Modelling AI HL [59 marks]

1. [Maximum mark: 5] EXM.1.AHL.TZ0.13

It is believed that two variables, m and p are related. Experimental values of m and p are obtained. A graph of $\ln m$ against p shows a straight line passing through (2.1, 7.3) and (5.6, 2.4).

(a) Find the equation of the straight line, giving your answer in the form $\ln m = ap + b$, where $a,\ b \in \mathbb{R}$. [3]

Markscheme

gradient = −1.4 ***A1***

$\ln m - 7.3 = -1.4\,(p - 2.1)$ ***M1***

$\ln m = -1.4p + 10.24$ ***A1***

[3 marks]

Hence, find

(b.i) a formula for m in terms of p. [1]

Markscheme

$m = e^{-1.4p+10.24}\ \left(= 28000e^{-1.4p}\right)$ ***A1***

[1 mark]

(b.ii) the value of m when $p = 0$. [1]

Markscheme

28000 ***A1***

[1 mark]

2. [Maximum mark: 7] EXM.1.AHL.TZ0.14

It is believed that two variables, v and w are related by the equation $v = kw^n$, where $k,\ n \in \mathbb{R}$. Experimental values of v and w are obtained. A graph of $\ln v$ against $\ln w$ shows a straight line passing through (–1.7, 4.3) and (7.1, 17.5).

Find the value of k and of n. [7]

Markscheme

$\ln v = n \ln w + \ln k$ ***M1A1***

gradient $= \frac{17.5-4.3}{7.1+1.7}\ \ (= 1.5)$ ***M1***

$n = 1.5$ ***A1***

y-intercept $= 1.5 \times 1.7 + 4.3\ \ (= 6.85)$ ***M1***

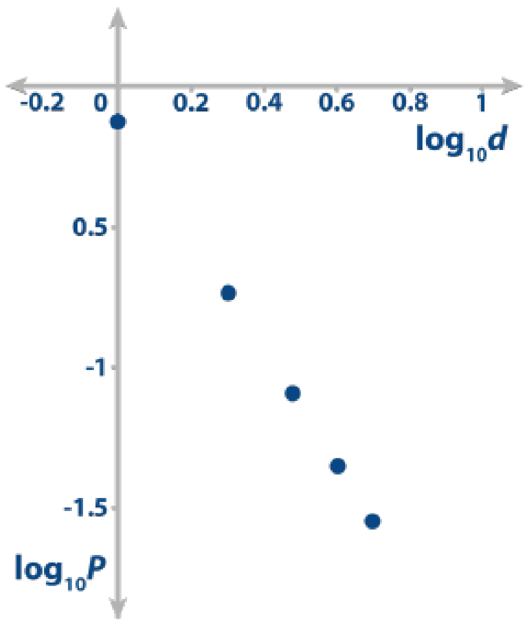
$k = e^{6.85} = 944$ ***M1A1***

[7 marks]

3. [Maximum mark: 7] EXN.1.AHL.TZ0.12

It is believed that the power P of a signal at a point d km from an antenna is inversely proportional to d^n where $n \in \mathbb{Z}^+$.

The value of P is recorded at distances of $1\,\mathrm{m}$ to $5\,\mathrm{m}$ and the values of $\log_{10} d$ and $\log_{10} P$ are plotted on the graph below.



(a) Explain why this graph indicates that P is inversely proportional to d^n.

[2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

a straight line with a negative gradient **A1A1**

[2 marks]

The values of $\log_{10} d$ and $\log_{10} P$ are shown in the table below.

$\log_{10} d$	0	0.301	0.477	0.602	0.699
$\log_{10} P$	-0.127	-0.740	-1.10	-1.36	-1.55

(b) Find the equation of the least squares regression line of $\log_{10} P$ against $\log_{10} d$.

[2]

Markscheme

$\log P = -2.040\ldots \log d - 0.12632\ldots \approx -2.04 \log d - 0.126$ **A1A1**

Note: A1 for each correct term.

[2 marks]

(c.i) Use your answer to part (b) to write down the value of n to the nearest integer.

[1]

Markscheme

$n = 2$ **A1**

[1 mark]

(c.ii) Find an expression for P in terms of d.

[2]

Markscheme

$P = 10^{-0.126\ldots}d^{-2}$ **(M1)**

$\approx 0.748d^{-2}$ **A1**

[2 marks]

4. [Maximum mark: 10] EXM.1.AHL.TZ0.15

Adesh wants to model the cooling of a metal rod. He heats the rod and records its temperature as it cools.

Time, t (seconds)	0	30	60	90	120	150
Temperature, T (°C)	75.6	62.2	53.3	47.4	42.3	38.5

He believes the temperature can be modeled by $T\,(t) = a\mathrm{e}^{bt} + 25$, where $a,\ b \in \mathbb{R}$.

(a) Show that $\ln\,(T - 25) = bt + \ln a$. [2]

Markscheme

$\ln\,(T - 25) = \ln\left(a\mathrm{e}^{bt}\right)$ ***M1***

$\ln\,(T - 25) = \ln a + \ln\left(\mathrm{e}^{bt}\right)$ ***A1***

$\ln\,(T - 25) = bt + \ln a$ ***AG***

[2 marks]

(b) Find the equation of the regression line of $\ln\,(T - 25)$ on t. [3]

Markscheme

$\ln\,(T - 25) = -0.00870t + 3.89$ ***M1A1A1***

[3 marks]

Hence

(c.i) find the value of a and of b. [3]

Markscheme

$b = -0.00870$ ***A1***

$a = e^{3.89\ldots} = 49.1$ ***M1A1***

[3 marks]

(c.ii) predict the temperature of the metal rod after 3 minutes. [2]

Markscheme

$T\left(180\right) = 49.1e^{-0.00870(180)} + 25 = 35.2$ ***M1A1***

[2 marks]

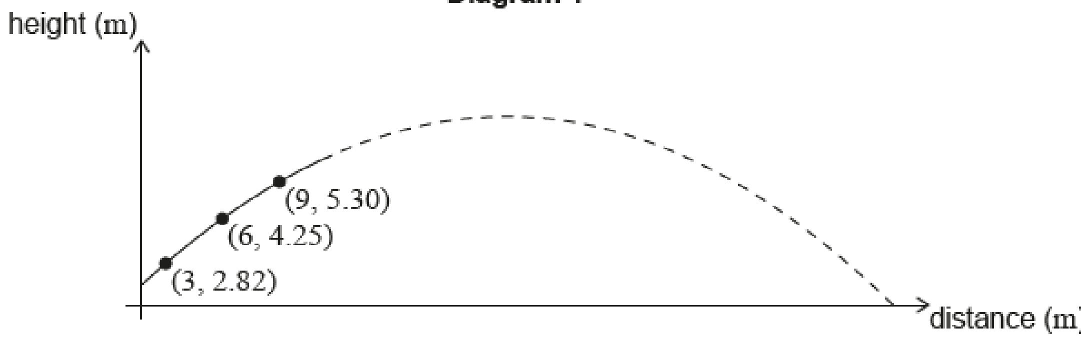
5. [Maximum mark: 9] 23N.1.AHL.TZ0.5

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3,\ 2.36)$, $(10,\ 5)$ and $(17,\ 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



(a) Use the coordinates $(3,\ 2.36)$ to write down an equation in terms of a, b, and c. [1]

Markscheme

$2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

[1 mark]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

Markscheme

finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
OR $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***

$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

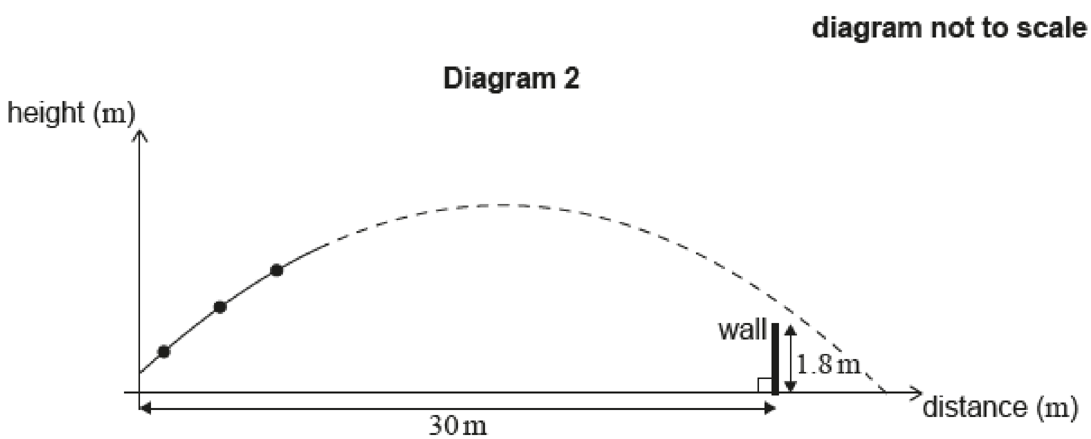
$(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots)$

$(f(x) = -\frac{6}{1255}x^2 + \frac{108}{245}x + \frac{53}{49})$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.



(c) Show that the model predicts that the ball will go over the fence. [3]

Markscheme

attempt to substitute 80 into their equation ***(M1)***

$(f(80) =)5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

Markscheme

setting their equation equal to zero, graph *(M1)*

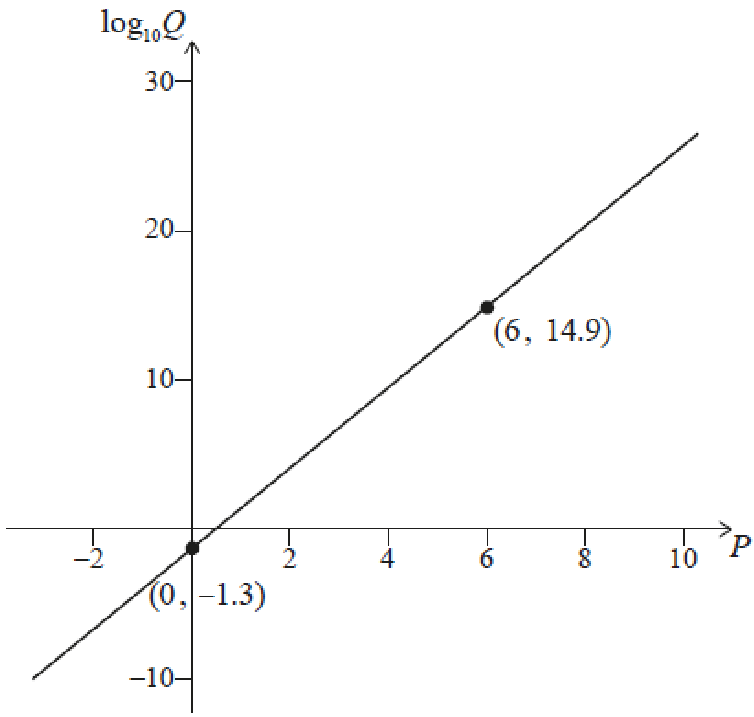
$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

$92.4\ (92.3902\ldots)\ (\text{m})$ *A1*

[2 marks]

6. [Maximum mark: 6] 22N.1.AHL.TZ0.13

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, \ -1.3)$ and $(6, \ 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Markscheme

Gradient $= \frac{14.9+1.3}{6} (= 2.7)$ *(M1)*

$\log_{10} Q = 2.7P - 1.3$ *(A1)*

$Q = 10^{2.7P-1.3}$ **OR**
$Q = 0.0501 \times 10^{2.7P} \ (= 0.0501187\ldots \times 10^{2.7P})$ *A1*

[3 marks]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

Markscheme

$\ln R$ on one axis and Q on the other axis ***A1***

[1 mark]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]

Markscheme

$\log_{10}(4.3 \ln R + 12.1) = 2.7P - 1.3$ **OR**
$10^{2.7P-1.3} = 4.3 \ln R + 12.1$ ***(M1)***

$P = \frac{\log_{10}(4.3 \ln R + 12.1) + 1.3}{2.7}$ ***A1***

[2 marks]

7. [Maximum mark: 7] 22N.1.AHL.TZ0.10

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5\log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Markscheme

$m = 1 - 2.5\log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ \ (4.19960\ldots)$ ***A1***

[2 marks]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

Markscheme

attempt to solve $7 = 1 - 2.5\log_{10}(b)$ ***(M1)***

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398 \ \ (0.00398107\ldots)$ ***A1***

[2 marks]

The star Proxima Centauri has a greater magnitude than the planet Neptune. The difference in their magnitudes is 3.2.

(c) Find how many times brighter Neptune is compared to Proxima Centauri. [3]

Markscheme

$-3.2 = (1 - 2.5\log_{10}(b_n)) - (1 - 2.5\log_{10}(b_p))$ ***M1***

$-3.2 = -2.5\log_{10}\left(\frac{b_n}{b_p}\right)$ ***A1***

$\frac{b_n}{b_p} = 19.1 \ (19.0546\ldots)$ ***A1***

[3 marks]

8. [Maximum mark: 8] 22M.1.AHL.TZ1.12

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

Markscheme

$\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

Markscheme

EITHER

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M}\right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \quad (= 10^5)$ ***A1***

[2 marks]

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(c) Find the average number of earthquakes in a year with a magnitude of at least 7.2. [1]

Markscheme

$N = \frac{10^5}{10^{7.2}} = 0.00631 \quad (0.0063095\ldots)$ ***A1***

Note: Do not accept an answer of $10^{-2.2}$.

[1 mark]

The number of earthquakes in a given year with a magnitude of at least 7.2 can be modelled by a Poisson distribution, with mean N. The number of earthquakes in one year is independent of the number of earthquakes in any other year.

Let Y be the number of years between the earthquake of magnitude 7.2 and the next earthquake of at least this magnitude.

(d) Find $\mathrm{P}(Y > 100)$. [3]

Markscheme

METHOD 1

$Y > 100 \Rightarrow$ no earthquakes in the first 100 years ***(M1)***

EITHER

let X be the number of earthquakes of at least magnitude 7.2 in a year

X~$\mathrm{Po}(0.0063095\ldots)$

$(\mathrm{P}(X=0))^{100}$ ***(M1)***

OR

let X be the number of earthquakes in 100 years

X~$\mathrm{Po}(0.0063095\ldots\times 100)$ ***(M1)***

$\mathrm{P}(X=0)$

THEN

$0.532\ \ (0.532082\ldots)$ ***A1***

METHOD 2

$Y>100\Rightarrow$no earthquakes in the first 100 years ***(M1)***

let X be the number of earthquakes in 100 years

since n is large and p is small

X~$\mathrm{B}(100,\ 0.0063095\ldots)$ ***(M1)***

$\mathrm{P}(X=0)$

$0.531\ \ (0.531019\ldots)$ ***A1***

[3 marks]