

- Q4.** (i)  $3\sqrt{13}$   
(ii)  $(-5, 1)$
- (iii)  $(-8, -3.5)$  and  $(-2, 5.5)$

**calculator questions**

- Q5.** (i)  $A(-6, -2.5), B(4, 2.5)$   
(ii)  $(1, 5)$   
(iii)  $x - 2y + 9 = 0$  or  $y = \frac{1}{2}x + \frac{9}{2}$   
(iv)  $(-3.54, 2.73)$   
(v) 25.9  
(vi) 3.58  
(vii) 29.1
- (ii)  $\pm \begin{pmatrix} 1.75 \\ 8.77 \end{pmatrix}$   
(iii)  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$   
(iv)  $\begin{pmatrix} -4 \\ -8 \end{pmatrix}$
- Q6.** (i)  $\pm \begin{pmatrix} 6.84 \\ 13.7 \end{pmatrix}$
- Q7.** (i)  $A(12.2, 0), B(0.935)$   
(ii)  $(6.12, 4.68)$   
(iii) 114

## Chapter 4

# Functions

### 4.1 Basic properties

- |  |  |   |        |        |         |
|--|--|---|--------|--------|---------|
| <b>Q1.</b> (1) yes   | (2) yes  | (3) yes   | (4) no | (5) no | (6) yes |
| <b>Q2.</b> (1) (i) $-4 < x \leq 4$<br>(ii) $-3 \leq y \leq 3$<br>(iii) $-2, 2.5$<br>(iv) $] -5, 1]$<br>(v) $[1, 4]$<br>(vi) —<br>(vii) $0, 1.5$                    | (5) (i) $x \geq -5$<br>(ii) $-2 \leq y \leq 2$<br>(iii) $-4$<br>(iv) $[-5, -3]$<br>(v) $[-3, -1]$<br>(vi) $x \geq -1$<br>(vii) $-3.5 < x < -1$         | (ii) $[-1, 2[$<br>(iii) $-\frac{10}{3}, -\frac{1}{2}, 4$<br>(iv) $[-4, -2[, [-2, 1[$<br>(v) $[3, 5[$<br>(vi) $[1, 3[$<br>(vii) $-\frac{8}{3}, 3$                |        |        |         |
| (2) (i) $-5 \leq x < 5$<br>(ii) $-3 \leq y < 2$<br>(iii) 3<br>(iv) $[-5, -1], [2, 5[$<br>(v) —<br>(vi) $[-1, 2]$<br>(vii) $[-1, 2]$                                | (6) (i) $-4 < x \leq 2$<br>(ii) $-3 < y \leq 3$<br>(iii) $-1$<br>(iv) $-4 < x \leq 2$<br>(v) —<br>(vi) —<br>(vii) $-4 < x \leq -2$                     | (10) (i) $x \leq 5$<br>(ii) $y \leq 2$<br>(iii) $-3.5, -1.5$<br>(iv) $x \leq -3$<br>(v) $[-2, -1[, [-1, 0]$<br>(vi) $[-3, -2] \cup [0, 5]$<br>(vii) $x \leq -4$ |        |        |         |
| (3) (i) $-5 < x \leq 5$<br>(ii) $1 \leq y < 2$<br>(iii) —<br>(iv) —<br>(v) $] -5, -3[, ] -3, -1[, ] -1, 1[, ] 1, 3[, ] 3, 5[$<br>(vi) —<br>(vii) —                 | (7) (i) $-5 \leq x \leq 2$<br>(ii) $-2 \leq y \leq 3$<br>(iii) $-3.5, -1$<br>(iv) $[-5, -2[, [-2, 2]$<br>(v) —<br>(vi) —<br>(vii) 1                    | (11) (i) $x \geq -4$<br>(ii) $y \leq 4$<br>(iii) 3<br>(iv) $[-2, -1]$<br>(v) $[-4, -2], [-1, +\infty[$<br>(vi) —<br>(vii) $-4, -1.5, 0$                         |        |        |         |
| (4) (i) $-5 \leq x < 5$<br>(ii) $\{-2, -1, 0, 1, 2, \}$<br>(iii) $[-1, 1[$<br>(iv) —<br>(v) —<br>(vi) $[-5, -3[, ] -3, -1[, ] -1, 1[, ] 1, 3[, ] 3, 5[$<br>(vii) — | (8) (i) $x \geq -5$<br>(ii) $y \geq -1$<br>(iii) $-3, -1.5$<br>(iv) $[-2, -1], [4, +\infty[$<br>(v) $[-5, -2]$<br>(vi) $[-1, 4]$<br>(vii) $[-3, -1.5]$ | (12) (i) $-4 < x \leq 5$<br>(ii) $-2 \leq y < 3$<br>(iii) $-2.5, -1, 2$<br>(iv) $[-2, 1], [4, 5]$<br>(v) $] -4, -2[, [1, 4]$<br>(vi) —<br>(vii) $-2, 3, 5$      |        |        |         |
| (9) (i) $[-4, 5[$  | (13) (i) $x \leq 4$<br>(ii) $y \leq 3$   |   |        |        |         |

(iii) $-4, -2$	(iii) —	(ii) $-3 \leq y \leq 2$
(iv) $]-\infty, -3], [-2, 0], [3, 4]$	(iv) $]-5, -3], [-1, 1]$	(iii) $-1, 0, 3$
(v) $[-3, -2], [0, 3]$	(v) $[-3, -1], [1, 3]$	(iv) $[-4, -1], [0, 1]$
(vi) —	(vi) —	(v) $] -1, 0], [1, 4]$
(vii) $-3, -1, 1, 3.5$	(vii) $]-5, -3] \cup ] -3, 0] \cup ] 2, 3]$	(vi) —
(14) (i) $-4 \leq x < 5$	(20) (i) $-5 \leq x \leq 3$	(vii) $[-4, 1] \cup [1, 2]$
(ii) $-2 < y \leq 2$	(ii) $-2 \leq y \leq 2$	(26) (i) $-4 < x \leq 3$
(iii) $-2, -1, 4$	(iii) 2, between $-4$ and $-3$ , between $-2$ and $-1$	(ii) $-2 \leq y \leq 2$
(iv) $[2, 3]$	(iv) $[-3, 0], [2, 3]$	(iii) $-3.5, 2$
(v) $[-4, -3[, [-3, -2], ] - 2, -1], ] - 1, 0[, [0, 2], [3, 5]$	(v) $[-5, -3], [0, 2]$	(iv) $]-4, 1], ] - 1, 1]$
(vi) —	(vi) —	(v) $[1, 3]$
(vii) $-4, -3, 0, 3$	(vii) $]-5, -1] \cup ] 1, 3[$	(vi) —
(15) (i) $-5 < x < 3$	(21) (i) $-4 < x \leq 3$	(vii) $-4 < x \leq 3$
(ii) $-2 \leq y \leq 2$	(ii) $0 \leq y \leq 2$	(27) (i) $-3 \leq x < 2,$ $2 < x \leq 3$
(iii) $-4, -1, 2$	(iii) 3	(ii) $y = -1, 0 \leq y \leq 2$
(iv) $]-5, -3], [1, 3[$	(iv) $]-4, -3]$	(iii) $-2$
(v) $[-3, 1]$	(v) $[-3, -2], [1, 3]$	(iv) $[-2, -1]$
(vi) —	(vi) $[-2, 1[$	(v) $[1, 2[$
(vii) $-3$	(vii) $[-2, 1] \cup \{2\}$	(vi) $[-3, -2[, [-1, 1], ] 2, 3]$
(16) (i) $-4 < x < 4$	(22) (i) $x \leq 2$	(vii) $[-3, -2] \cup ] 2, 3]$
(ii) $-3 < y \leq 3$	(ii) $y \geq -1$	(28) (i) $-4 < x \leq -1,$ $0 < x \leq 2$
(iii) $-3, 1$	(iii) $-3, -1$	(ii) $-3 < y \leq 2$
(iv) $]-4, -1], [2, 3]$	(iv) $[-2, 0]$	(iii) $-2, 2$
(v) $[-1, 2], [3, 4[$	(v) $]-\infty, -2]$	(iv) $]-4, -1], ] 0, 1]$
(vi) —	(vi) $[0, 2]$	(v) $[1, 2]$
(vii) —	(vii) $]-3, -1[$	(vi) —
(17) (i) $-4 \leq x \leq 3$	(23) (i) $-4 < x \leq 4$	(vii) $]-2, -1] \cup ] 0, 2[$
(ii) $-2 \leq y \leq 3$	(ii) $-1 < y \leq 1$	(29) (i) $x > -4$
(iii) $-3.5, 1$	(iii) $-3, -1, 1, 3$	(ii) $y \geq -1$
(iv) $[-4, -2]$	(iv) $]-4, -2[, ] -2, 0[, ] 0, 2[, ] 2, 4]$	(iii) 0
(v) $[-2, 3]$	(v) —	(iv) $]-3, -1], [1, +\infty[$
(vi) —	(vi) —	(v) $]-4, -3], ] 0, 1]$
(vii) $[-4, 3] \cup ] 0, 3]$	(vii) $\{-2, 0, 2, 4\}$	(vi) —
(18) (i) $-4 < x \leq 3$	(24) (i) $-3 \leq x < 1$	(vii) $\{-1\} \cup ] 0, +\infty[$
(ii) $-2 \leq y \leq 2$	(ii) $\{-1, 0, 1, 2\}$	(30) (i) $-2 < x \leq 3$
(iii) $-2, 0, 3$	(iii) $[-1, 0[$	(ii) $-2 \leq y \leq 1$
(iv) $]-4, -1], [1, 3]$	(iv) —	(iii) 2
(v) $[-1, 1]$	(v) —	(iv) $[1, 3]$
(vi) —	(vi) $[-3, -2[, [-2, -1[, ] -1, 0[, ] 0, 1[$	(v) $]-2, -1]$
(vii) $]-4, -3] \cup ] -0.5, 2[$	(vii) $[-1, 1[$	(vi) $[-1, 0], ] 0, 1]$
(19) (i) $-5 < x \leq 3$	(25) (i) $-4 \leq x \leq 4$	(vii) $[-1, 0]$
(ii) $1 \leq y \leq 3$		

- Q3.** (1) domain:  $\mathbb{R}$ , range:  $[0, +\infty[$       (4) domain:  $[0, +\infty[$ , range:  $[0, +\infty[$   
 (2) domain:  $\mathbb{R}$ , range:  $\mathbb{R}$       (5) domain:  $\mathbb{R}$ , range:  $\mathbb{R}$   
 (3) domain:  $\mathbb{R} \setminus \{0\}$ , range:  $\mathbb{R} \setminus \{0\}$       (6) domain:  $\mathbb{R}$ , range:  $[0, +\infty[$

- Q4.** (1)  $-\frac{5}{2}$       (4)  $\pm 3$       (7)  $-\frac{3\pi}{2}$       (10)  $\pm 2$       (13)  $-2\sqrt{3}$       (16) 1  
 (2) 3      (5)  $\frac{1}{2}$       (8)  $-\frac{1}{3}$       (11)  $-3, 1$       (14) 4.5      (17)  $\frac{1}{2}, \frac{7}{2}$   
 (3)  $\frac{1}{3}$       (6)  $-3, 1$       (9)  $-5.5$       (12) —      (15) 2      (18) 9

- Q5.** (1) domain:  $\mathbb{R}$ , range:  $\mathbb{R}$   
 (2) domain:  $]-\infty, 3]$ , range:  $[0, +\infty[$   
 (3) domain:  $\mathbb{R} \setminus \{1\}$ , range:  $\mathbb{R} \setminus \{3\}$   
 (4) domain:  $\mathbb{R}$ , range:  $[-4, +\infty[$   
 (5) domain:  $\mathbb{R}$ , range:  $[-2, +\infty[$   
 (6) domain:  $\mathbb{R}$ , range:  $\mathbb{R}$   
 (7) domain:  $[-\frac{1}{3}, +\infty[$ , range:  $[0, +\infty[$   
 (8) domain:  $\mathbb{R} \setminus \{-4\}$ , range:  $\mathbb{R} \setminus \{-2\}$
- (9) domain:  $\mathbb{R}$ , range:  $[-9, +\infty[$   
 (10) domain:  $\mathbb{R}$ , range:  $[3, +\infty[$   
 (11) domain:  $\mathbb{R}$ , range:  $\mathbb{R}$   
 (12) domain:  $[0, +\infty[$ , range:  $[-3, +\infty[$   
 (13) domain:  $\mathbb{R}$ , range:  $[0, +\infty[$   
 (14) domain:  $\mathbb{R}$ , range:  $] -\infty, 3]$   
 (15) domain:  $[0, \frac{1}{4}[ \cup ]\frac{1}{4}, +\infty[$ , range:  $] -\infty, 4[ \cup ]7, +\infty[$

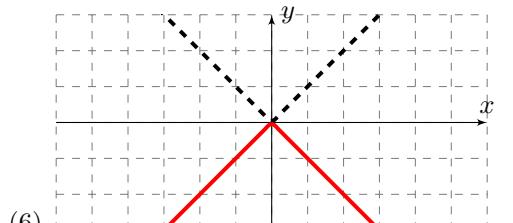
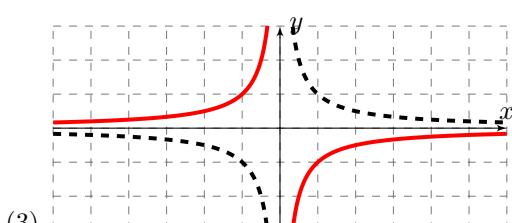
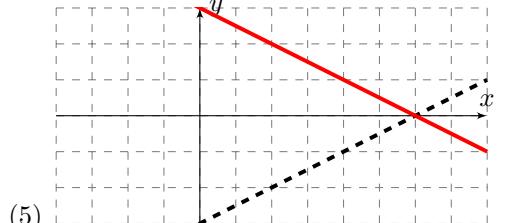
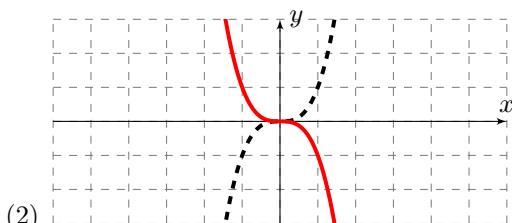
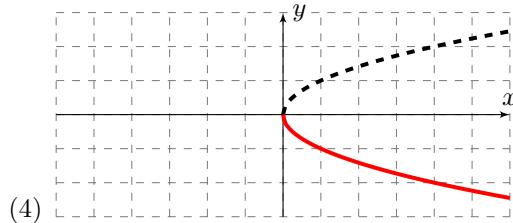
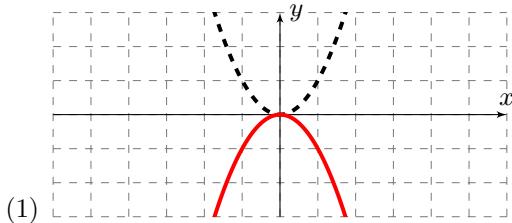
- Q6.** (1) 1 (4)  $-40$  (7)  $5\pi$  (10)  $-\frac{11}{9}$  (13) 18 (16)  $-27$   
 (2) 3 (5) 16 (8) 5 (11)  $\frac{13}{9}$  (14)  $2\sqrt{6} - 3$  (17)  $\frac{2}{3}$   
 (3) 3.5 (6) 0 (9)  $-3$  (12) 4.2 (15)  $-16$  (18)  $-\frac{1}{3}$

- Q7.** (1)  $f(-x) = -2x + 5$  (7)  $f(-x) = -\frac{2}{3}x + \pi$  (13)  $f(-x) = -\sqrt{3}x + 6$   
 (2)  $f(-x) = \sqrt{3+x}$  (8)  $f(-x) = \sqrt{1-3x}$  (14)  $f(-x) = \sqrt{-2x} - 3$   
 (3)  $f(-x) = 3 - \frac{2}{x+1}$  (9)  $f(-x) = \frac{3}{x-4} - 2$  (15)  $f(-x) = -x^3 - 8$   
 (4)  $f(-x) = 9 - x^2$  (10)  $f(-x) = x^2 - 4$  (16)  $f(-x) = -(x+1)^3$   
 (5)  $f(-x) = (2x+1)^2$  (11)  $f(-x) = (1-x)^2 - 4$  (17)  $f(-x) = 3 - |2x+4|$   
 (6)  $f(-x) = |1-x| - 2$  (12)  $f(-x) = 3 + |4+2x|$  (18)  $f(-x) = \frac{\sqrt{-x}-3}{2\sqrt{-x-1}}$

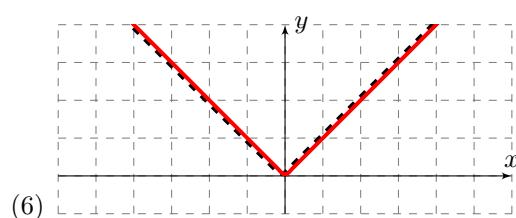
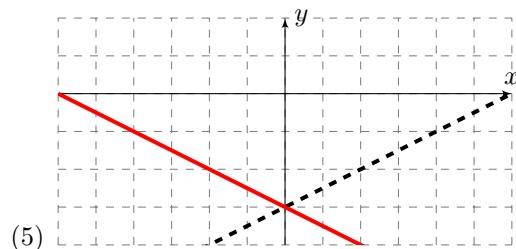
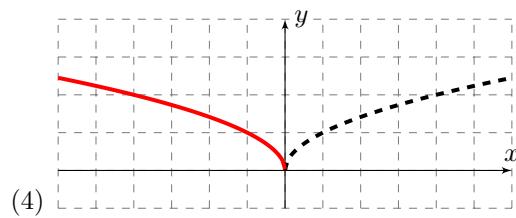
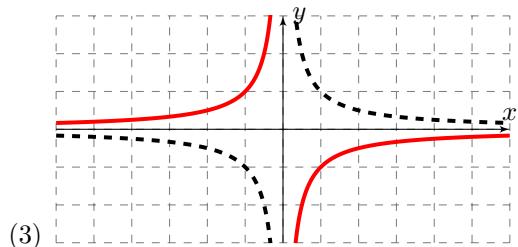
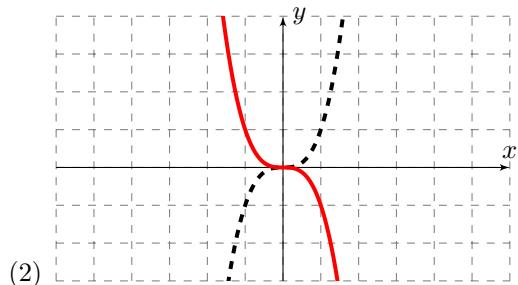
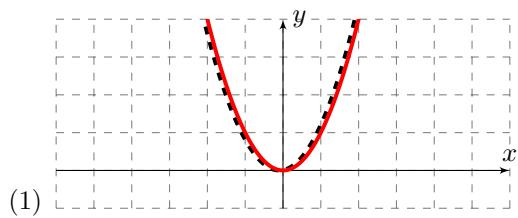
- Q8.** (1) O (4) E (7) O (10) E (13) E (16) N  
 (2) N (5) E (8) O (11) N (14) O (17) E  
 (3) O (6) N (9) N (12) E (15) E (18) N

## 4.2 Transformations of graphs of functions

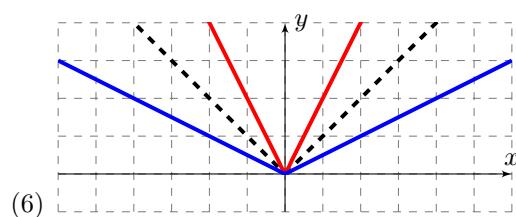
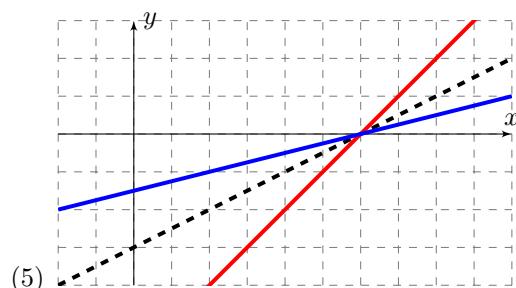
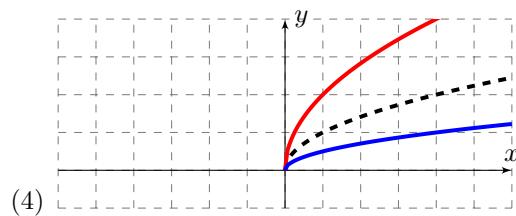
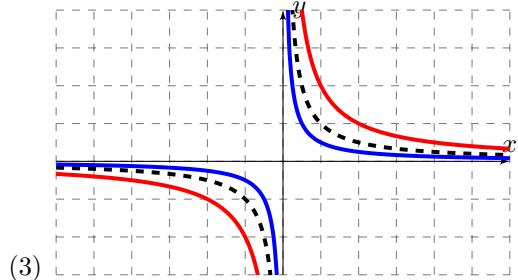
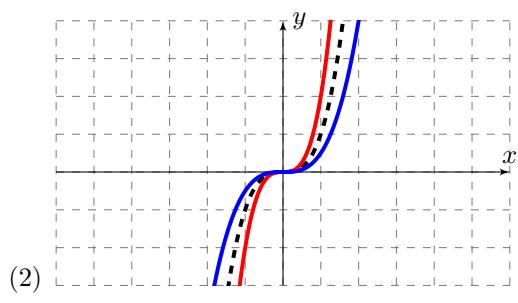
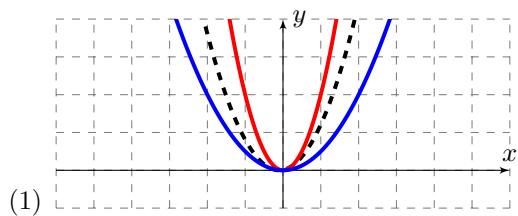
- Q9.** Graphs of  $y = f(x)$  (black, dashed) and  $y = -f(x)$  (red, solid)



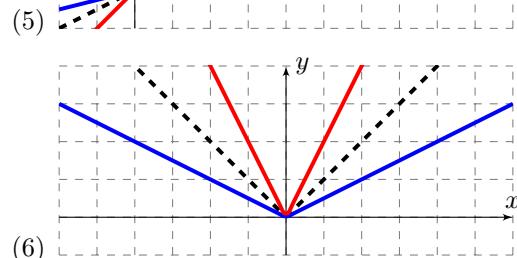
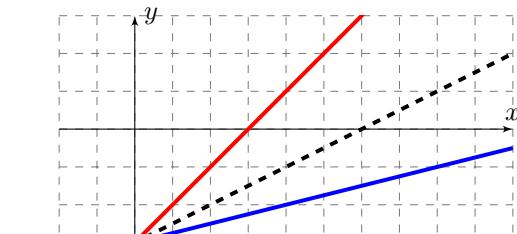
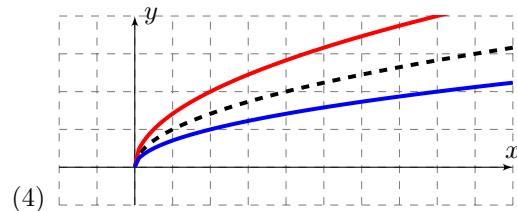
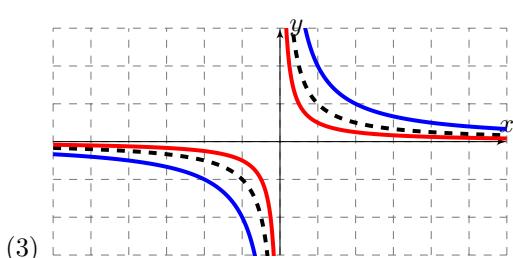
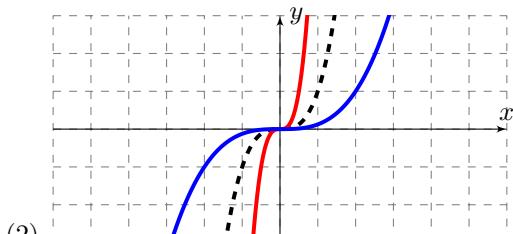
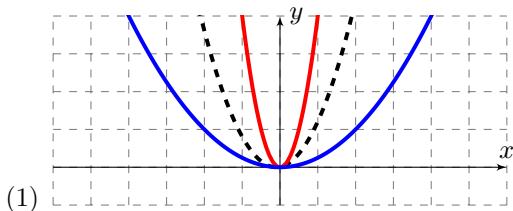
- Q10.** Graphs of  $y = f(x)$  (black, dashed) and  $y = f(-x)$  (red, solid)



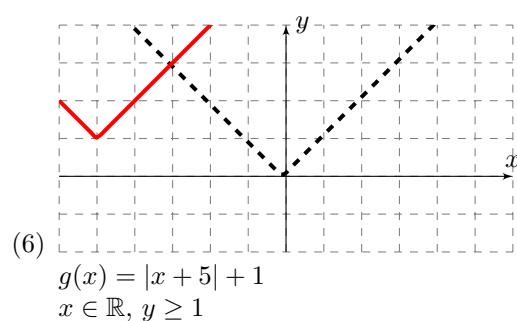
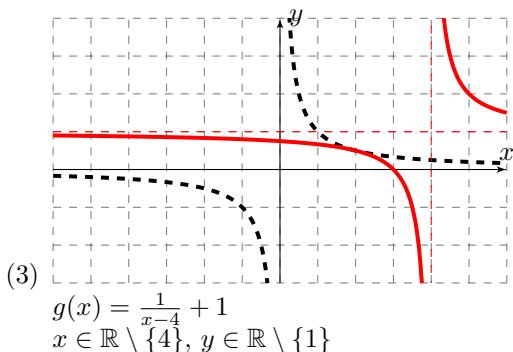
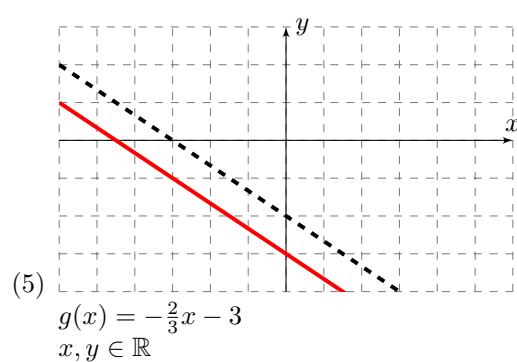
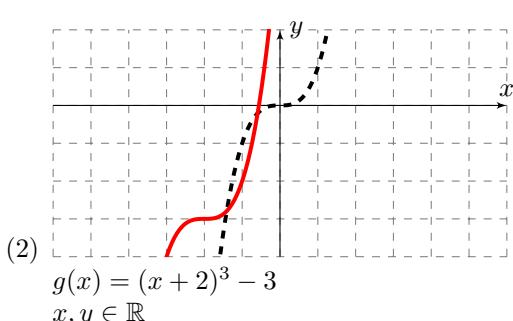
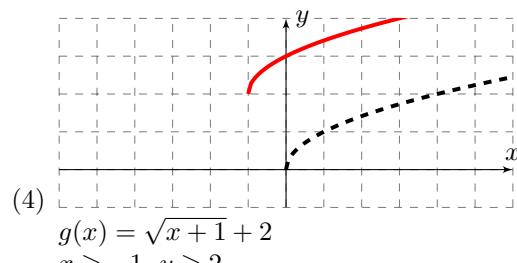
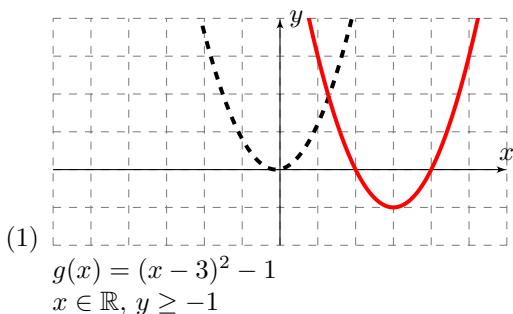
**Q11.** Graphs of  $y = f(x)$  (black, dashed),  $y = 2f(x)$  (red, solid) and  $y = \frac{1}{2}f(x)$  (blue, solid)

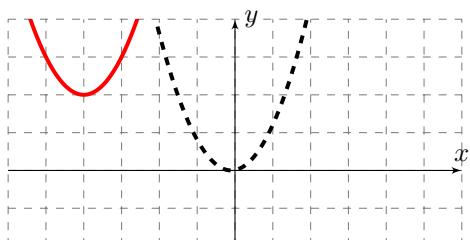


**Q12.** Graphs of  $y = f(x)$  (black, dashed),  $y = f(2x)$  (red, solid) and  $y = f(\frac{1}{2}x)$  (blue, solid)

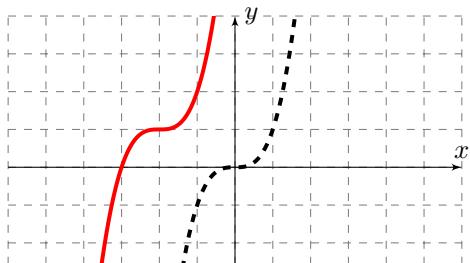


**Q13.** Graphs of  $y = f(x)$  (black, dashed) and  $y = g(x)$  (red, solid).

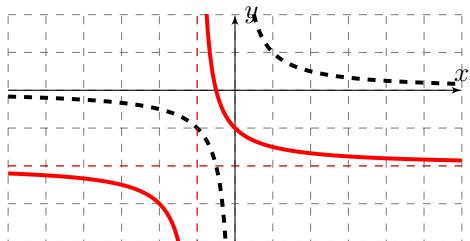




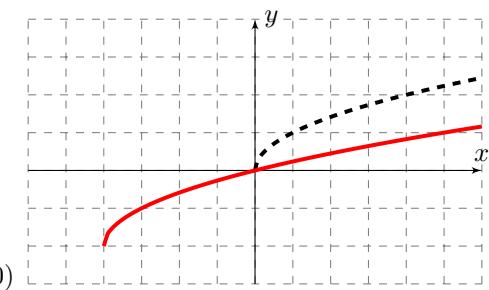
(7)  $g(x) = (x + 4)^2 + 2$   
 $x \in \mathbb{R}, y \geq 2$



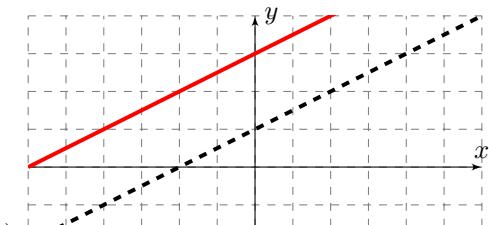
(8)  $g(x) = (x + 2)^3 + 1$   
 $x, y \in \mathbb{R}$



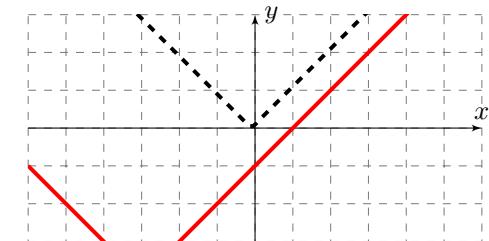
(9)  $g(x) = \frac{1}{x+1} - 2$   
 $x \in \mathbb{R} \setminus \{-1\}, x \in \mathbb{R} \setminus \{-2\}$



(10)  $g(x) = \sqrt{x + 4} - 2$   
 $x \geq -4, y \geq -2$

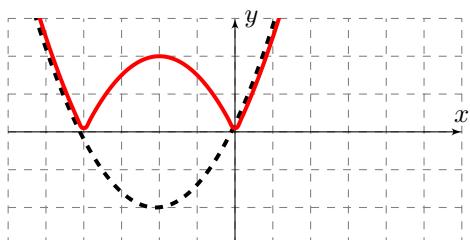


(11)  $g(x) = \frac{1}{2}x + 3$   
 $x, y \in \mathbb{R}$

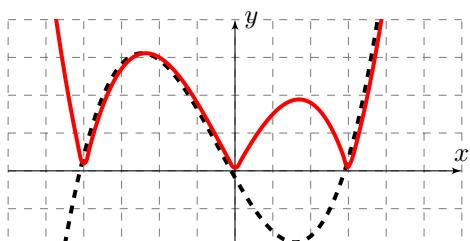


(12)  $g(x) = |x + 3| - 4$   
 $x \in \mathbb{R}, y \geq -4$

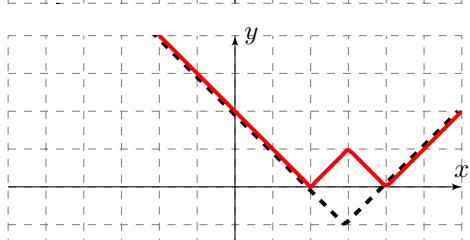
Q14.  $y = f(x)$  (black, dashed),  $y = |f(x)|$  (red, solid)



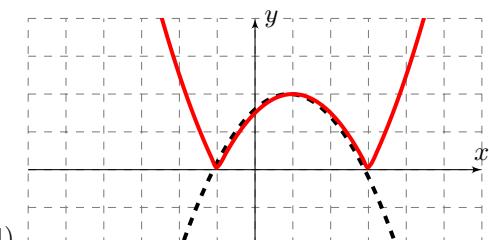
(1)



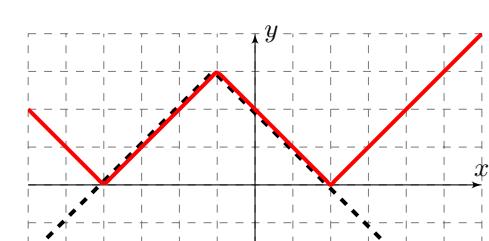
(2)



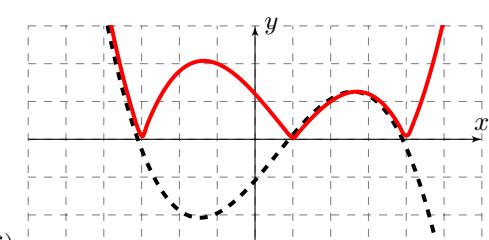
(3)



(4)

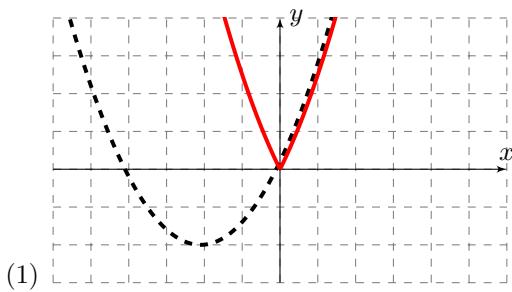


(5)

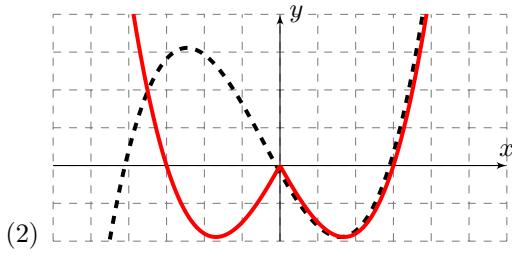


(6)

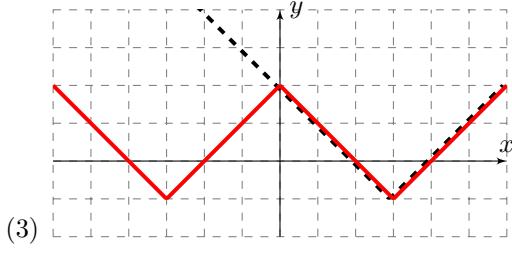
**Q15.** graph of  $y = f(x)$  (dashed, black) and of  $y = f(|x|)$  (solid, red)



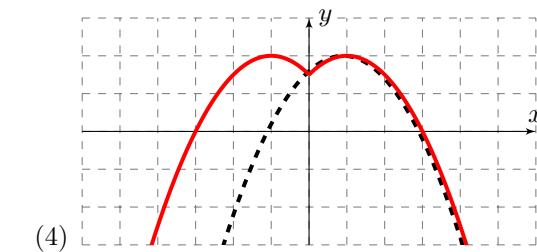
(1)



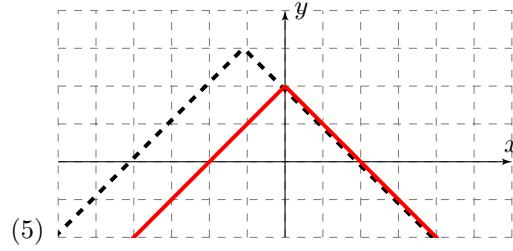
(2)



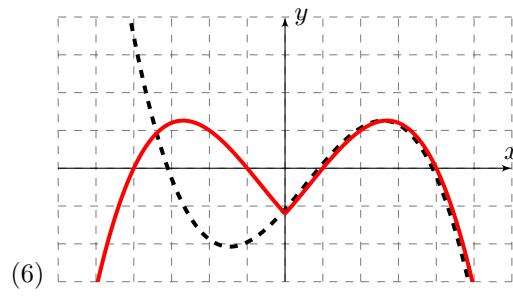
(3)



(4)



(5)



(6)

**Q16.** (1) (i)  $y = x^2$

$$\text{(ii) translation by } \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\text{(iii) } y = (x - 3)^2 + 1$$

(2) (i)  $y = x^2$

$$\text{(ii) translation by } \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\text{(iii) } y = (x - 2)^2 - 4$$

(3) (i)  $y = x^2$

$$\text{(ii) translation by } \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

$$\text{(iii) } y = (x + 1)^2 - 4$$

(4) (i)  $y = x^3$

$$\text{(ii) translation by } \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\text{(iii) } y = (x - 4)^3 + 1$$

(5) (i)  $y = x^3$

$$\text{(ii) translation by } \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

$$\text{(iii) } y = (x + 5)^3 + 1$$

(6) (i)  $y = x^3$

(ii) reflection in  $x$ -axis or in  $y$ -axis

$$\text{(iii) } y = -x^3$$

(7) (i)  $y = x^3$

(ii) reflection in  $x$ -axis or in  $y$ -axis

followed by translation by  $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$$\text{(iii) } y = -(x + 5)^3 + 1$$

(8) (i)  $y = \sqrt{x}$

$$\text{(ii) translation by } \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

$$\text{(iii) } y = \sqrt{x + 5} - 2$$

(9) (i)  $y = \sqrt{x}$

(ii) reflection in  $y$ -axis

$$\text{(iii) } y = \sqrt{-x}$$

(10) (i)  $y = \sqrt{x}$

(ii) reflection in  $y$ -axis

followed by translation by  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

$$\text{(iii) } y = \sqrt{-x} - 3$$

(11) (i)  $y = |x|$

$$\text{(ii) translation by } \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\text{(iii) } y = |x - 2| - 3$$

(12) (i)  $y = |x|$

(ii) reflection in  $x$  axis

followed by translation by  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$\text{(iii) } y = -|x - 2| + 1$$

(13) (i)  $y = \sqrt{x}$

$$\text{(ii) translation by } \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\text{(iii) } y = \sqrt{x + 2} - 2$$

(14) (i)  $y = \sqrt{x}$

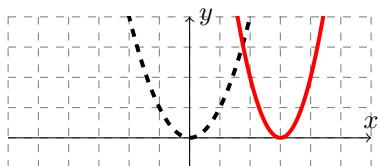
(ii) reflection in  $y$ -axis

followed by translation by  $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

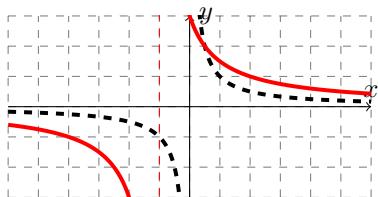
- or translation by  $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$   
followed by reflection in  $y$ -axis
- (iii)  $y = \sqrt{-x+2} - 2$
- (15) (i)  $y = \sqrt{x}$   
 (ii) reflection in  $x$ -axis followed by shift 4 left and 1 up  
 (iii)  $y = -\sqrt{x+4} + 1$
- (16) (i)  $y = \frac{1}{x}$   
 (ii) reflection in  $x$ -axis or in  $y$ -axis  
 (iii)  $y = -\frac{1}{x}$
- (17) (i)  $y = \frac{1}{x}$   
 (ii) translation 2 right  
 (iii)  $y = \frac{1}{x-2}$
- (18) (i)  $y = \frac{1}{x}$   
 (ii) translation by  $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$   
 (iii)  $y = \frac{1}{x+2} - 1$
- (19) (i)  $y = \frac{1}{x}$   
 (ii) reflection in  $x$ -axis or in  $y$ -axis  
 followed by translation by  $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$   
 (iii)  $y = -\frac{1}{x+2} - 1$
- (20) (i)  $y = \frac{1}{x}$   
 (ii) reflection in  $x$ -axis or in  $y$ -axis  
 followed by translation by  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$   
 (iii)  $y = -\frac{1}{x-1} + 2$
- (21) (i)  $y = x^2$   
 (ii) vertical stretch by  $-2$   
 followed by translation by  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$   
 (iii)  $y = -2(x-3)^2 + 3$
- (22) (i)  $y = x^2$   
 (ii) vertical stretch by  $\frac{1}{2}$   
 followed by translation by  $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$
- (23) (i)  $y = |x|$   
 (ii) vertical stretch by  $-\frac{2}{3}$   
 followed by translation by  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$   
 (iii)  $y = -\frac{2}{3}|x-3| + 2$
- (24) (i)  $y = x^3$   
 (ii) vertical stretch by  $-\frac{1}{4}$   
 followed by translation by  $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$   
 (iii)  $y = -\frac{1}{4}(x+3)^3 + 2$
- (25) (i)  $y = \sqrt{x}$   
 (ii) horizontal dilation by  $-\frac{1}{2}$   
 followed by translation 2 down  
 (iii)  $y = \sqrt{-2x} - 2$
- (26) (i)  $y = \sqrt{x}$   
 (ii) vertical stretch by 2  
 followed by translation by  $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$   
 (iii)  $y = 2\sqrt{x+1} - 3$
- (27) (i)  $y = \sqrt{x}$   
 (ii) reflection in  $x$ -axis followed by horizontal stretch by 2  
 and translation 3 up  
 (iii)  $y = -\sqrt{\frac{x}{2}} + 3$
- (28) (i)  $y = \frac{1}{x}$   
 (ii) vertical / horizontal dilation by  $\frac{1}{2}$   
 followed by translation 1 down  
 (iii)  $y = \frac{1}{2x} - 1$
- (29) (i)  $y = \frac{1}{x}$   
 (ii) vertical / horizontal stretch by  $-2$   
 followed by shift 2 right 1 up  
 (iii)  $y = \frac{-2}{x-2} + 1$
- (30) (i)  $y = x^2$   
 (ii) shift 3 right and 4 down followed by reflection of the part below  $x$ -axis in the axis  
 (iii)  $y = |(x-3)^2 - 4|$

**Q17.** Graphs of  $y = f(x)$  (black, dashed) and  $y = g(x)$  (red, dotted).

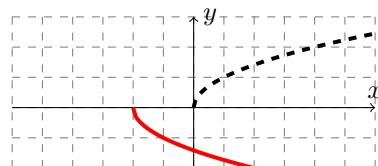
- (1) vertical stretch by 2, translation 3 right



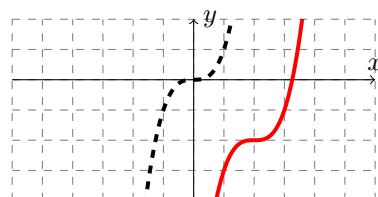
- (2) vertical stretch by 3, translation 1 left



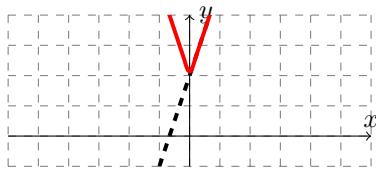
- (3) reflection in  $x$ -axis, translation 2 left



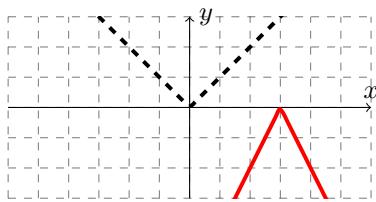
- (4) translation 2 right, 2 down



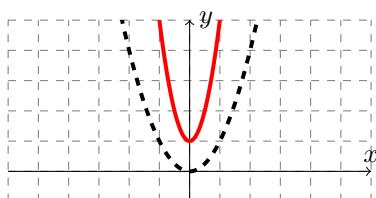
- (5) reflection in  $y$  axis of the part for  $x > 0$



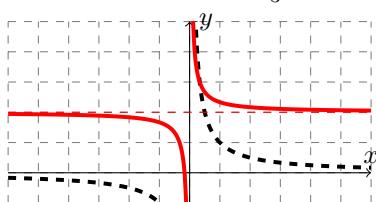
- (6) vertical stretch by  $-2$ , translation 3 right



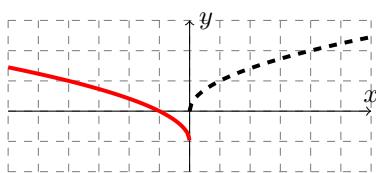
- (7) horizontal dilation by  $\frac{1}{2}$ , translation 1 up



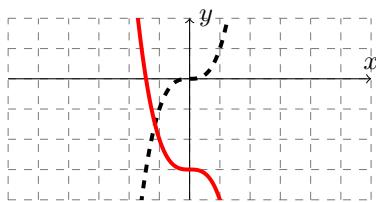
- (8) horizontal dilation by  $\frac{1}{3}$ , translation 2 up



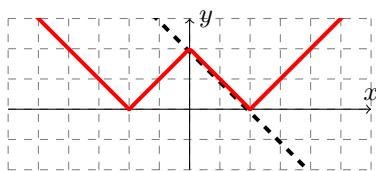
- (9) reflection in  $y$ -axis, translation 1 down



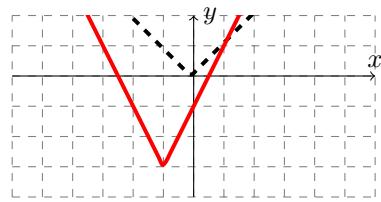
- (10) reflection in  $x/y$ -axis followed by translation 3 down



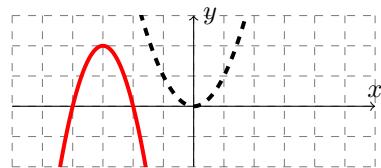
- (11) reflection in  $y$ -axis of the part  $x > 0$  followed by reflection in  $x$ -axis of the part  $y < 0$



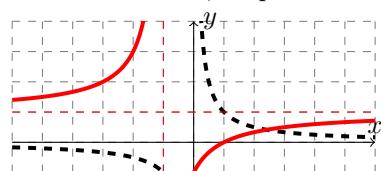
- (12) vertical stretch by 2 followed by translation 1 left, 3 down



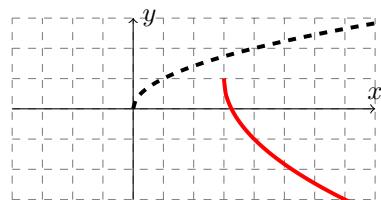
- (13) vertical stretch by  $-2$  followed by translation 3 left, 2 up



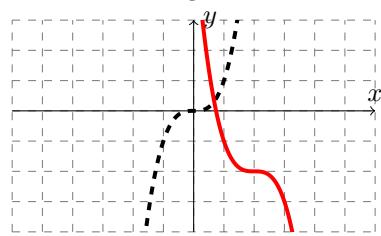
- (14) vertical dilation by  $-2$  followed by translation 1 left, 1 up



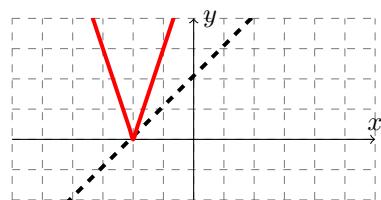
- (15) vertical dilation by  $-2$  followed by translation 3 right 1 up



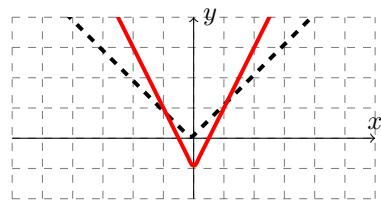
- (16) reflection in  $x$ -axis followed by translation 2 right 2 down



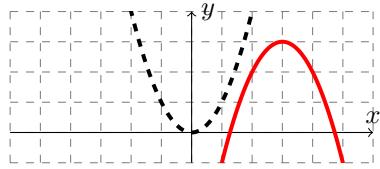
- (17) reflection in  $x$ -axis of the part  $y < 0$  followed by vertical stretch by 2



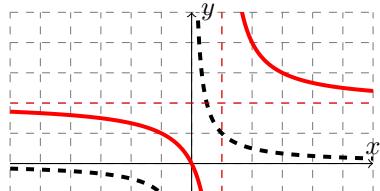
- (18) horizontal dilation by  $\frac{1}{2}$ , translation 1 down



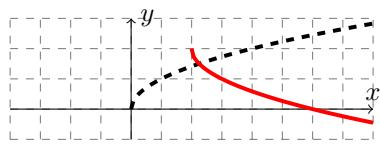
- (19) reflection in  $x$ -axis followed by translation 3 right, 3 up



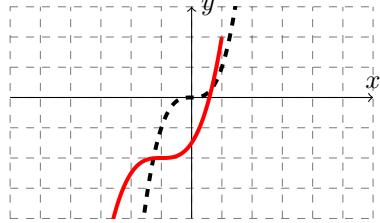
- (20) vertical stretch by 2 followed by translation 1 right, 2 up



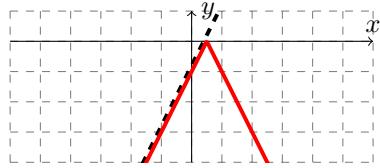
- (21) reflection in  $x$ -axis followed by translation 2 right 2 up



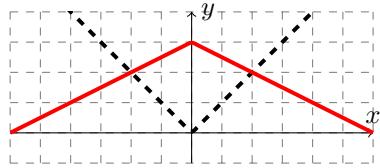
- (22) vertical dilation by  $\frac{1}{2}$  followed by translation 1 left 2 down



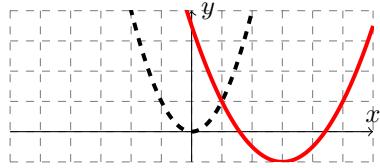
- (23) reflection in  $x$ -axis of the part  $y < 0$  followed by reflection in  $x$ -axis



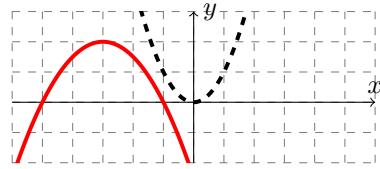
- (24) reflection in  $x$ -axis and horizontal dilation by 2 followed by translation 3 up



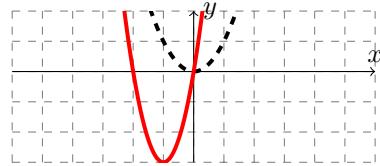
- (25) vertical dilation by  $\frac{1}{2}$  followed by translation 3 right, 1 down



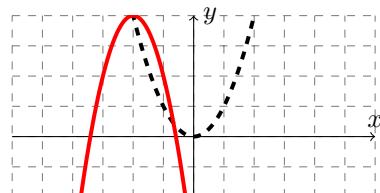
- (26) vertical dilation by  $-\frac{1}{2}$  followed by translation 3 left, 2 up



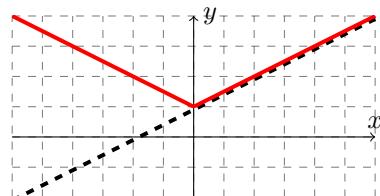
- (27) vertical dilation by 3 followed by translation 1 left, 3 down



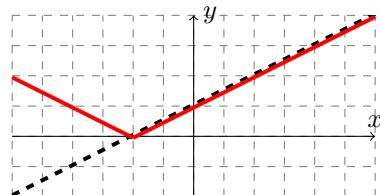
- (28) vertical dilation by  $-2$  followed by translation 2 left, 4 up



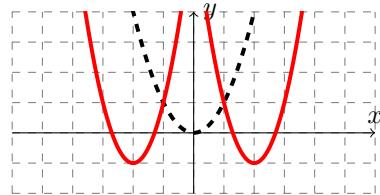
- (29) reflection of the part left of  $y$ -axis in the axis



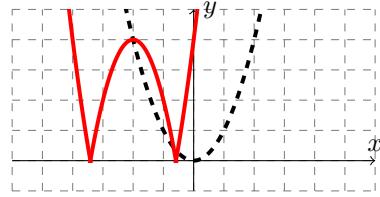
- (30) reflection of the part below  $x$ -axis in the axis



- (31) vertical stretch by 2, then shift 2 right and 1 down and reflection of the part right of  $y$ -axis in the axis



- (32) vertical stretch by 2, then shift by  $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$  and reflection of the part below  $x$ -axis in the axis

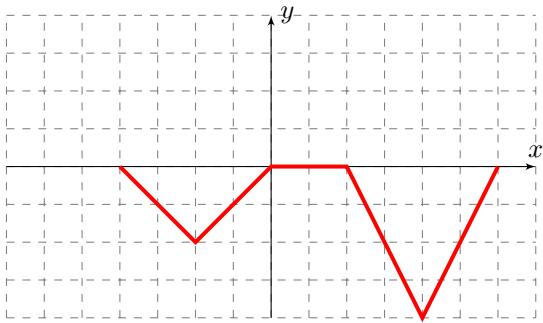


- Q18.** (1)  $A' = (4, 4)$ ,  
 (2)  $A' = (-3, 2)$ ,

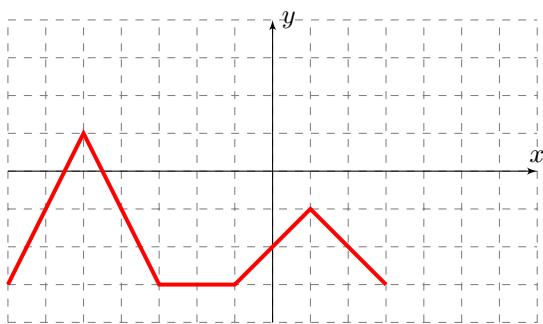
- (3)  $A' = (-1.5, 3)$ ,  
 (4)  $A' = (-3, -4)$ ,

- (5)  $A' = (2, 7)$ ,  
 (6)  $A' = (-8, -5)$ .

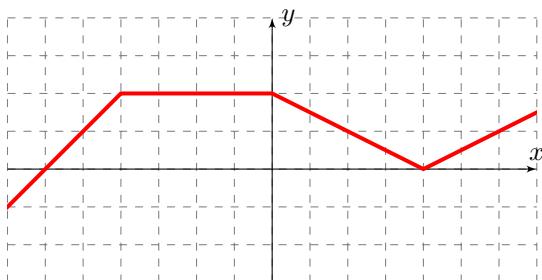
- Q19.** (i) (1)  $y = f(-x) - 2$



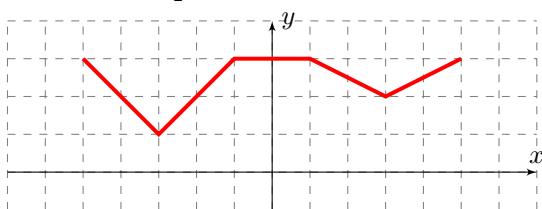
(2)  $y = -f(x + 1) - 1$



- (3)  $y = f(\frac{x}{2})$



(4)  $y = \frac{1}{2}f(x - 1) + 2$



- (ii) (1)  $y = -f(x) - 1$   
 (2)  $y = f(2x) - 1$   
 (3)  $y = \frac{1}{2}f(-2x)$

### 4.3 Equations and inequalities

- Q20.** (1)  $-1, \frac{1}{3}$       (3)  $-\frac{7}{4}, \frac{3}{4}$       (5) 2      (7) 11      (9)  $-1$       (11) 3  
 (2)  $-1, 2$       (4)  $-\frac{8}{3}, \frac{4}{3}$       (6)  $-1$       (8)  $\frac{4}{3}$       (10) 1      (12)  $\frac{1}{2}$

- Q21.** (i)  $-2, 2$   
 (ii)  $0, 4$

- Q22.** (i)  $-3$   
 (ii)  $-5$

- Q23.** (i)  $\frac{1}{2}$   
 (ii)  $\frac{7}{2}$

- Q24.** (i) 9  
 (ii) 4.5

- Q25.** (1) 0: —  
 (2) 1:  $-1.31$   
 (3) 2:  $-1, 1.54$   
 (4) 1:  $0.0605$       (5) 3:  $-0.0644, 3.17, 4.89$   
 (6) 1:  $2.21$       (7) 1:  $-1.52$   
 (8) 3:  $-0.481, 1.31, 3.17$       (9) 2:  $-1.15, 1.15$   
 (10) 2:  $-1.22, 0.549$   
 (11) 2:  $0.780, 5.55$   
 (12) 2:  $-5.24, -0.764$

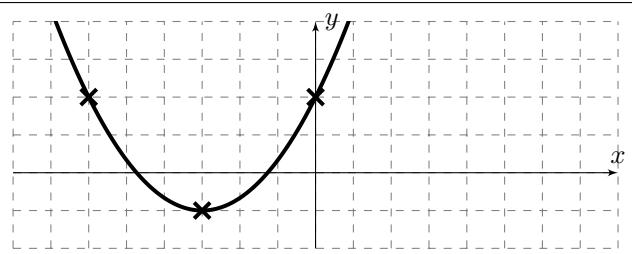
- Q26.** (1)  $x \leq -0.861$  or  $0.746 \leq x \leq 3.11$   
 (2)  $-4.59 < x < -0.887$  or  $x > 1.47$   
 (3)  $-0.535 \leq x \leq 0.444$  or  $x \geq 3.69$   
 (4)  $-1.65 < x < 1.27$  or  $2 < x < 2.38$   
 (5)  $-0.618 < x < 0$  or  $1.62 < x \leq 2$   
 (6)  $0 \leq x \leq 1$   
 (7)  $-1.88 \leq x < -1$  or  $0.347 \leq x \leq 1.53$       (8)  $-2 \leq x \leq -1.41$  or  $-1 < x \leq 1.41$   
 (9)  $-3 < x \leq -2$  or  $-1.41 \leq x < -1$   
 or  $1.41 \leq x < 3$   
 (10)  $-0.562 < x < 1$  or  $3.56 < x \leq 4$   
 (11)  $1 \leq x \leq 3.56$   
 (12)  $-2.41 < x < -0.305$   
 (13)  $0.918 \leq x \leq 2.66$

### 4.4 Chapter review

#### non-calculator questions

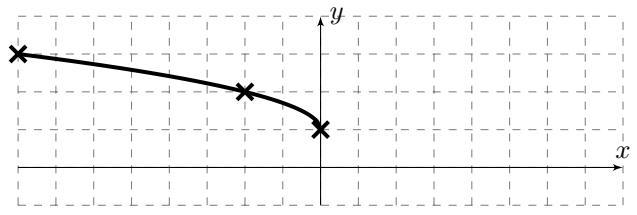
Q1. (1)

vertical dilation by  $\frac{1}{3}$   
*followed by*  
shift 3 left and 1 down



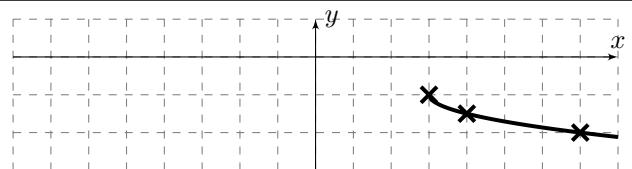
(2)

horizontal dilation by  $-2$   
*followed by*  
shift 1 up



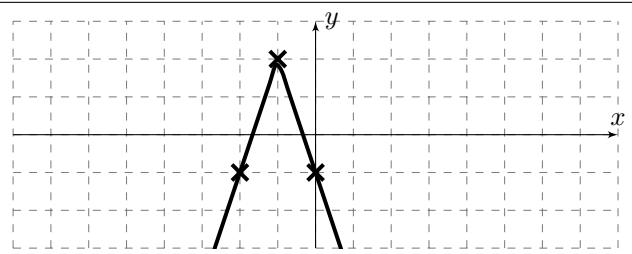
(3)

vertical dilation by  $-\frac{1}{2}$   
*followed by*  
shift 3 right and 1 down



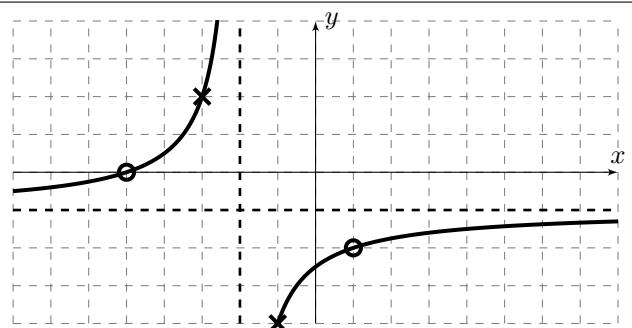
(4)

vertical dilation by  $-3$   
*followed by*  
shift 1 left and 2 up



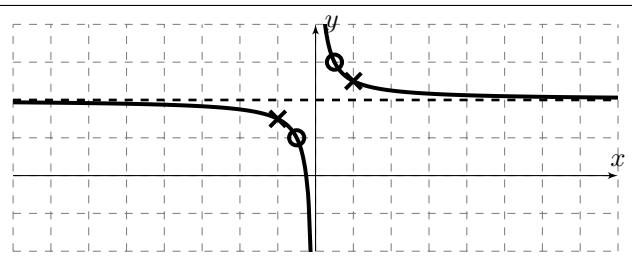
(5)

vertical (or horizontal) dilation by  $-3$   
*followed by*  
shift 2 left and 1 down

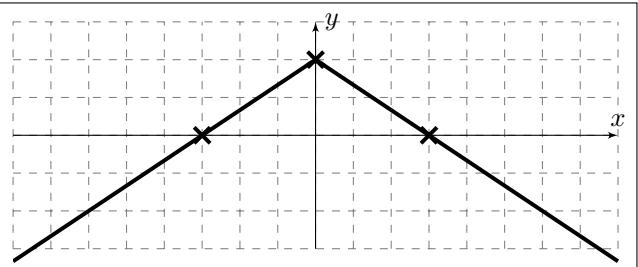


(6)

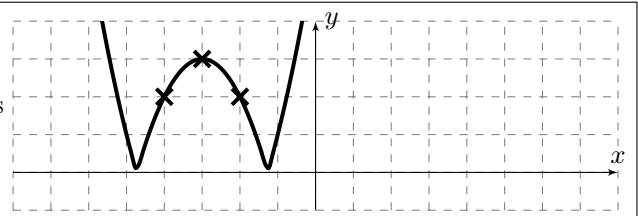
vertical (or horizontal) dilation by  $-\frac{1}{2}$   
*followed by*  
shift 2 up



(7)

reflection of the part right of  $y$ -axis in the axis

(8)

shift 3 left and 3 down  
followed by  
reflection of the part below the  $x$ -axis in the axis

Q2. (1)  $y = -2(x + 3)^2 + 3$

(2)  $y = \frac{-2}{x-1} - 1$

(3)  $y = \frac{1}{2}(x - 2)^3 + 1$

(4)  $y = -\frac{2}{3}|x + 3| + 2$

(5)  $y = \sqrt{-2x} - 1$

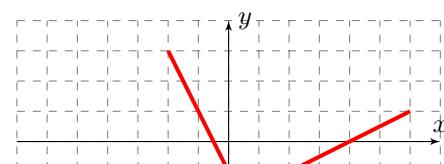
(6)  $y = ||x + 3| - 2|$

Q3. (1) even

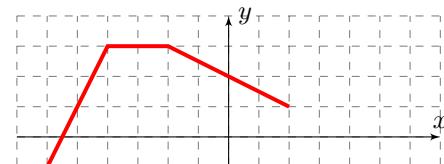
(2) odd

(3) neither

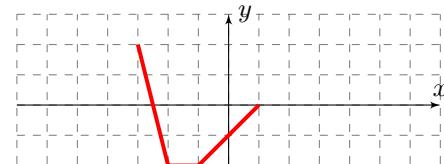
Q4. (i) (1)



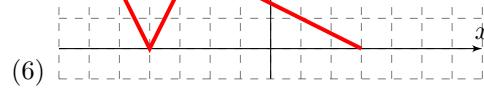
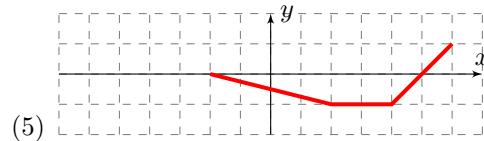
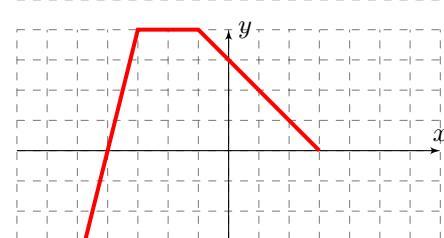
(2)



(3)



(4)

(ii) (1)  $y = f(x - 2) + 1$ (2)  $y = -f(x + 1) + 1$ (3)  $y = -2f(x - 1)$ (4)  $y = \frac{1}{2}f(x) - 1$ (5)  $y = |f(x) + 1|$ (6)  $y = f(2x) + 1$ (7)  $y = f(|x|)$ 

Q5. DOMAIN:

$f_1: x \leq -\frac{3}{2}$

$f_2: x \in \mathbb{R}$

$f_3: x \in \mathbb{R}$

$f_4: x \in \mathbb{R}$

$f_5: x \in \mathbb{R}, x \neq -1$

$f_6: x \in \mathbb{R}$

$f_7: x > -3$

$f_8: x \leq 3, x \neq -3$

RANGE:

$f_1: y \leq 0$

$f_2: y \leq 4$

$f_3: y \geq -1$

$f_4: y \geq -5$

$f_5: y \in \mathbb{R}, y \neq 3$

$f_6: y \leq 2$

function	(1)	(2)	(3)
domain	$[-5, -1] \cup [1, 5]$	$] -5, 4[$	$[-5, 5]$
range	$[0, 2]$	$] -4, 2]$	$[-2, 2]$
zeroes	$-4, 4$	$-3, 0, 3$	$-4, 0, 4$
decreasing	$[-5, -4], [2, 4]$	$[-1, 1]$	$[-5, -4], [-2, 2], [4, 5]$
increasing	$[-4, -2], [4, 5]$	$] -5, -1], [1, 4[$	$[-4, -2], [2, 4]$
constant	$[-2, -1[, ]1, 2]$	$\emptyset$	$\emptyset$
even	yes	no	no
odd	no	no	yes
one-to-one	no	no	no

**calculator questions**

**Q7.** (1)  $(-3.59, -0.279), (-0.549, -1.82), (10.1, 0.0986)$

(2)  $(4.24, 9.48), (8.83, 18.7)$

(3)  $(-13.4, -0.590), (0.561, 20.1), (15.3, -0.416)$

(4)  $(-16.4, -6.82), (0.382, 0.159), (16.0, 6.66)$

**Q8.** (1)  $x \in [-20, -3.59] \cup -0.549, 0 \cup ]10.1, 20]$  /  $-20 \leq x < -3.59$  or  $-0.549 < x < 0$  or  $10.1 < x \leq 20$

(2)  $x \in [-20, 4.24] \cup ]8.83, 20]$  /  $-20 \leq x < 4.24$  or  $8.83 < x \leq 20$

(3)  $x \in [-20, -13.4] \cup ]0.5, 0.561] \cup [15.3, 20]$  /  $-20 \leq x \leq -13.4$  or  $0.5 < x \leq 0.561$  or  $15.3 \leq x \leq 20$

(4)  $x \in [-20, -16.4] \cup [0.382, 16.0]$  /  $-20 \leq x \leq -16.4$  or  $0.382 \leq x \leq 20$

(answer can be given in any of the two forms shown above)

## Chapter 5

# Quadratics

### 5.1 Solving quadratic equations

#### Factorisation

- |                                 |                        |                        |                         |
|---------------------------------|------------------------|------------------------|-------------------------|
| <b>Q1.</b> (1) $x^2 + 3x + 2$   | (5) $x^2 - 6x + 8$     | (9) $x^2 + 2x - 8$     | (13) $x^2 - 2x - 24$    |
| (2) $x^2 + 4x + 3$              | (6) $x^2 - 13x + 12$   | (10) $x^2 - 9$         | (14) $x^2 + 5x - 24$    |
| (3) $x^2 + 7x + 10$             | (7) $x^2 + x - 6$      | (11) $x^2 - x - 12$    | (15) $x^2 + 4x - 21$    |
| (4) $x^2 - 4x + 3$              | (8) $x^2 - x - 6$      | (12) $x^2 + 4x - 12$   |                         |
| <b>Q2.</b> (1) $2x^2 + 3x + 1$  | (5) $2x^2 - x - 1$     | (9) $6x^2 + 13x - 5$   | (13) $15x^2 - 17x - 4$  |
| (2) $2x^2 + 9x + 10$            | (6) $3x^2 + x - 2$     | (10) $6x^2 - 13x + 5$  | (14) $15x^2 + 4x - 4$   |
| (3) $2x^2 - 7x + 3$             | (7) $3x^2 - 5x - 2$    | (11) $6x^2 + 7x - 5$   |                         |
| (4) $2x^2 - 5x + 3$             | (8) $3x^2 - x - 2$     | (12) $15x^2 - 16x + 4$ | (15) $12x^2 - 25x + 12$ |
| <b>Q3.</b> (1) $(x)(x - 2)$     | (7) $(x + 4)(x + 2)$   | (13) $(x - 1)(x - 3)$  | (19) $(x - 3)(x - 4)$   |
| (2) $(x + 2)(x + 1)$            | (8) $(x + 4)(x + 3)$   | (14) $(x - 1)(x - 2)$  | (20) $(x - 2)(x - 6)$   |
| (3) $(x + 3)(x + 2)$            | (9) $(x + 5)(x + 1)$   | (15) $(x - 2)(x - 3)$  | (21) $(x - 1)(x - 12)$  |
| (4) $(x + 3)(x + 1)$            | (10) $(x + 5)(x + 2)$  | (16) $(x - 1)(x - 6)$  |                         |
| (5) $(x + 4)(x)$                | (11) $(x + 6)(x + 2)$  | (17) $(x - 2)(x - 4)$  |                         |
| (6) $(x + 4)(x + 1)$            | (12) $(x + 1)(x + 12)$ | (18) $(x - 1)(x - 8)$  |                         |
| <b>Q4.</b> (1) $(x + 3)(x - 2)$ | (5) $(x + 1)(x - 8)$   | (9) $(x + 2)(x - 6)$   | (13) $(x + 2)(x - 12)$  |
| (2) $(x + 2)(x - 3)$            | (6) $(x + 3)(x - 3)$   | (10) $(x + 6)(x - 2)$  | (14) $(x + 8)(x - 3)$   |
| (3) $(x + 4)(x - 2)$            | (7) $(x + 6)(x - 1)$   | (11) $(x + 4)(x - 3)$  |                         |
| (4) $(x + 2)(x - 4)$            | (8) $(x + 3)(x - 4)$   | (12) $(x + 4)(x - 6)$  | (15) $(x + 24)(x - 1)$  |