let $x$ = distance from observer to rocket
let $h$ = the height of the rocket above the ground

**METHOD 1**

$\frac{dh}{dt} = 300$ when $h = 800$ A1

$x = \sqrt{h^2 + 360000} = (h^2 + 360000)^{\frac{1}{2}}$ M1

$\frac{dx}{dh} = \frac{h}{\sqrt{h^2 + 360000}}$ A1

when $h = 800$

$\frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt}$ M1

$= \frac{300h}{\sqrt{h^2 + 360000}}$ A1

$= 240$ (m s$^{-1}$) A1

**METHOD 2**

$h^2 + 600^2 = x^2$ M1

$2h = 2x\frac{dx}{dh}$ A1

$\frac{dx}{dh} = \frac{h}{x}$

$= \frac{800}{1000}\left(= \frac{4}{5}\right)$ A1

$\frac{dh}{dt} = 300$ A1

$\frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt}$ M1

$= \frac{4}{5} \times 300$

$= 240$ (m s$^{-1}$) A1

**METHOD 3**

$x^2 = 600^2 + h^2$ M1

$2x\frac{dx}{dt} = 2h\frac{dh}{dt}$ A1A1

when $h = 800$, $x = 1000$

$\frac{dx}{dt} = \frac{800}{1000} \times \frac{dh}{dt}$ M1A1

$= 240$ m s$^{-1}$ A1

(a) let $H\hat{P}Q = \theta$

$\tan\theta = \dfrac{h}{40}$

$\sec^2\theta \dfrac{d\theta}{dt} = \dfrac{1}{40}\dfrac{dh}{dt}$ M1

$\dfrac{d\theta}{dt} = \dfrac{1}{4\sec^2\theta}$ (A1)

$= \dfrac{16}{4 \times 25}$ $\left(\sec\theta = \dfrac{5}{4} \text{ or } \theta = 0.6435\right)$ A1

$= 0.16$ radians per second AG

(b) $x^2 = h^2 + 1600$, where $PH = x$

$2x\dfrac{dx}{dt} = 2h\dfrac{dh}{dt}$ M1

$\dfrac{dx}{dt} = \dfrac{h}{x} \times 10$ A1

$= \dfrac{10h}{\sqrt{h^2 + 1600}}$ (A1)

$h = 30$, $\dfrac{dx}{dt} = 6 \text{ m s}^{-1}$ A1

**Note:** Accept solutions that begin $x = 40\sec\theta$ or use $h = 10t$.

**[7]**

(a) the distance of the spot from P is $x = 500\tan\theta$ A1

the speed of the spot is

$\dfrac{dx}{dt} = 500\sec^2\theta\dfrac{d\theta}{dt}$ $(= 4000\pi\sec^2\theta)$ M1A1

when $x = 2000$, $\sec^2\theta = 17$ $(\theta = 1.32581\ldots)$ $\left(\dfrac{d\theta}{dt} = 8\pi\right)$

$\Rightarrow \dfrac{dx}{dt} = 500 \times 17 \times 8\pi$ M1A1

speed is 214000 (metres per minute) AG

**Note:** If their displayed answer does not round to 214 000, they lose the final A1.

(b) $\dfrac{d^2x}{dt^2} = 8000\pi\sec^2\theta\tan\theta\dfrac{d\theta}{dt}$ or $500 \times 2\sec^2\theta\tan\theta\left(\dfrac{d\theta}{dt}\right)^2$ M1A1

$\left(\text{since } \dfrac{d^2\theta}{dt^2} = 0\right)$

$= 43000000$ $(= 4.30 \times 10^7)$ (metres per minute$^2$) A1

**[8]**

(a) AQ = $\sqrt{x^2+4}$ (km) (A1)

QY = $(2-x)$ (km) (A1)

$T = 5\sqrt{5}\,\text{AQ} + 5\text{QY}$ (M1)

$= 5\sqrt{5}\sqrt{(x^2+4)} + 5(2-x)$ (mins) A1

(b) Attempting to use the chain rule on $5\sqrt{5}\sqrt{(x^2+4)}$ (M1)

$\frac{\mathrm{d}}{\mathrm{d}x}\left(5\sqrt{5}\sqrt{(x^2+4)}\right) = 5\sqrt{5}\times\frac{1}{2}(x^2+4)^{-\frac{1}{2}}\times 2x$ A1

$\left(= \frac{5\sqrt{5}x}{\sqrt{x^2+4}}\right)$

$\frac{\mathrm{d}}{\mathrm{d}x}(5(2-x)) = -5$ A1

$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{5\sqrt{5}x}{\sqrt{x^2+4}} - 5$ AG N0

(c) (i) $\sqrt{5}\,x = \sqrt{x^2+4}$ or equivalent A1

Squaring both sides and rearranging to obtain $5x^2 = x^2 + 4$ M1

$x = 1$ A1 N1

**Note:** Do not award the final A1 for stating a negative solution in final answer.

(ii) $T = 5\sqrt{5}\sqrt{1+4} + 5(2-1)$ M1

$= 30$ (mins) A1 N1

**Note:** Allow FT on incorrect $x$ value.

(iii) **METHOD 1**

Attempting to use the quotient rule M1

$u = x, v = \sqrt{x^2+4}, \frac{\mathrm{d}u}{\mathrm{d}x} = 1$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = x(x^2+4)^{-\frac{1}{2}}$ (A1)

$\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = 5\sqrt{5}\left[\frac{\sqrt{x^2+4} - \frac{1}{2}(x^2+4)^{-\frac{1}{2}}\times 2x^2}{(x^2+4)}\right]$ A1

Attempt to simplify (M1)

$$= \frac{5\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}}\left[x^2+4-x^2\right] \text{ or equivalent}$$ A1

$$= \frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}}$$ AG

When $x = 1$, $\frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}} > 0$ and hence $T = 30$

is a minimum R1 N0

**Note:** Allow FT on incorrect $x$ value, $0 \le x \le 2$.

**METHOD 2**

Attempting to use the product rule M1

$u = x, v = \sqrt{x^2+4}, \frac{du}{dx} = 1$ and $\frac{dv}{dx} = x\left(x^2+4\right)^{-\frac{1}{2}}$ (A1)

$$\frac{d^2T}{dx^2} = 5\sqrt{5}\left(x^2+4\right)^{-\frac{1}{2}} - \frac{5\sqrt{5}x}{2}\left(x^2+4\right)^{-\frac{3}{2}} \times 2x$$ A1

$$\left(= \frac{5\sqrt{5}}{\left(x^2+4\right)^{\frac{1}{2}}} - \frac{5\sqrt{5}\,x^2}{\left(x^2+4\right)^{\frac{3}{2}}}\right)$$

Attempt to simplify (M1)

$$= \frac{5\sqrt{5}\left(x^2+4\right) - 5\sqrt{5}\,x^2}{\left(x^2+4\right)^{\frac{3}{2}}} \quad \left(= \frac{5\sqrt{5}\left(x^2+4-x^2\right)}{\left(x^2+4\right)^{\frac{3}{2}}}\right)$$ A1

$$= \frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}}$$ AG

When $x = 1$, $\frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}} > 0$ and hence $T = 30$ is a

minimum R1 N0

**Note:** Allow FT on incorrect $x$ value, $0 \le x \le 2$.

**[18]**

(a) Area of hexagon = $6 \times \frac{1}{2} \times x \times x \times \sin 60°$ M1

$= \frac{3\sqrt{3}x^2}{2}$ AG

(b) Let the height of the box be $h$

Volume = $\frac{3\sqrt{3}hx^2}{2} = 90$ M1

Hence $h = \frac{60}{\sqrt{3}x^2}$ A1

Surface area, $A = 3\sqrt{3}\,x^2 + 6hx$ M1

$= 3\sqrt{3}x^2 + \frac{360}{\sqrt{3}}x^{-1}$ A1

$\frac{dA}{dx} = 6\sqrt{3}x - \frac{360}{\sqrt{3}}x^{-2}$ A1

$\left(\frac{dA}{dx} = 0\right)$

$6\sqrt{3}x^3 = \frac{360}{\sqrt{3}}$ M1

$x^3 = 20$

$x = \sqrt[3]{20}$ AG

$\frac{d^2A}{dx^2} = 6\sqrt{3} + \frac{720x^{-3}}{\sqrt{3}}$

which is positive when $x = \sqrt[3]{20}$, and hence gives a minimum value. R1

**[8]**

(a) OP = $\sqrt{a^2 + (a^2 - 5)^2}$ M1

$= \sqrt{a^2 + a^4 - 10a^2 + 25}$ A1

$= \sqrt{a^4 - 9a^2 + 25}$ AG

(b) **EITHER**

Let $s = \sqrt{a^4 - 9a^2 + 25}$

$\Rightarrow s^2 = a^4 - 9a^2 + 25$

$\frac{\mathrm{d}s^2}{\mathrm{d}a} = 4a^3 - 18a = 0$ M1A1

$\frac{\mathrm{d}s^2}{\mathrm{d}a} = 0$ for minimum (M1)

$\Rightarrow 2a(2a^2 - 9) = 0$

$\Rightarrow a^2 = \frac{9}{2}$

$\Rightarrow a = \pm\frac{3}{\sqrt{2}} \left(= \pm\frac{3\sqrt{2}}{2}\right)$ A1A1

**OR**

$s = (a^4 - 9a^2 + 25)^{\frac{1}{2}}$

$\frac{\mathrm{d}s}{\mathrm{d}a} = \frac{1}{2}(a^4 - 9a^2 + 25)^{-\frac{1}{2}}(4a^3 - 18a)$ M1A1

$\frac{\mathrm{d}s}{\mathrm{d}a} = 0$ for a minimum (M1)

$4a^3 - 18a = 0$

$\Rightarrow 2a(2a^2 - 9) = 0$

$\Rightarrow a^2 = \frac{9}{2}$

$\Rightarrow a = \pm\frac{3}{\sqrt{2}} \left(= \pm\frac{3\sqrt{2}}{2}\right)$ A1A1

**[7]**