

Probability - basics (preDP2) [104 marks]

1. [Maximum mark: 4]

23N.1.SL.TZ1.3

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

(a) Find $P(A \cup B)$.

[2]

Markscheme

$$(P(A \cup B) =) 0.7 + 0.75 - 0.55 \quad (A1)$$

$$= 0.9 \quad A1$$

[2 marks]

(b) Hence, otherwise find $P(A' \cap B')$.

[2]

Markscheme

recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$

(region/value may be seen in a correctly shaded/labeled Venn diagram) (M1)

$$(= 1 - 0.9)$$

$$= 0.1 \quad A1$$

Note: For the final mark, 0.1 must be stated as the candidate's answer, or labeled as $P(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.1 in the correct region of their diagram earns **M1A0**.

[2 marks]

2. [Maximum mark: 5]

23M.1.AHL.TZ2.3

Events A and B are such that $P(A) = 0.4$, $P(A|B) = 0.25$ and $P(A \cup B) = 0.55$.

Find $P(B)$.

[5]

Markscheme

substitutes into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to form

$$0.55 = 0.4 + P(B) - P(A \cap B) \text{ (or equivalent)} \quad (A1)$$

substitutes into $P\left(A|B\right) = \frac{P(A \cap B)}{P(B)}$ to form $0.25 = \frac{P(A \cap B)}{P(B)}$ (or equivalent) (A1)

attempts to combine their two probability equations to form an equation in $P(B)$ (M1)

Note: The above two A marks are awarded independently.

correct equation in $P(B)$ A1

$$0.55 = 0.4 + P(B) - 0.25P(B) \text{ OR } \frac{P(B) - 0.15}{P(B)} = 0.25 \text{ OR} \\ P(B) - 0.15 = 0.25P(B)$$

(or equivalent)

$$P(B) = \frac{15}{75} \left(= \frac{1}{5} = 0.2 \right) \quad A1$$

[5 marks]

3. [Maximum mark: 6]

20N.1.SL.TZ0.T_14

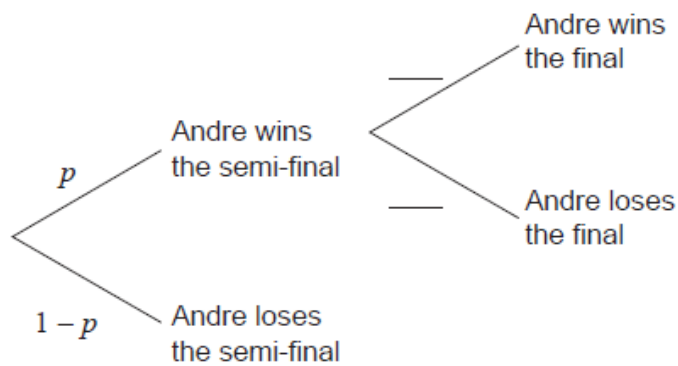
Andre will play in the semi-final of a tennis tournament.

If Andre wins the semi-final he will progress to the final. If Andre loses the semi-final, he will **not** progress to the final.

If Andre wins the final, he will be the champion.

The probability that Andre will win the semi-final is p . If Andre wins the semi-final, then the probability he will be the champion is 0.6 .

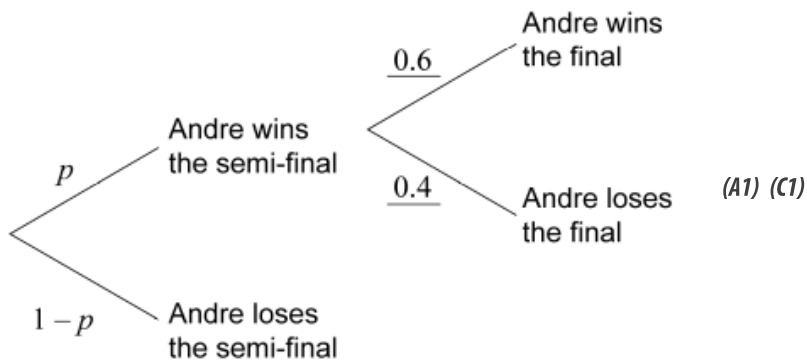
(a) Complete the values in the tree diagram.



[1]

Markscheme

*This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.



Note: Award (A1) for the correct pair of probabilities.

[1 mark]

The probability that Andre will not be the champion is 0.58.

(b) Find the value of p .

[2]

Markscheme

$$p \times 0.4 + (1 - p) = 0.58 \quad (M1)$$

Note: Award (M1) for multiplying and adding correct probabilities for losing equated to 0.58.

OR

$$p \times 0.6 = 1 - 0.58 \quad (M1)$$

Note: Award (M1) for multiplying correct probabilities for winning equated to $1 - 0.58$ or 0.42.

$$(p =) 0.7 \quad (A1)(ft) \quad (C2)$$

Note: Follow through from their part (a). Award the final (A1)(ft) only if their p is within the range $0 < p < 1$.

[2 marks]

(c) Given that Andre did not become the champion, find the probability that he lost in the semi-final.

[3]

Markscheme

$$\frac{0.3}{0.58} \left(\frac{1-0.7}{0.58} \right) \quad (A1)(ft)(A1)$$

Note: Award (A1)(ft) for their correct numerator. Follow through from part (b). Award (A1) for the correct denominator.

OR

$$\frac{0.3}{0.3+0.7 \times 0.4} \quad (A1)(ft)(A1)(ft)$$

Note: Award (A1)(ft) for their correct numerator. Follow through from part (b). Award (A1)(ft) for their correct calculation of Andre losing the semi-final or winning the semi-final and then losing in the final. Follow through from their parts (a) and (b).

$$\frac{15}{29} \quad (0.517, 0.517241\dots, 51.7\%) \quad (A1)(ft) \quad (C3)$$

Note: Follow through from parts (a) and (b).

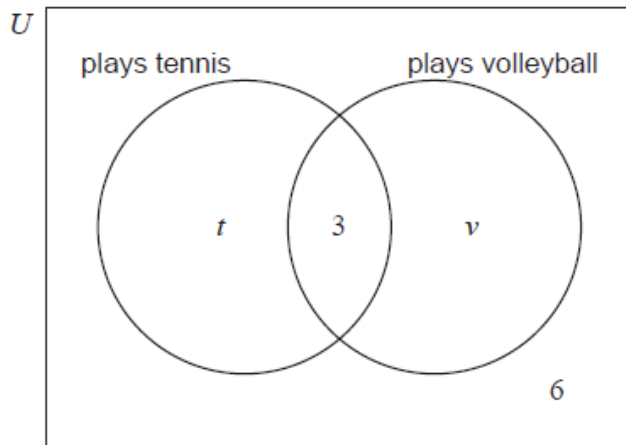
[3 marks]

4. [Maximum mark: 6]

20N.1.SL.TZ0.S_1

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport.

The following Venn diagram shows the events “plays tennis” and “plays volleyball”. The values t and v represent numbers of students.



(a.i) Find the value of t .

[2]

Markscheme

valid approach to find t (M1)

$$\text{eg } t + 3 = 19, 19 - 3$$

$$t = 16 \text{ (may be seen on Venn diagram) } \quad \mathbf{A1\ N2}$$

[2 marks]

(a.ii) Find the value of v .

[2]

Markscheme

valid approach to find v (M1)

$$\text{eg } t + 3 + v + 6 = 30, 30 - 19 - 6$$

$$v = 5 \text{ (may be seen on Venn diagram) } \quad \mathbf{A1\ N2}$$

[2 marks]

- (b) Find the probability that a randomly selected student from the class plays tennis or volleyball, but not both.

[2]

Markscheme

valid approach (M1)

eg $16 + 5, 21$ students, $1 - \frac{3+6}{30}$,



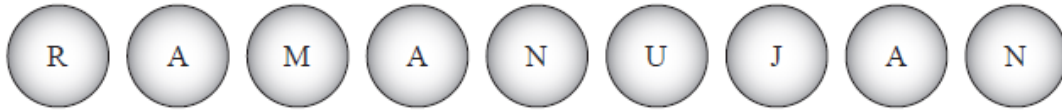
$\frac{21}{30}$ ($= \frac{7}{10}$) A1 N2

[2 marks]

5. [Maximum mark: 6]

20N.1.SL.TZ0.T_6

Srinivasa places the nine labelled balls shown below into a box.



Srinivasa then chooses two balls at random, one at a time, from the box. The first ball is **not replaced** before he chooses the second.

(a.i) Find the probability that the first ball chosen is labelled **A**.

[1]

Markscheme
$\frac{3}{9}$ ($\frac{1}{3}$, 0.333, 0.333333..., 33.3%) (A1) (C1)
[1 mark]

(a.ii) Find the probability that the first ball chosen is labelled **A** or labelled **N**.

[1]

Markscheme
$\frac{5}{9}$ (0.556, 0.555555..., 55.6%) (A1) (C1)
[1 mark]

(b) Find the probability that the second ball chosen is labelled **A**, given that the first ball chosen was labelled **N**.

[2]

Markscheme
$\frac{3}{8}$ (0.375, 37.5%) (A1)(A1) (C2)
Note: Award (A1) for correct numerator, (A1) for correct denominator.
[2 marks]

(c) Find the probability that both balls chosen are labelled **N**.

[2]

Markscheme

$$\frac{2}{9} \times \frac{1}{8} \quad (M1)$$

Note: Award (M1) for a correct compound probability calculation seen.

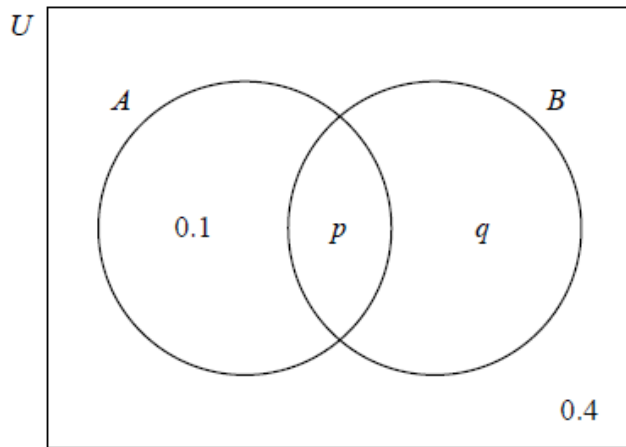
$$\frac{2}{72} \left(\frac{1}{36}, 0.0278, 0.0277777 \dots, 2.78\% \right) \quad (A1) (C2)$$

[2 marks]

6. [Maximum mark: 6]

19M.1.SL.TZ1.S_1

The following Venn diagram shows the events A and B , where $P(A) = 0.3$. The values shown are probabilities.



(a) Find the value of p .

[2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach (M1)

eg $0.30 - 0.1, p + 0.1 = 0.3$

$p = 0.2$ A1 N2

[2 marks]

(b) Find the value of q .

[2]

Markscheme

valid approach (M1)

eg $1 - (0.3 + 0.4), 1 - 0.4 - 0.1 - p$

$q = 0.3$ A1 N2

[2 marks]

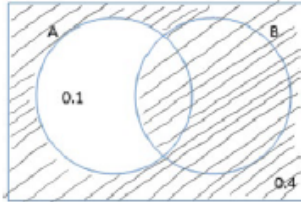
(c) Find $P(A' \cup B)$.

[2]

Markscheme

valid approach (M1)

eg $0.7 + 0.5 - 0.3$, $p + q + 0.4$, $1 - 0.1$,
 $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$



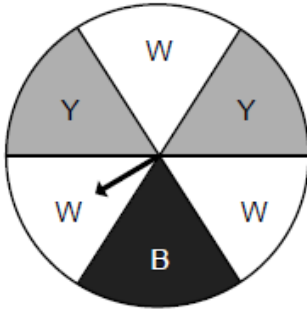
$$P(A' \cup B) = 0.9 \quad A1N2$$

[2 marks]

7. [Maximum mark: 6]

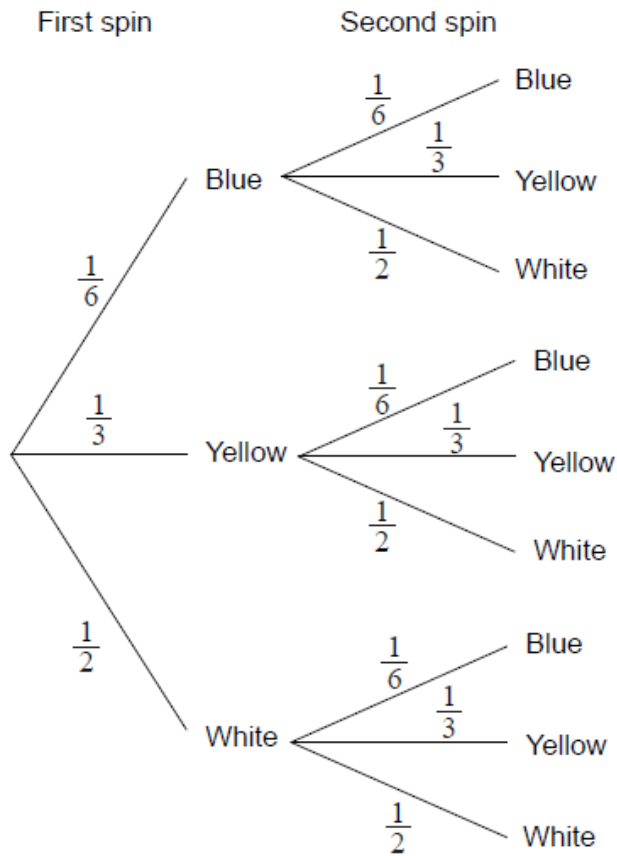
19M.1.SL.TZ1.T_12

The diagram shows a circular horizontal board divided into six equal sectors. The sectors are labelled white (W), yellow (Y) and blue (B).



A pointer is pinned to the centre of the board. The pointer is to be spun and when it stops the colour of the sector on which the pointer stops is recorded. The pointer is equally likely to stop on any of the six sectors.

Eva will spin the pointer twice. The following tree diagram shows all the possible outcomes.



(a) Find the probability that both spins are yellow.

[2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$\frac{1}{3} \times \frac{1}{3} \text{ OR } \left(\frac{1}{3}\right)^2 \quad (M1)$$

Note: Award (M1) for multiplying correct probabilities.

$$\frac{1}{9} \quad (0.111, 0.111111\dots, 11.1\%) \quad (A1) \quad (C2)$$

[2 marks]

- (b) Find the probability that at least one of the spins is yellow.

[3]

Markscheme

$$\left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right) + \frac{1}{3} \quad (M1)(M1)$$

Note: Award (M1) for $\left(\frac{1}{2} \times \frac{1}{3}\right)$ and $\left(\frac{1}{6} \times \frac{1}{3}\right)$ or equivalent, and (M1) for $\frac{1}{3}$ **and** adding only the three correct probabilities.

OR

$$1 - \left(\frac{2}{3}\right)^2 \quad (M1)(M1)$$

Note: Award (M1) for $\frac{2}{3}$ seen and (M1) for subtracting $\left(\frac{2}{3}\right)^2$ from 1. This may be shown in a tree diagram with "yellow" and "not yellow" branches.

$$\frac{5}{9} \quad (0.556, 0.55555\dots, 55.6\%) \quad (A1)(ft) \quad (C3)$$

Note: Follow through marks may be awarded if their answer to part (a) is used in a correct calculation.

[3 marks]

- (c) Write down the probability that the second spin is yellow, given that the first spin is blue.

[1]

Markscheme

$\frac{1}{3}$ (0.333, 0.333333..., 33.3%) (A1) (C1)

[1 mark]

8. [Maximum mark: 6]

19M.1.SL.TZ2.T_5

A school café sells three flavours of smoothies: mango (M), kiwi fruit (K) and banana (B).
85 students were surveyed about which of these three flavours they like.

35 students liked mango, 37 liked banana, and 26 liked kiwi fruit

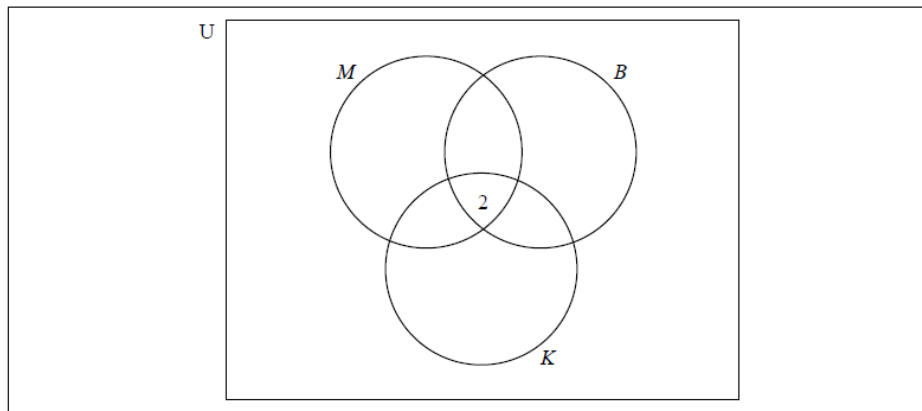
2 liked all three flavours

20 liked both mango and banana

14 liked mango and kiwi fruit

3 liked banana and kiwi fruit

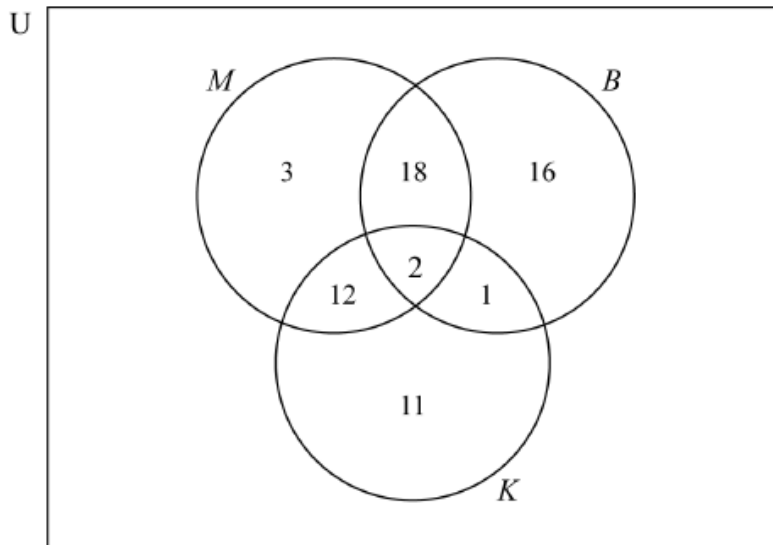
(a) Using the given information, complete the following Venn diagram.



[2]

Markscheme

*This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(A1)(A1)(C2)

Note: Award (A1) for 18, 12 and 1 in correct place on Venn diagram, (A1) for 3, 16 and 11 in correct place on Venn diagram.

[2 marks]

- (b) Find the number of surveyed students who did not like any of the three flavours.

[2]

Markscheme

$$85 - (3 + 16 + 11 + 18 + 12 + 1 + 2) \quad (M1)$$

Note: Award (M1) for subtracting the sum of their values from 85.

$$22 \quad (A1)(ft) \quad (C2)$$

Note: Follow through from their Venn diagram in part (a).

If any numbers that are being subtracted are negative award (M1)(A0).

[2 marks]

- (c) A student is chosen at random from the surveyed students.

Find the probability that this student likes kiwi fruit smoothies given that they like mango smoothies.

[2]

Markscheme

$$\frac{14}{35} \quad \left(\frac{2}{5}, 0.4, 40\% \right) \quad (A1)(ft)(A1)(ft) \quad (C2)$$

Note: Award (A1) for correct numerator; (A1) for correct denominator. Follow through from their Venn diagram.

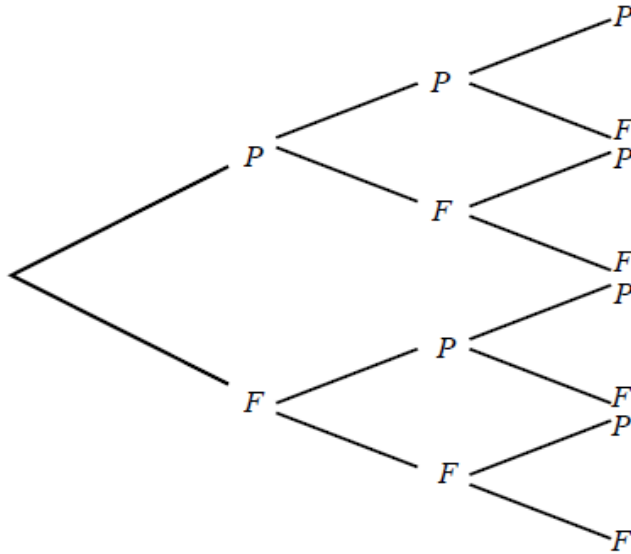
[2 marks]

9. [Maximum mark: 8]

19M.2.AHL.TZ2.H_3

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

- (a) Complete the given probability tree diagram for Iqbal's three attempts, labelling each branch with the correct probability.



[3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

A1A1A1

Note: Award *A1* for each correct column of probabilities.

[3 marks]

- (b) Calculate the probability that Iqbal passes at least two of the papers he attempts.

[2]

Markscheme

probability (at least twice) =

EITHER

$$(0.6 \times 0.7 \times 0.8) + (0.6 \times 0.7 \times 0.2) + (0.6 \times 0.3 \times 0.6) + (0.4 \times 0.6 \times 0.7)$$

(M1)

OR

$$(0.6 \times 0.7) + (0.6 \times 0.3 \times 0.6) + (0.4 \times 0.6 \times 0.7) \quad (M1)$$

Note: Award *M1* for summing all required probabilities.

THEN

$$= 0.696 \quad A1$$

[2 marks]

- (c) Find the probability that Iqbal passes his third paper, given that he passed only one previous paper.

[3]

Markscheme

P(passes third paper given only one paper passed before)

$$= \frac{P(\text{passes third AND only one paper passed before})}{P(\text{passes once in first two papers})} \quad (M1)$$

$$= \frac{(0.6 \times 0.3 \times 0.6) + (0.4 \times 0.6 \times 0.7)}{(0.6 \times 0.3) + (0.4 \times 0.6)} \quad A1$$

$$= 0.657 \quad A1$$

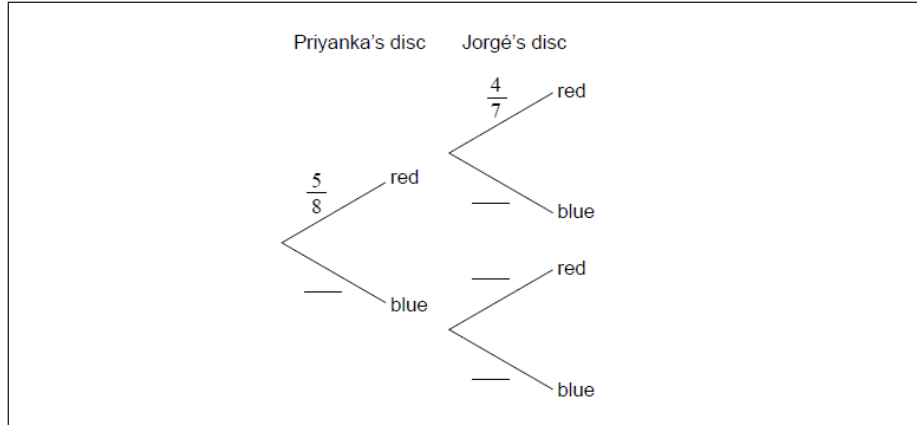
[3 marks]

10. [Maximum mark: 6]

18N.1.SL.TZ0.T_8

A bag contains 5 red and 3 blue discs, all identical except for the colour. First, Priyanka takes a disc at random from the bag and then Jorgé takes a disc at random from the bag.

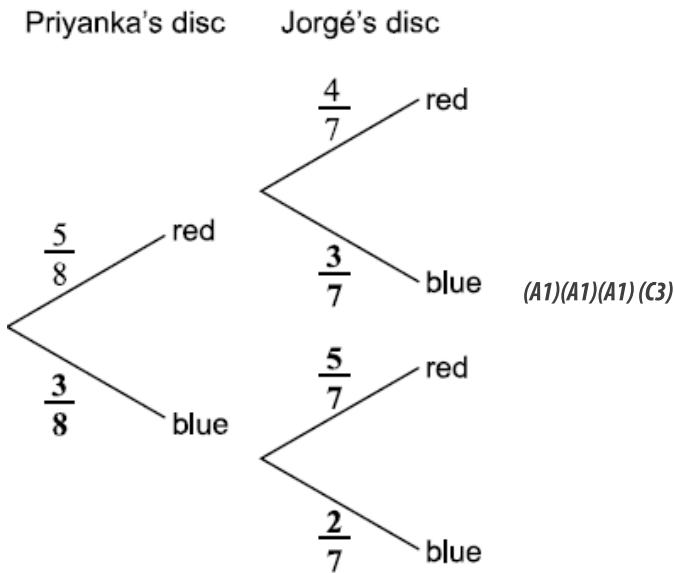
(a) Complete the tree diagram.



[3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



Note: Award (A1) for each correct pair of branches.

[3 marks]

(b) Find the probability that Jorgé chooses a red disc.

[3]

Markscheme

$$\frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{5}{7} \quad (A1)(ft)(M1)$$

Note: Award *(A1)(ft)* for **their** two correct products from their tree diagram. Follow through from part (a), award *(M1)* for adding their two products. Award *(M0)* if additional products or terms are added.

$$= \frac{5}{8} \left(\frac{35}{56}, 0.625, 62.5\% \right) \quad (A1)(ft)(C3)$$

Note: Follow through from their tree diagram, only if probabilities are [0,1].

[3 marks]

11. [Maximum mark: 14]

18N.2.SL.TZ0.T_2

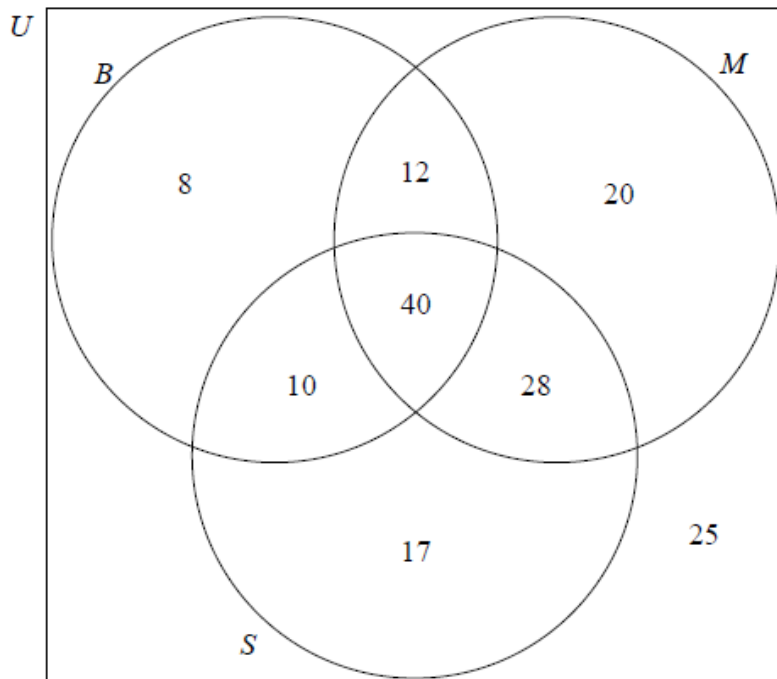
160 students attend a dual language school in which the students are taught only in Spanish or taught only in English.

A survey was conducted in order to analyse the number of students studying Biology or Mathematics. The results are shown in the Venn diagram.

Set S represents those students who are **taught** in Spanish.

Set B represents those students who **study** Biology.

Set M represents those students who **study** Mathematics.



(a.i) Find the number of students in the school that are taught in Spanish.

[2]

Markscheme

$$10 + 40 + 28 + 17 \quad (M1)$$

$$= 95 \quad (A1)(G2)$$

Note: Award (M1) for each correct sum (for example: $10 + 40 + 28 + 17$) seen.

[2 marks]

(a.ii) Find the number of students in the school that study Mathematics in English.

[2]

Markscheme

$$20 + 12 \quad (M1)$$

$$= 32 \quad (A1)(G2)$$

Note: Award (M1) for each correct sum (for example: $10 + 40 + 28 + 17$) seen.

[2 marks]

(a.iii) Find the number of students in the school that study both Biology and Mathematics.

[2]

Markscheme

$$12 + 40 \quad (M1)$$

$$= 52 \quad (A1)(G2)$$

Note: Award (M1) for each correct sum (for example: $10 + 40 + 28 + 17$) seen.

[2 marks]

(b.i) Write down $n(S \cap (M \cup B))$.

[1]

Markscheme

$$78 \quad (A1)$$

[1 mark]

(b.ii) Write down $n(B \cap M \cap S')$.

[1]

Markscheme
12 (A1)
[1 mark]

A student from the school is chosen at random.

(c.i) Find the probability that this student studies Mathematics.

[2]

Markscheme
$\frac{100}{160}$ ($\frac{5}{8}$, 0.625, 62.5 %) (A1)(A1)(G2)
Note: Throughout part (c), award (A1) for correct numerator, (A1) for correct denominator. All answers must be probabilities to award (A1).
[2 marks]

(c.ii) Find the probability that this student studies neither Biology nor Mathematics.

[2]

Markscheme
$\frac{42}{160}$ ($\frac{21}{80}$, 0.263 (0.2625), 26.3 % (26.25 %)) (A1)(A1)(G2)
Note: Throughout part (c), award (A1) for correct numerator, (A1) for correct denominator. All answers must be probabilities to award (A1).
[2 marks]

(c.iii) Find the probability that this student is taught in Spanish, given that the student studies Biology.

[2]

Markscheme

$\frac{50}{70}$ ($\frac{5}{7}$, 0.714 (0.714285...), 71.4% (71.4285...%)) (A1)(A1)(G2)

Note: Throughout part (c), award (A1) for correct numerator, (A1) for correct denominator. All answers must be probabilities to award (A1).

[2 marks]

12. [Maximum mark: 6]

18M.1.SL.TZ2.T_7

In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			Total
		Portuguese	Mandarin	Hindi	
Participants	Boys	20	18	5	43
	Girls	18	7	12	37
Total		38	25	17	80

(a) State the number of boys who answered questions in Portuguese.

[1]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

20 (A1)(C1)

[1 mark]

A boy is chosen at random.

(b) Find the probability that the boy answered questions in Hindi.

[2]

Markscheme

$\frac{5}{43}$ (0.11627..., 11.6279...%) (A1)(A1)(C2)

Note: Award (A1) for correct numerator, (A1) for correct denominator.

[2 marks]

(c) Two girls are selected at random.

Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi.

[3]

Markscheme

$$\frac{7}{37} \times \frac{12}{36} + \frac{12}{37} \times \frac{7}{36} \quad (A1)(M1)$$

Note: Award *(A1)* for first or second correct product seen, *(M1)* for adding their two products or for multiplying their product by two.

$$= \frac{14}{111} \quad (0.12612\dots, 12.6126\%) \quad (A1)(C3)$$

[3 marks]

13. [Maximum mark: 14]

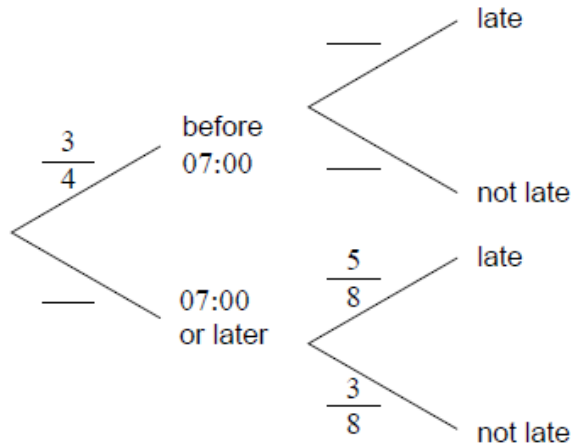
18M.1.SL.TZ2.S_8

Pablo drives to work. The probability that he leaves home before 07:00 is $\frac{3}{4}$.

If he leaves home before 07:00 the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at 07:00 or later the probability he will be late for work is $\frac{5}{8}$.

(a) **Copy** and complete the following tree diagram.



[3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Note: Award *A1* for each bold fraction.

[3 marks]

(b) Find the probability that Pablo leaves home before 07:00 and is late for work.

[2]

Markscheme

multiplying along correct branches (A1)

$$\text{eg } \frac{3}{4} \times \frac{1}{8}$$

$$P(\text{leaves before 07:00} \cap \text{late}) = \frac{3}{32} \quad \text{A1 N2}$$

[2 marks]

- (c) Find the probability that Pablo is late for work.

[3]

Markscheme

multiplying along other "late" branch (M1)

$$\text{eg } \frac{1}{4} \times \frac{5}{8}$$

adding probabilities of two mutually exclusive late paths (A1)

$$\text{eg } \left(\frac{3}{4} \times \frac{1}{8}\right) + \left(\frac{1}{4} \times \frac{5}{8}\right), \frac{3}{32} + \frac{5}{32}$$

$$P(L) = \frac{8}{32} \left(= \frac{1}{4}\right) \quad \text{A1 N2}$$

[3 marks]

- (d) Given that Pablo is late for work, find the probability that he left home before 07:00.

[3]

Markscheme

recognizing conditional probability (seen anywhere) (M1)

$$\text{eg } P(A|B), P(\text{before 7}|\text{late})$$

correct substitution of **their** values into formula (A1)

$$\text{eg } \frac{\frac{3}{32}}{\frac{1}{4}}$$

$$P(\text{left before 07:00}|\text{late}) = \frac{3}{8} \quad \text{A1 N2}$$

[3 marks]

- (e) Two days next week Pablo will drive to work. Find the probability that he will be late at least once.

[3]

Markscheme

valid approach **(M1)**

eg $1 - P(\text{not late twice}), P(\text{late once}) + P(\text{late twice})$

correct working **(A1)**

eg $1 - \left(\frac{3}{4} \times \frac{3}{4}\right), 2 \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}$

$\frac{7}{16}$ **A1 N2**

[3 marks]

14. [Maximum mark: 6]

23N.2.AHL.TZ2.7

A junior baseball team consists of six boys and three girls.

The team members are to be placed in a line to have their photograph taken.

(a) In how many ways can the team members be placed if

(a.i) there are no restrictions.

[1]

Markscheme
$(9! =) 362880$ A1
Note: Accept $9!$ or 363000 .
[1 mark]

(a.ii) the girls must be placed next to each other.

[2]

Markscheme
attempt to consider girls as a single object (M1)
$(3! \times 7! =) 30240$ A1
Note: Accept 30200 .
[2 marks]

(b) Five members of the team are selected to attend a baseball summer camp. Find the number of possible selections that contain at least two girls.

[3]

Markscheme
METHOD 1
recognition of the two different cases for 2 girls and 3 girls (M1)
exactly 2 girls is ${}^6C_3 \times {}^3C_2 = 60$ and exactly 3 girls $({}^3C_3x) {}^6C_2 = 15$ (A1)
total $(= 60 + 15) = 75$ A1
METHOD 2

recognition of the three different cases: total choices, 1 girl and no girl (M1)

total choices ${}^9C_5 = 126$, one girl case ${}^3C_1 \times {}^6C_4 = 45$, no girl case ${}^6C_5 = 6$
(A1)

total $(= 126 - 45 - 6) = 75$ A1

[3 marks]

15. [Maximum mark: 5]

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A team of four is to be chosen from a group of four boys and four girls.

(a) Find the number of different possible teams that could be chosen.

[3]

Markscheme

*This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

$$\binom{8}{4} \quad (A1)$$
$$= \frac{8!}{4!4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 7 \times 2 \times 5 \quad (M1)$$
$$= 70 \quad A1$$

METHOD 2

recognition that they need to count the teams with 0 boys, 1 boy... 4 boys *M1*

$$1 + \binom{4}{1} \times \binom{4}{3} + \binom{4}{2} \times \binom{4}{2} + \binom{4}{1} \times \binom{4}{3} + 1$$
$$= 1 + (4 \times 4) + (6 \times 6) + (4 \times 4) + 1 \quad (A1)$$
$$= 70 \quad A1$$

[3 marks]

(b) Find the number of different possible teams that could be chosen, given that the team must include at least one girl and at least one boy.

[2]

Markscheme

EITHER

recognition that the answer is the total number of teams minus the number of teams with all girls or all boys *(M1)*

70 – 2

OR

recognition that the answer is the total of the number of teams with 1 boy,

2 boys, 3 boys (M1)

$$\binom{4}{1} \times \binom{4}{3} + \binom{4}{2} \times \binom{4}{2} + \binom{4}{1} \times \binom{4}{3} = (4 \times 4) + (6 \times 6) + (4 \times 4)$$

THEN

= 68 A1

[2 marks]