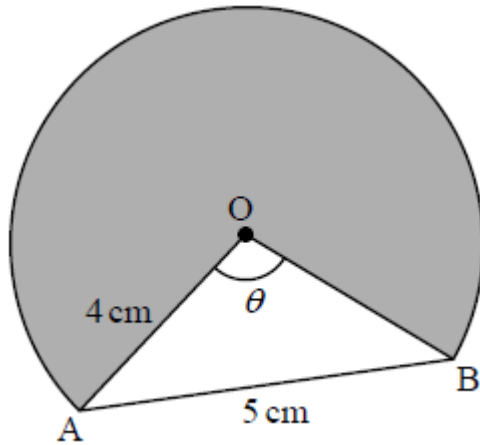
1. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius 4 cm.

Chord AB has a length of 5 cm and AÔB = θ.

(a)   Find the value of θ, giving your answer in radians.   [3]

(b)   Find the area of the shaded region.   [3]

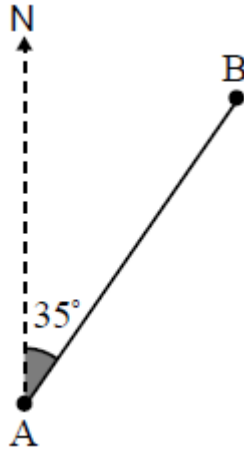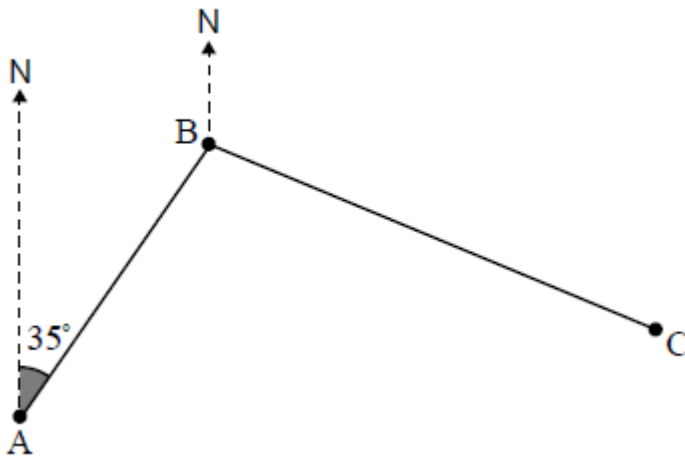**2.** [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a)   Find the distance from point A to point B. [2]

Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.

(b.i)   Show that $A\hat{B}C$ is 101°. [2]

(b.ii)   Find the distance from the camp to point C. [3]

(c)   Find $B\hat{C}A$.   [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d)   Find the bearing that Jacob must take to point C.   [3]

(e)   Jacob hikes at an average speed of 3.9 km/h.

       Find, to the nearest minute, the time it takes for Jacob to reach point C.   [3]

3. [Maximum mark: 13]

Consider a function f, such that $f(x) = 5.8 \sin\left(\frac{\pi}{6}(x+1)\right) + b, 0 \le x \le 10, b \in \mathbb{R}$.

(a)   Find the period of f.   [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b.i)   Find the value of b.   [2]

(b.ii)   Hence, find the value of $f(6)$.   [2]

A second function g is given by $g(x) = p \sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q, 0 \le x \le 10; p, q \in \mathbb{R}$.

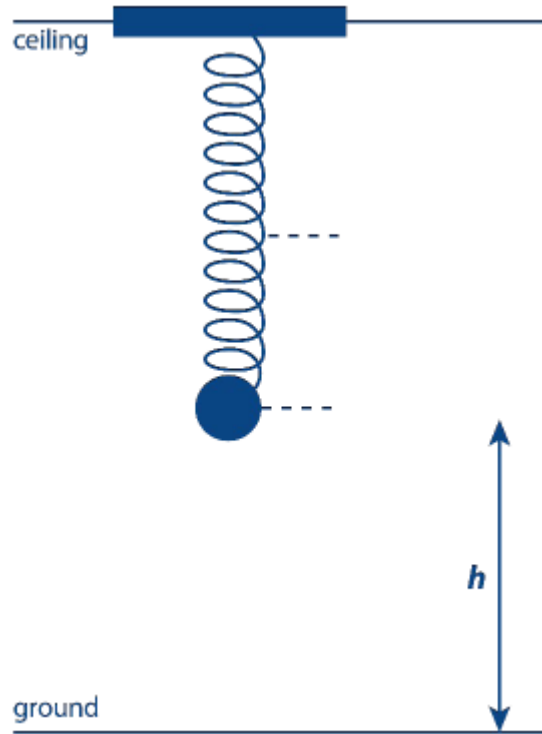The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c)   Find the value of p and the value of q.   [5]

(d)   Find the value of x for which the functions have the greatest difference.   [2]

4. [Maximum mark: 11]

The following diagram shows a ball attached to the end of a spring, which is suspended from a ceiling.


The height, h metres, of the ball above the ground at time t seconds after being released can be modelled by the function $h(t) = 0.4\cos(\pi t) + 1.8$ where $t \geq 0$.

(a) Find the height of the ball above the ground when it is released. [2]

(b) Find the minimum height of the ball above the ground. [2]

(c) Show that the ball takes 2 seconds to return to its initial height above the ground for the first time. [2]

(d) For the first 2 seconds of its motion, determine the amount of time that the ball is less than $1.8 + 0.2\sqrt{2}$ metres above the ground. [5]

5. [Maximum mark: 7]

The following diagram shows a circle with centre O and radius 3.
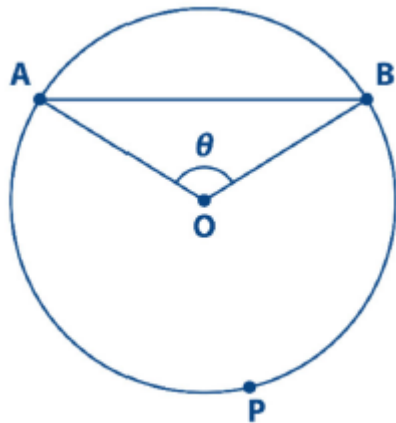
Points A, P and B lie on the circumference of the circle.

Chord $[AB]$ has length L and $A\widehat{O}B = \theta$ radians.

(a) Show that arc APB has length $6\pi - 3\theta$. [2]
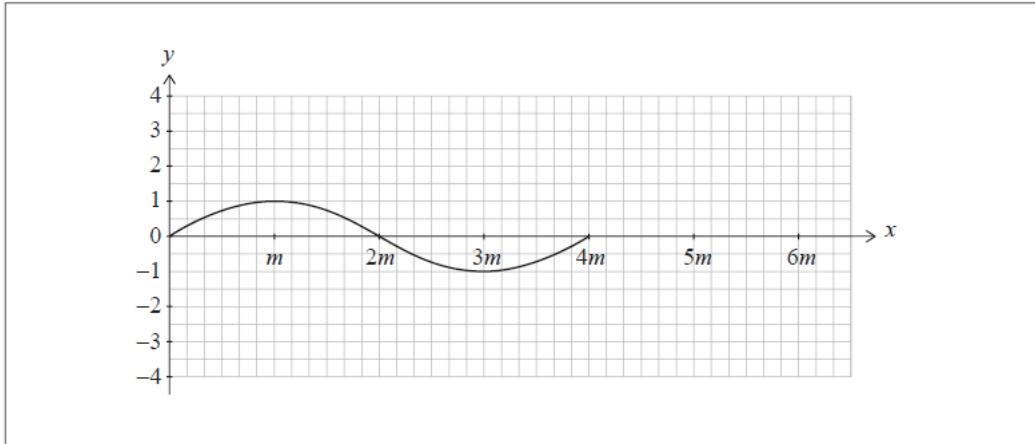
(b) Show that $L = \sqrt{18 - 18 \cos \theta}$. [2]

(c) Arc APB is twice the length of chord $[AB]$.

Find the value of θ. [3]

6.

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \leq x \leq 4m$, where x is in radians. There are x-intercepts at $x = 0, 2m$ and $4m$.

(a)   Find an expression for m in terms of q.   [2]

The function g is defined by $g\left(x\right) = 3 \sin \frac{2qx}{3}$, for $0 \leq x \leq 6m$.
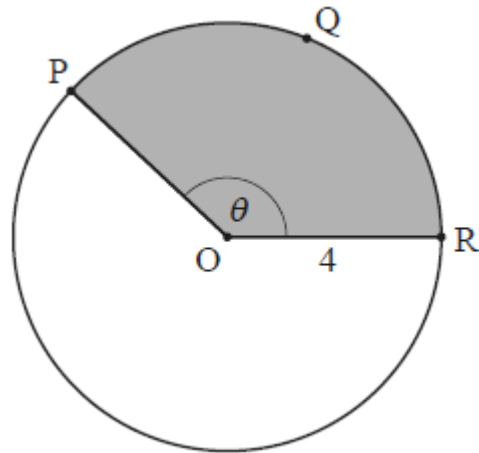
(b)   On the axes above, sketch the graph of g.   [4]

7. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 4 cm.

diagram not to scale

The points P, Q and R lie on the circumference of the circle and $P\widehat{O}R = \theta$, where θ is measured in radians.
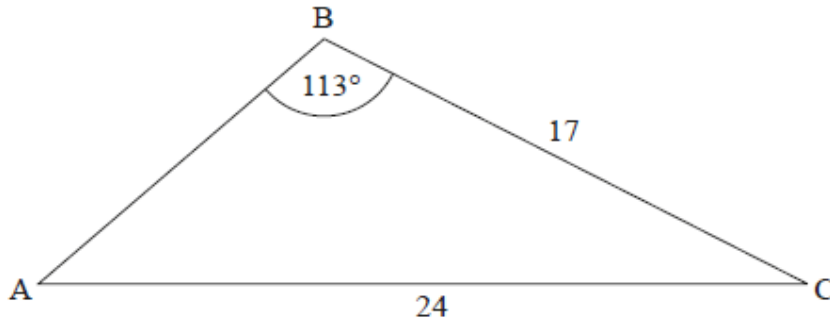
The length of arc PQR is 10 cm.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

8. [Maximum mark: 6]

The following diagram shows triangle ABC, with $AC = 24, BC = 17$, and $A\widehat{B}C = 113°$.

diagram not to scale

(a)   Find $B\widehat{A}C$.   [3]

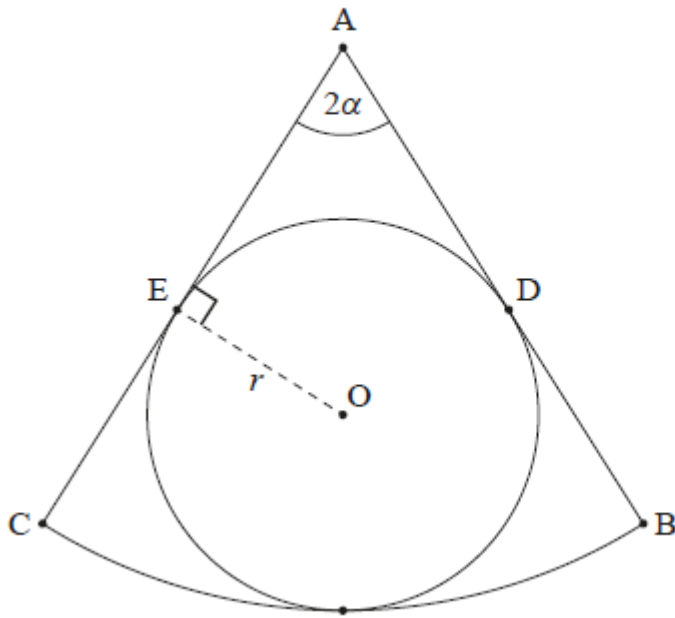(b)   Find AB.   [3]

9. [Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The

angle $B\widehat{A}C = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $O\widehat{E}A = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.
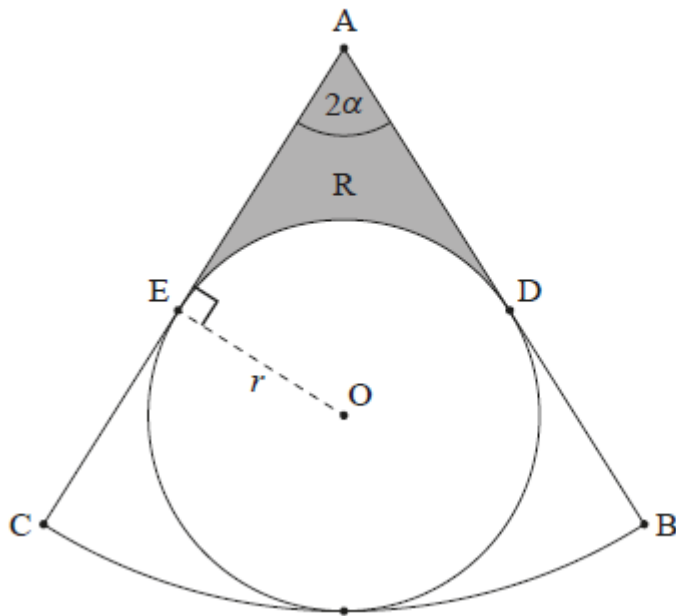
diagram not to scale

(a)   Show that the area of the quadrilateral $ADOE$ is $\dfrac{r^2}{\tan \alpha}$.   [4]

R represents the shaded region shown in the following diagram.

diagram not to scale

(b.i)   Find $D\widehat{O}E$ in terms of α.   [2]

(b.ii)   Hence or otherwise, find an expression for the area of R.   [3]

(c)   Find the value of α for which the area of R is equal to the area
         of the circle of centre O and radius r.   [4]

10. [Maximum mark: 8]

Consider a quadrilateral $ABCD$ such that $AB = 2$, $BC = 4$, $CD = 6$ and $DA = 8$, as shown in the following diagram. Let $\alpha = A\widehat{B}C$ and $\beta = A\widehat{D}C$.

diagram not to scale
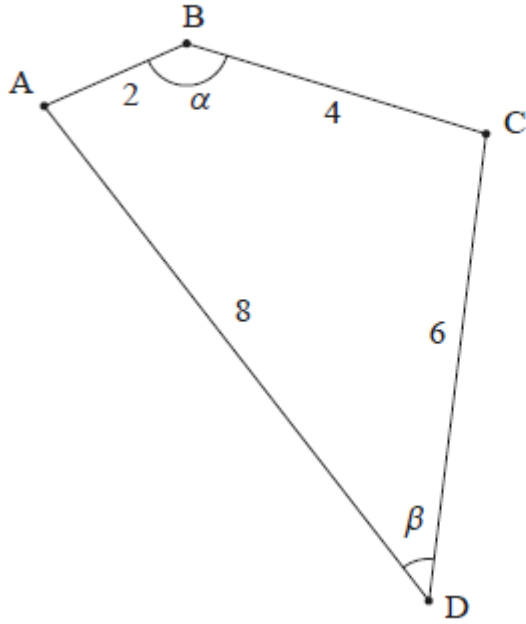
(a.i) Find AC in terms of α. [2]

(a.ii) Find AC in terms of β. [1]

(a.iii) Hence or otherwise, find an expression for α in terms of β. [1]

(b) Find the maximum area of the quadrilateral $ABCD$. [4]

11.

Consider $f(x) = 4 \sin x + 2.5$ and

$g(x) = 4 \sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a)   Describe these two transformations.   [2]

(b)   The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r.   [5]

12.

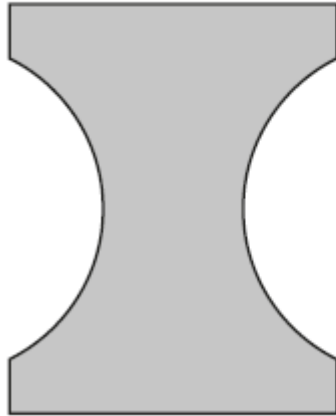Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$.   [5]

13. [Maximum mark: 6]   22M.2.SL.TZ1.3
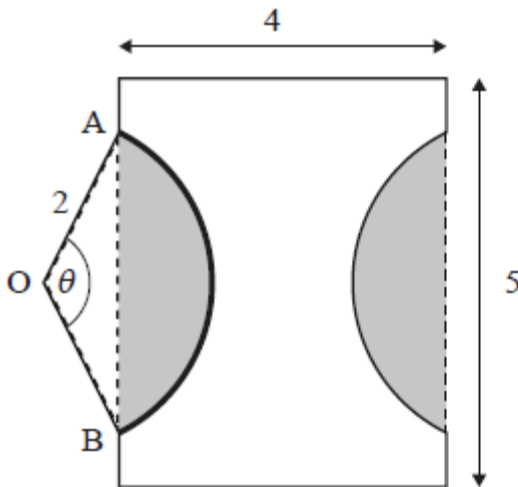
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale

The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $A\hat{O}B = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale

(a)   Find the area of one of the shaded segments in terms of θ.   [3]

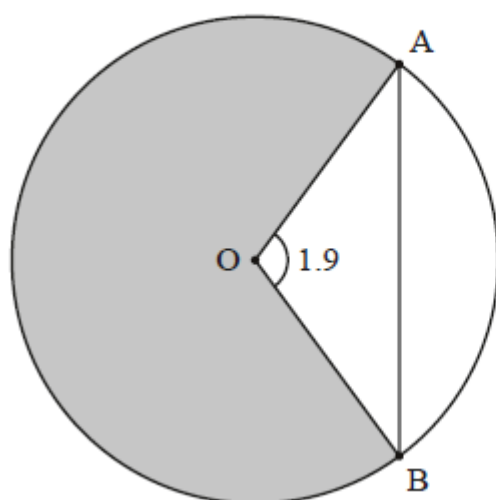(b)   Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ

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14. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\widehat{O}B = 1.9$ radians.

diagram not to scale

(a)   Find the length of the chord $[AB]$.   [3]
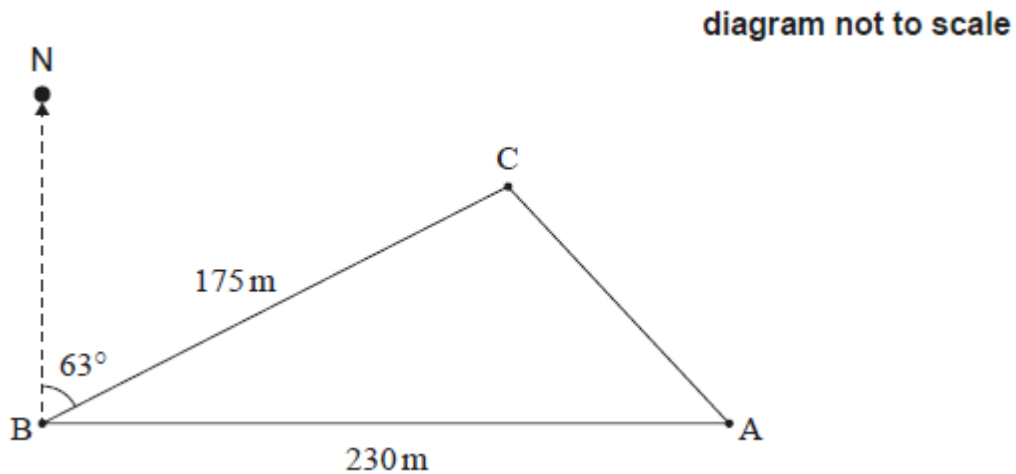
(b)   Find the area of the shaded sector.   [3]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of $063°$ to reach point C.

This is shown in the following diagram.

diagram not to scale

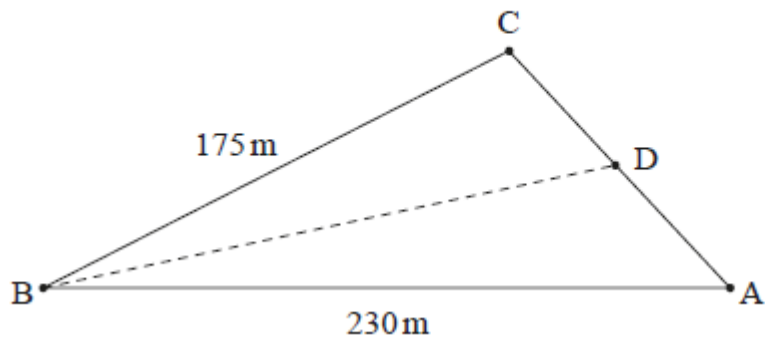(a)   Find the distance from point A to point C.   [4]

(b)   Find the area of this piece of land.   [2]

(c)   Find $\hat{CAB}$.   [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale

(d)   Find the distance from point B to point D.   [5]