1. A rocket is rising vertically at a speed of 300 m s^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.

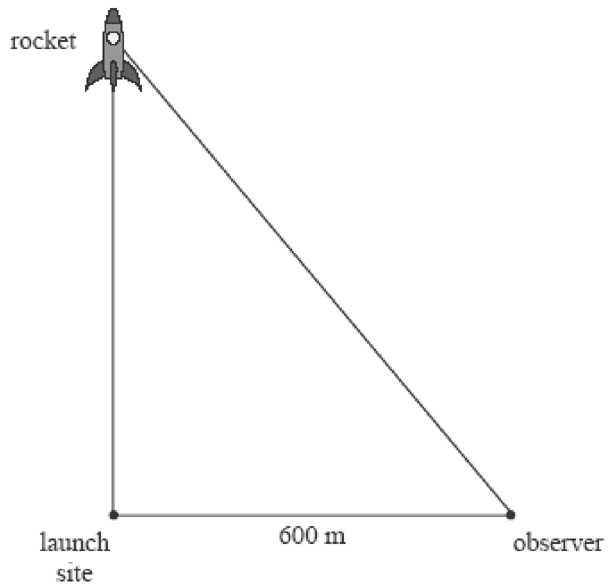


diagram not to scale

(Total 6 marks)

2. A helicopter H is moving vertically upwards with a speed of 10 m s^{-1}. The helicopter is h m directly above the point Q, which is situated on level ground. The helicopter is observed from the point P, which is also at ground level, and PQ = 40 m.
This information is represented in the diagram below.

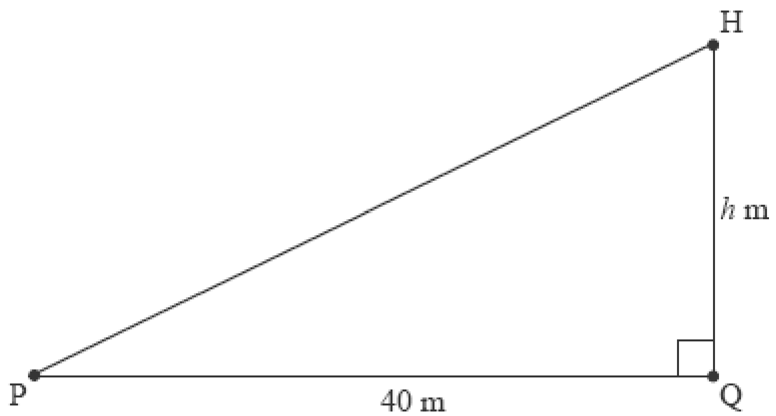


diagram not to scale

When h = 30,

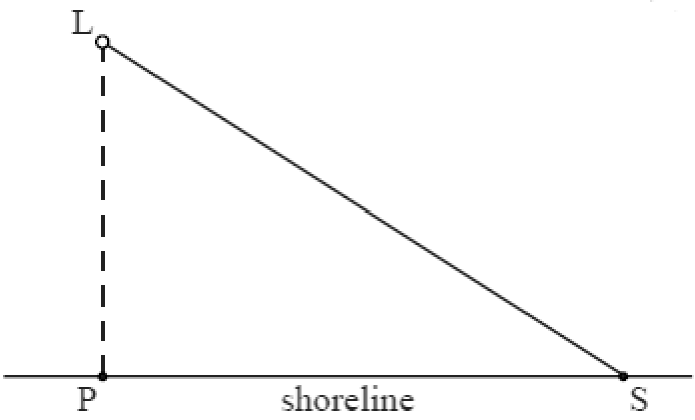
(a) show that the rate of change of $\mathrm{H\hat{P}Q}$ is 0.16 radians per second;

(3)

(b) find the rate of change of PH.

(4)
(Total 7 marks)

3. A lighthouse L is located offshore, 500 metres from the nearest point P on a long straight shoreline. The narrow beam of light from the lighthouse rotates at a constant rate of 8π radians per minute, producing an illuminated spot S that moves along the shoreline. You may assume that the height of the lighthouse can be ignored and that the beam of light lies in the horizontal plane defined by sea level.



When S is 2000 metres from P,

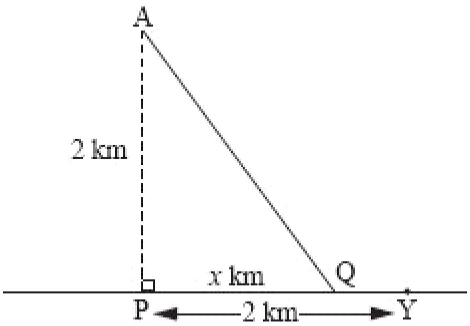
(a) show that the speed of S, correct to three significant figures, is 214 000 metres per minute;

(5)

(b) find the acceleration of S.

(3)
(Total 8 marks)

4. André wants to get from point A located in the sea to point Y located on a straight stretch of beach. P is the point on the beach nearest to A such that AP = 2 km and PY = 2 km. He does this by swimming in a straight line to a point Q located on the beach and then running to Y.



When André swims he covers 1 km in $5\sqrt{5}$ minutes. When he runs he covers 1 km in 5 minutes.

(a) If PQ = x km, $0 \le x \le 2$, find an expression for the time T minutes taken by André to reach point Y.

(4)

(b) Show that $\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{5\sqrt{5}\,x}{\sqrt{x^2+4}} - 5$.

(3)

(c) (i) Solve $\frac{\mathrm{d}T}{\mathrm{d}x} = 0$.

(ii) Use the value of x found in **part (c) (i)** to determine the time, T minutes, taken for André to reach point Y.

(iii) Show that $\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = \frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}}$ and **hence** show that the time found in **part (c) (ii)** is a minimum.

(11)
(Total 18 marks)

5. A packaging company makes boxes for chocolates. An example of a box is shown below. This box is closed and the top and bottom of the box are identical regular hexagons of side x cm.

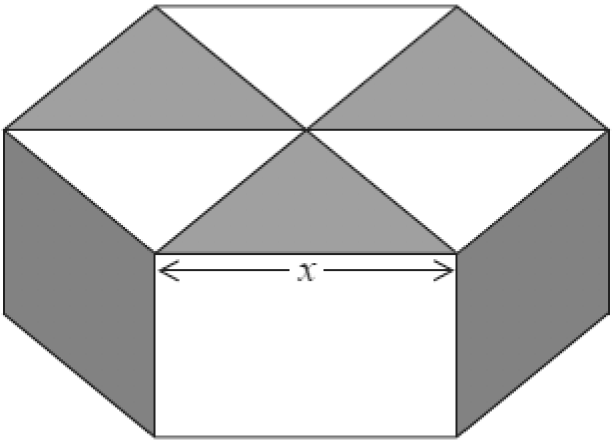


diagram not to scale

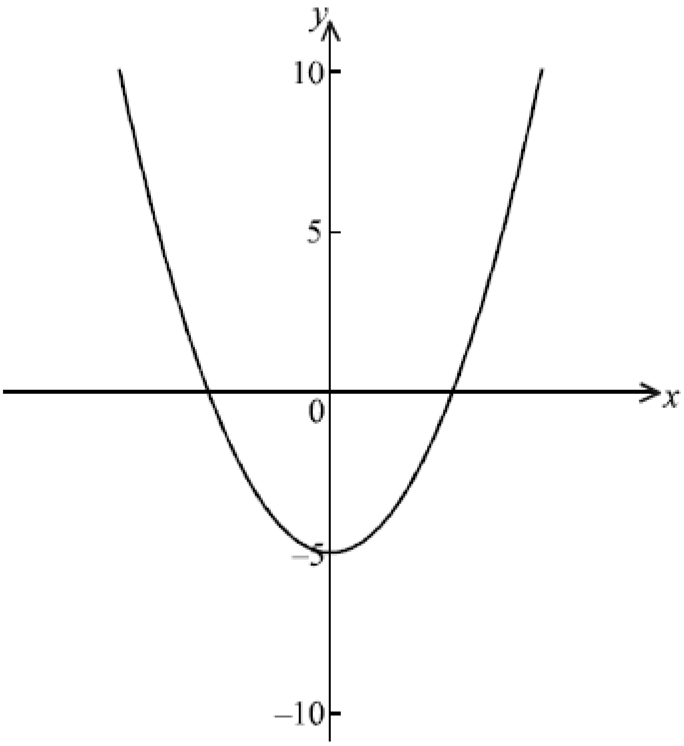
(a) Show that the area of each hexagon is $\dfrac{3\sqrt{3}x^2}{2}$ cm^2.

(1)

(b) Given that the volume of the box is 90 cm^3, show that when $x = \sqrt[3]{20}$ the total surface area of the box is a minimum, justifying that this value gives a minimum.

(7)
(Total 8 marks)

6. The curve $y = x^2 - 5$ is shown below.



A point P on the curve has x-coordinate equal to a.

(a) Show that the distance OP is $\sqrt{a^4 - 9a^2 + 25}$.

(2)

(b) Find the values of a for which the curve is closest to the origin.

(5)
(Total 7 marks)