Modelling AI HL [59 marks]

1. [Maximum mark: 5] EXM.1.AHL.TZ0.13

It is believed that two variables, m and p are related. Experimental values of m and p are obtained. A graph of $\ln m$ against p shows a straight line passing through (2.1, 7.3) and (5.6, 2.4).

(a) Find the equation of the straight line, giving your answer in the form $\ln m = ap + b$, where $a,\ b \in \mathbb{R}$. [3]

Hence, find

(b.i) a formula for m in terms of p. [1]

(b.ii) the value of m when $p = 0$. [1]

2. [Maximum mark: 7] EXM.1.AHL.TZ0.14

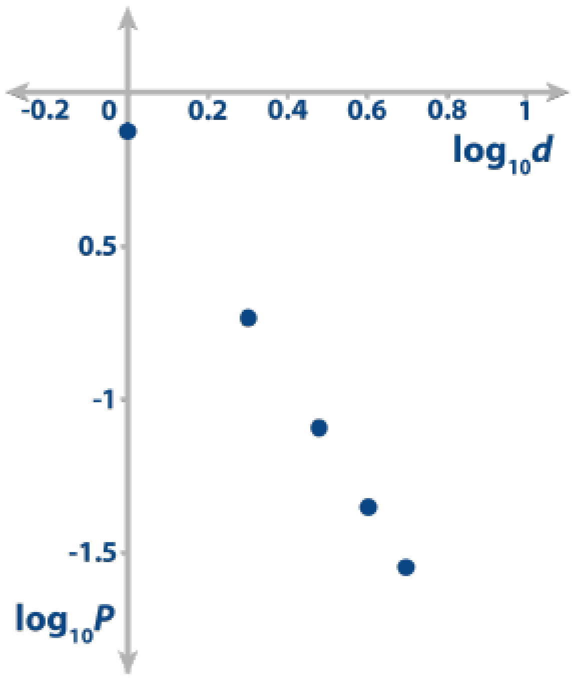
It is believed that two variables, v and w are related by the equation $v = kw^n$, where $k,\ n \in \mathbb{R}$. Experimental values of v and w are obtained. A graph of $\ln v$ against $\ln w$ shows a straight line passing through (−1.7, 4.3) and (7.1, 17.5).

Find the value of k and of n. [7]

3. [Maximum mark: 7] EXN.1.AHL.TZ0.12

It is believed that the power P of a signal at a point d km from an antenna is inversely proportional to d^n where $n \in \mathbb{Z}^+$.

The value of P is recorded at distances of $1\ \mathrm{m}$ to $5\ \mathrm{m}$ and the values of $\log_{10} d$ and $\log_{10} P$ are plotted on the graph below.



(a) Explain why this graph indicates that P is inversely proportional to d^n.

[2]

The values of $\log_{10} d$ and $\log_{10} P$ are shown in the table below.

$\log_{10} d$	0	0.301	0.477	0.602	0.699
$\log_{10} P$	-0.127	-0.740	-1.10	-1.36	-1.55

(b) Find the equation of the least squares regression line of $\log_{10} P$ against $\log_{10} d$.

[2]

(c.i) Use your answer to part (b) to write down the value of n to the nearest integer. [1]

(c.ii) Find an expression for P in terms of d. [2]

4. [Maximum mark: 10] EXM.1.AHL.TZ0.15

Adesh wants to model the cooling of a metal rod. He heats the rod and records its temperature as it cools.

Time, t (seconds)	0	30	60	90	120	150
Temperature, T (°C)	75.6	62.2	53.3	47.4	42.3	38.5

He believes the temperature can be modeled by $T(t) = ae^{bt} + 25$, where $a,\ b \in \mathbb{R}$.

(a) Show that $\ln(T-25) = bt + \ln a$. [2]

(b) Find the equation of the regression line of $\ln(T-25)$ on t. [3]

Hence

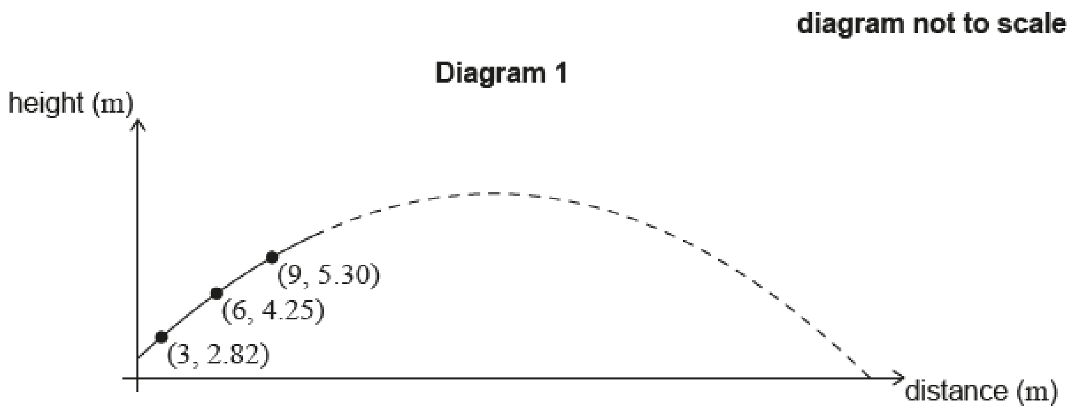
(c.i) find the value of a and of b. [3]

(c.ii) predict the temperature of the metal rod after 3 minutes. [2]

5. [Maximum mark: 9] 23N.1.AHL.TZ0.5

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3,\ 2.36)$, $(10,\ 5)$ and $(17,\ 7.16)$, as shown in **Diagram 1**.



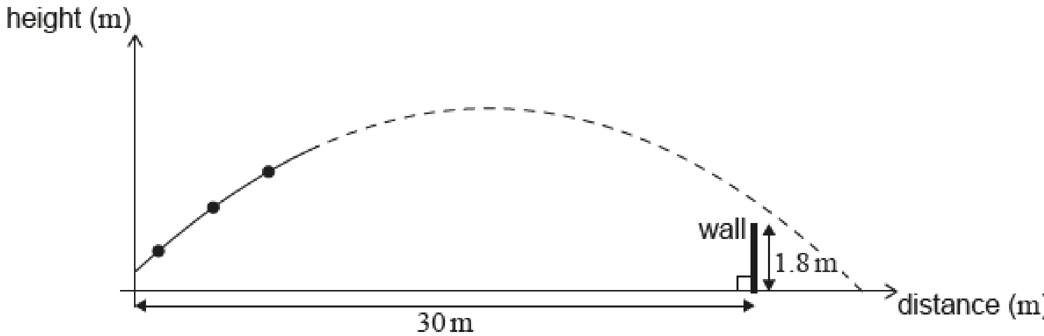
(a) Use the coordinates $(3,\ 2.36)$ to write down an equation in terms of a, b, and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2

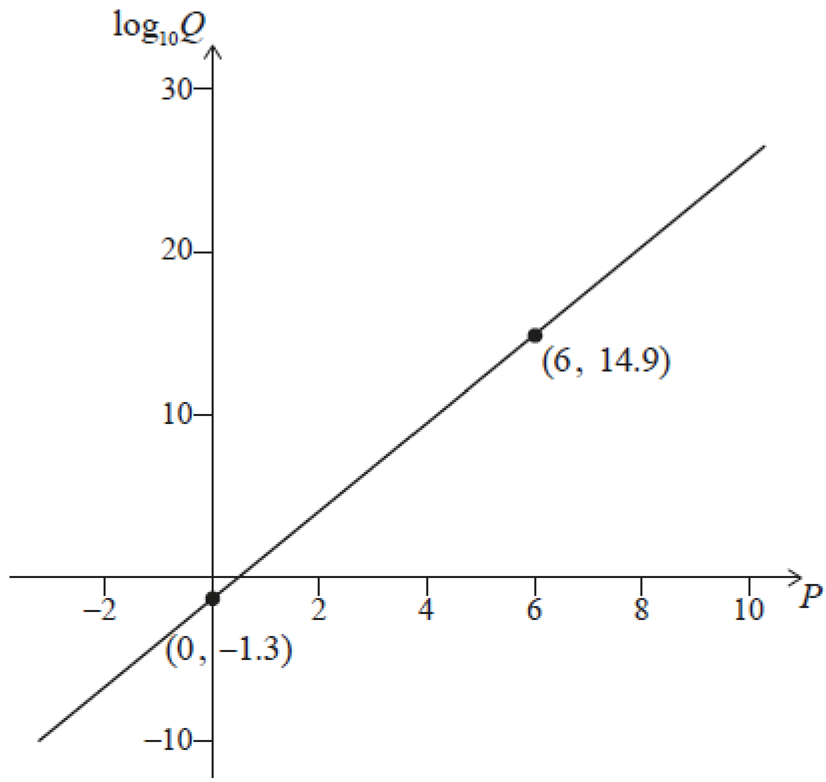


(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

6. [Maximum mark: 6] 22N.1.AHL.TZ0.13

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, \ -1.3)$ and $(6, \ 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]

7. [Maximum mark: 7] 22N.1.AHL.TZ0.10

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

The star Proxima Centauri has a greater magnitude than the planet Neptune. The difference in their magnitudes is 3.2.

(c) Find how many times brighter Neptune is compared to Proxima Centauri. [3]

8. [Maximum mark: 8] 22M.1.AHL.TZ1.12

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$\log_{10} N = a - M$, for some $a \in \mathbb{R}$.

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(c) Find the average number of earthquakes in a year with a magnitude of at least 7.2. [1]

The number of earthquakes in a given year with a magnitude of at least 7.2 can be modelled by a Poisson distribution, with mean N. The number of earthquakes in one year is independent of the number of earthquakes in any other year.

Let Y be the number of years between the earthquake of magnitude 7.2 and the next earthquake of at least this magnitude.

(d) Find $\mathrm{P}(Y > 100)$. [3]