Real World Applications of the Prisoner's Dilemma

The world and concepts of mathematics is presented to us everywhere, sometimes even when we don't recognize it. An example of this circumstance is game theory, a branch of mathematics dealing with competitive situations in where strategies of participants are analyzed as the outcome of their choice of action is critically dependent on the other participants' action. Game theory has been applied to various contexts, including economics, business, politics and biology, but the most well known problem that falls under the context of law enforcement, is the prisoner's dilemma. Developed in 1950 by mathematicians Merrill Flood and Melvin Dresher (both employed in the RAND Corporation) and then formalized by Albert W. Tucker, another great mathematician[9], the concept of the prisoner's dilemma, along with my research into the idea, raised some questions such as "what would be the best options for each prisoner?", "how can psychology be used to assist in this situation?", and "what would affect each prisoner's decision to testify or remain silent?"; this opened some thought about the logistics of the situation – exploring the likelihood of each outcome happening, and how these affect each prisoner, thinking about the psychological aspects of this and the mathematics incorporated for this problem. This lead to further research of the notations used in game theory, including dominant strategy equilibrium, Pareto optimality, Nash equilibrium and strategy profiles, all with appropriate applications to the concept. Also, as mentioned earlier, this problem in game theory can be applied in the context of the real world around us; so what exactly is the prisoner's dilemma, and how can its versatility be useful in our everyday lives?

In researching various concepts in mathematics, I landed upon a video explaining strategies you can use to win at the game of rock, paper, scissors, which I thought was fascinating, so I decided to go into further research about where this idea came from. I discovered the branch of game theory and its most famous problem, the prisoner's dilemma[2], and decided to explore more about this subject. This lead me to a video[15] explaining the concept of the dilemma, illustrating how the problem worked, and presenting quite a few ideas that I found really interesting. I was first drawn by the psychological aspects of the problem, thinking of how each "player" in the dilemma would be thinking in order to maximize their benefits in the given situation. I grew curious in game theory and the different strategies to come up with the best possible outcome, or "winning the game". Not only is this an intriguing concept, but has links across a large number of disciplines[2], and can be applied to real life, regular activities or even games. There has been and there still are now a number of decisions in many aspects of humanity that can be "simplified to a prisoner's dilemma"[11]; this includes decisions in politics, sports, economics, biology and psychology. Concepts under game theory have a "selective usefulness for studying certain types of problems and obtaining certain types of conclusions"[3] and analyzing these can even lead to better decision making. Through this exploration, I will research and explain the concept of the prisoner's dilemma, and situations sharing similar processes and concepts. I will explain how it relates to game theory, and how it can be applied to circumstances in the real world, under different disciplines.

The "mathematical theory of games" was invented by John von Neumann and Oskar Morgenstern in 1944[10]. This theory describes and analyzes "situations in which the decision of one entity and the decision of an opposing or competing entity result in a number of predetermined outcomes"[11], and has been applied to contexts in war, business and biology. However, limitations in the the mathematical framework only made it applicable to limited and special conditions initially. This framework has been deepened and generalized over the past six decades, and since at least the late 1970s, it has been possible to say that game theory is the "most important and useful tool in the analyst's kit" whenever confronted by situations in which what is one agent's best action (for themselves) is dependent on the expectation about what the other agents will do, and vice versa. Game theory has been "rendered mathematically and logically systematic" only since 1944, but the game-theoretic concepts can be traced back to ancient times, such as in Plato's the *Laches* and the *Symposium*, Socrates recalls a time from the Battle of Delium that some commentators have interpreted as involving a situation that includes a concept dealing with "which strategy would be most beneficial to an individual, taking account all the circumstances"[10]. This reflects concepts in the prisoner's dilemma, proving how this can be applied to real life situations, and that similar concepts apply to different circumstances involving the best outcome for an individual at the expense of the other[13], even dating back long ago.

The situation of the prisoner's dilemma goes as so: two suspects are arrested for a crime, however the police do not have enough evidence to convict them, therefore they need a confession. The suspects are separated into different rooms and are each given the opportunity to testify against the other or remain silent. If suspect A testifies (defects) against suspect B, while B remains silent (cooperates), suspect A goes free and B is sentenced to 3 years in prison. If both suspects cooperate, each will serve a 1-year sentence, however is both suspects testify, each will serve 2 years in prison[15]. This can be represented through a payoff matrix such as the following:

	B – cooperate	B – defect
A – cooperate	1, 1	3, 0
A – defect	0, 3	2, 2

Each row or column of the payoff matrix represents both an action, and a pure strategy, which is when a player selects a single action and plays it[4]. So what should each prisoner do? It seems best for the group for both suspects to cooperate, giving only a 1-year sentence to each, however from an individual's point of view, they are better off defecting as they can either go free if the other stays silent, or serve only 2 years if the other suspect defects too versus 3 years if they stay silent and the other defects. So as a whole, both suspects would be seen as better off cooperating, but from an individual stand-point, they can always gain by defecting (if they have no control over the other's decision), therefore they both end up with 2 years each rather then 1 year if they both cooperate[15].

	B – cooperate	B – defect
A – cooperate	1, 1	3, 0
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This problem also brings up an interesting moral dilemma, as we as humans often seek opportunities that would maximize our own benefit, hence wanting to better our own situation as can be reflected through this problem if both prisoners decide to defect. This again shows how from their individual stand-points they are worse off than if they both cooperated[15]. The prisoner's dilemma is also an example of a situation of imperfect information, where neither agent knows the other's move or decision until after both of the agents have moved[4]. It is important to note how this problem uses the assumption that each player is a rational agent, meaning they are acting as "computers" and will hypothetically always choose the option that they think will be best for them, disregarding the gains of others. This again reflects the general human thinking process as they want what is best for themselves. However, when taken to a deeper level, this model can be inaccurate as when applied to real people, there can be various external factors affecting each person's decision. This can include the relationship between the two players, and their closeness and trust on each other, which can lead both choosing to cooperate instead of the predicted defecting. Thus the point of the prisoner's dilemma is to address the possible problem faced in certain situations where the rational agents would end up ironically hurting themselves when trying to better their own situation rather than choosing the benefit of the group as a whole[15].

This same concept can be applied to different situations, such as in the game *Golden Balls* during "Split or Steal" where two contestants get to decide whether they want to "split" or "steal" the jackpot money. In this case, if both choose to split, each contestant receives half the jackpot, if one chooses to split and one chooses to steal, the one who steals will receive the whole jackpot, and if both choose to steal, neither of them get the money[2].

https://ncnblogger.wordpress.com/2011/11/01/golden-balls-split-or-steal-women-more-likely-to-steal/

Another more well-known context where the prisoner's dilemma can be applied is under economics in marketing, considering competition between companies. One example is regarding two cigarette companies trying to make sales in a hypothetical society of 100 people who are all smokers[15]. Two cigarette companies A and B have the option to advertise to promote their product, so they have to think of the best way for them to be able to get maximum profit given the conditions. If both companies decide not to advertise, the predicted payoff of sales within the society is that half the population will buy company A's cigarettes and the other half will buy company B's, this equates to 50 customers per company. The cigarettes cost $2 per packet, thus each company will end up with a profit of $100. However, if a company decides to put money into advertising, which in this case costs $10, this will increase their sales to 80% of the population buying their product, and 20% buying from the other company. This equates to 80 customers for the advertising company, still selling each cigarette pack for $2 for a profit of $160, but spending $10 on advertising leading to a final profit of $150. The company which does not choose to advertise only ends up with 20 customers, and still selling $2 per pack ends up with a $40 profit. If both companies choose to advertise, again half of the population will buy from company A and half will buy from company B. Both companies spend money on advertising so both of their profits will end up being $90[15]. This can be illustrated in the payoff matrix below:

	B – nothing	B – advertise
A – nothing	50 x $2 = $100 50 x $2 = $100	20 x $2 = $40 80 x $2 - $10 = $150
A – advertise	80 x $2 - $10 = $150 20 x $2 = $40	50 x $2 - $10 = $90 50 x $2 - $10 = $90

In this case, it can be seen again that the option best for the group would be for both companies to not advertise, and each will still end up with a profit of $100, compared to only $90 if they both advertise. However, from an individual company's stand-point, they can gain more profit by advertising regardless if the other company advertises or not as depicted here:

	B – nothing	B – advertise
A – nothing	50 x $2 = $100 50 x $2 = $100	20 x $2 = $40 80 x $2 - $10 = $150
A – advertise	80 x $2 - $10 = $150 20 x $2 = $40	50 x $2 - $10 = $90 50 x $2 - $10 = $90

	B – nothing	B – advertise
A – nothing	50 x $2 = $100 50 x $2 = $100	20 x $2 = $40 80 x $2 - $10 = $150
A – advertise	80 x $2 - $10 = $150 20 x $2 = $40	50 x $2 - $10 = $90 50 x $2 - $10 = $90

This situation is clearly similar to the dilemma the prisoner's face, as each outcome is based on the decisions of both agents, and it can be seen that each company's decision to advertise ends up hurting them than if both decide not to advertise, despite their attempts to better their situation by gaining by advertising. However, unlike the prisoners, the companies are able to talk and coordinate with each other while making the decision, and can arrive to the conclusion that would best benefit the group, rather than just the individual[15]. This shows how the model of the prisoner's dilemma can be applied to real world contexts, but will not always follow the predicted outcomes as external factors can come into play that are not necessarily considered in the original mathematical concept itself.

The mathematics involved with the prisoner's dilemma includes solution concepts, such as the dominant strategy equilibrium and the Nash equilibrium, and the concept of Pareto optimality. Solution concepts are applied to games dealing with multiple agents, and the best strategy depends on others' choices[4]. A dominant strategy is the strategy among a player's strategy profile (the set of strategies determined by a game that players can put forward) that gives the player the largest payoff regardless of what the other players do[12]. A dominant strategy equilibrium is established when all players play their strictly dominant strategy[6]. In relation to the prisoner's dilemma, there is one dominant strategy equilibrium, which is when both players defect.

A Nash equilibrium in game theory is a solution concept of non-cooperative games[1], describing a "stable state of a system involving the interaction of different participants"[5], where no player has the incentive to move from or change their optimal strategy when given the other player's strategy[7]. It exists when there is "no unilateral profitable deviation" from any of the players in the game, as no player would change their action as long as all the other players stay the same. It is a self-enforcing concept for when players of a game are at Nash equilibrium, they are not inclined to move because they will be worse off. In the case of the prisoner's dilemma, there is only one Nash equilibrium, which is when both players defect[14]. This is the same as the dominant strategy equilibrium, meaning that all dominant strategy equilibria are Nash equilibria. However, there can be more than one Nash equilibrium in a system, but only one dominant strategy equilibrium. More Nash equilibria can occur when there is a finite number of agents and action profiles added in a game[4]. An example of how a Nash equilibrium is established in a simple game is shown below:

	L	R
T	a, b	c, d
B	e, f	g, h

In order for (T, L) to be a dominant strategy equilibrium (also a Nash equilibrium), the following must be true[14]:

- $a > e$
- $c > g$
- $b > d$
- $f > h$

And in order for (T, L) to be a Nash equilibrium, only this must be true[14]:

- $a \geq e$
- $b \geq d$

If in a game, every player plays their dominant strategy (assuming every player has one), then it will result in a Nash equilibrium. In the prisoner's dilemma example shown earlier, both players know that 2 years is better than 3, and that 0 years is better than 1, hence their dominant strategy is to testify and they will both choose to do so (defect). Since both players choose to defect, (2,2) is the consequence and also the Nash equilibrium. To check whether or not it is a Nash equilibrium, check if either of the suspects would desire to deviate from their position. Suspect A would not want to deviate because if he chooses to remain silent and suspect B still testifies, suspect A would increase prison time by 1 year[14]. Therefore, that Nash equilibrium is established.

Pareto optimality, named after Vilfredo Pareto, is a measure of efficiency for all the members involved in a game. The notion answers one question in game theory of whether one outcome can be better than any other. An outcome of a game is considered "Pareto optimal" if there is "no other outcome that makes every player at least as well off and at least one player strictly better off"; this means a Pareto optimal outcome cannot be improved upon without being at the expense of at least one other player[1]. In the prisoner's dilemma, the Pareto optimal outcomes are as so:

Pareto Optimal outcomes (C= cooperate, D = defect)[4]:

- (C, C)
  - no other profile gives both players a higher payoff
- (D, C)
  - no other profile gives suspect A a higher payoff
- (C, D)
  - no other profile gives suspect B a higher payoff
- (D, D) is Pareto dominated by (C, C)
  - ironically, (D, D) is the dominant strategy equilibrium

	B – cooperate	B – defect
A – cooperate	Pareto Optimal 1, 1	Pareto Optimal 3, 0
A – defect	Pareto Optimal 0, 3	Nash Equilibrium , 2 Dominant Strategy Equilibrium

As can be seen through this, the Nash equilibrium of the prisoner's dilemma does not correspond to its Pareto optimal outcome. This reflects one of the features of the Nash equilibrium, as it is not always the same as the socially optimal outcome. This means that in a game, it is possible for all the players to move improve their payoffs by collectively coordinating to change to strategies not in the Nash equilibrium. This is because some agents may diverge from the optimal strategy that has been agreed upon in order to maximize their own payoff more at the expense of the rest of the group. A Pareto optimal equilibrium is then established when the social optimum where no individual agent can improve their payoff without hurting at least one other agent and make them worse off. Though Pareto optimality is not a solution concept, unlike the dominant strategy equilibrium and Nash equilibrium, it can be an important feature used to determine what solution the agents should play, or learn to play over time. The interesting thing about the prisoner's dilemma in this case is that all outcomes are Pareto optimal except for the unique Nash equilibrium, which is when both prisoners defect. This strong dissimilarity between Nash equilibrium and Pareto optimality is what makes the prisoner's dilemma a central model of studying game theory. It also illustrates one of the core dynamics of game theory which is the concept of cooperation versus competition[1].

This can also be reflected in the marketing example explained earlier, however this particular example slightly differs from the conceptual prisoner's dilemma as the companies do have the chance to cooperate and choose to move to the Pareto optimal outcome, where they both do not advertise and gain what is best for the group. This applied context of whether companies should compete or cooperate with other businesses determines the success of the business in terms of the profit they end up making. This can also be the case for determining whether a business should raise prices or not, again using the same concept as the prisoner's dilemma. Raising prices can help gain profit, but lose customers; this becomes a logistic to be considered where there are competing companies in the same area, selling the same products[8].

Following the previous model, if all the similar businesses keep similar, regular prices, hypothetically, all should be making similar profit. However, if one business decides to lower prices, they will end up having more customers and making more profit than those that decide to keep the regular prices. This is when the businesses start to compete. If all the businesses decide to lower their prices, all will again end up with similar profits, but is lower than the previous profit the businesses were making by all keeping the original, regular prices. This shows how agents in a game similar to the prisoner's dilemma can have the incentive to better off their situation, but can end up hurting themselves in the process when they could all easily be better off by cooperating and agreeing to choose what's best for the group. However, in the case of marketing, there is some flexibility to the concept. Businesses can choose to set a certain price for a period of time, and change it afterwards for another set period of time. For example, a business can set a regular price for one month, and lower prices for another, then set it back to the original for the month after. This is where the idea of market sales comes from. Other competing companies can recognize this, and cooperate with the surrounding businesses to manage their selling prices accordingly. This can result in even distribution of raising and lowering prices that benefits the whole group of companies, giving the best payoff for the individual business as well as the group of competing businesses through time[8].

	B – Cooperate (keep prices high)	B – Compete (lower prices)
A – Cooperate (keep prices high)	High, equal profit	B gains more profit, A makes less profit
A – Compete (lower prices)	A gains more profit, B makes less profit	Lower, equal profit

What can be concluded from these examples and the concept of the prisoner's dilemma is that it is better off for the group if both suspects, companies, or agents cooperate, but from the individual's perspective, they seem better off defecting or competing against each other, if they have no control of what the other will do. The Pareto optimal outcome for the group is not as obvious from the individual stand-points, as the dominant strategy equilibrium and Nash equilibrium in this case is unique from it. As the prisoner's dilemma is just a model however, its real life applications do not always follow its predicted outcomes, for it does not take into account the various external factors than can affect the agents' decisions. This can include trust and a close relationship between two players or coordination between businesses to manage selling prices, which would theoretically result in each agent cooperating, and choosing to play what is best for the group.

Throughout this exploration, I personally had some difficulty fully understanding this concept in game theory as it is unfamiliar to me and had many notions involved in it that can be difficult to understand in themselves. The several equilibria and new notations and applications involved in this one problem gave me some obstacles to overcome and understand through my research on this subject, such as Pareto optimality. However, I do find it a very intriguing concept to think about, taking in all the factors and possibilities this situation can involve, and be applied to in real life situations. I also found the "ambiguity" of the model to be fascinating, in how some external factors can influence the predicted outcomes when applied to some real life situations. Game theory in itself is one of the fundamental branches of mathematics which offers several ideas to analyze and incorporate into daily life understandings, with the famous prisoner's dilemma being one of its underlying problems. Exploring these ideas in mathematics has made me realize the truly amazing things mathematics gives us to think about. Learning about the prisoner's dilemma and the wide variety of contexts its can be applied to really helped me appreciate the wonders of the complexity and versatility of of mathematics, and the several ideas and questions this particular concept raises continues to intrigue me. It's astounding how this one model proves its use in the real world, being a useful tool for analysts and players alike in game theory.

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