Modelling Radioactive decay and Applying it to the Fukushima accident

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I. Introduction

On 11th March, 2011, a huge nuclear disaster happened in Fukushima, Japan and the global effects it left are still present nowadays. A major earthquake and a 15-metre tsunami disabled the power supply and cooling of three nuclear reactors at the Fukushima Daiichi Nuclear Power Plant which led to massive releases of radioactive material into the atmosphere as well as the ocean in several days (World Nuclear Association, 2020). About 160,000 people were forced to evacuate in the radius of 20-30 km from the power plant and in 2019, the government eased the measure which allowed parts of Okuma to open for business, "but only a few hundred residents have moved home" (McCurry, 2019). I find this very interesting because clearly, the phenomenon and mathematics behind this situation are quite complicated therefore, a lot people are very worried and confused about the long-term effects of the radioactive materials released. Therefore, the following research question arose in my mind, *When is it safe for adults and infants to live normally in Fukushima again?* As an aspiring nuclear physicist, answering this question can help not only me but also the readers to understand how serious are the effects of radioactive materials when they are accidentally leaked into the environment. From that, we can have a more thorough understanding about the situation in Fukushima so that the old residents can feel more secure when thinking of going back to their old town.

The radioactive materials consist of radioactive nuclei and they are dangerous because they will all undergo radioactive decay, a process which emits energy in the form of ionising radiation and this radiation can either kill cells or damage the DNA, which can eventually lead to cancer (Kirschenbaum, 2016). In order to answer the research question, firstly I will need to investigate and generate a mathematical model of radioactive decay that can hypothesize the number of nuclei left (haven't undergone decay) at any particular time. 2 approaches will be used to test against each other: graphing and algebraic method. The collecting data process for graphing method will be done with the support of a radioactivity simulation because this type of experiment cannot be conducted at school lab. The model will then be applied to the accident. There were different radioactive isotopes released in the accident, which basically means that they have different half life (the time it takes for half of the radioactive samples to decay). By applying the model to different isotopes, I can find their activity at any particulate time in the future. For people to live there safely, the activity must be lower than the safety radiation dose limits. The radiation limits vary with age and gender, however this exploration will only investigate about infants below 1 year old, male and female adults.

II. The mathematical model of radioactive decay

1. <u>Graphing approach</u>

a) Collecting data

A virtual radioactive decay experiment was conducted on an *Alpha decay* simulation (PhET, 2016). The experiment started with 100 radioactive nuclei of the an isotope and they decay over time. Theoretically, the number of nuclei that haven't undergone decay (N) will never reach 0 since after 1 half life, half of the radioactive samples will decay and that goes on to infinity. The number of nuclei left were shown in the simulation and that was recorded every 5 seconds from the start of the experiment until it became constant at 1 (the simulation can only show an integer of nuclei so 1 is the smallest possible number of nuclei left to be shown). A 5-second separation was chosen because that was just enough time to note down the number as well as to see a detailed yet, distinct change in number of nuclei.

b) Result

Time (s)	N (nuclei)	Time (s)	N (nuclei)
0	100	55	8
5	78	60	6
10	60	65	4
15	46	70	3
20	40	75	3
25	31	80	2
30	25	85	2
35	19	90	1
40	16	95	1
45	13	100	1
50	11		

Table 1. Number of present nuclei every 5 seconds

The data in Table 1 were plotted to produce a scatter diagram with y-axis being N and x-axis being the time *t* (see Figure 1.) This was done in order to see which model is best suitable for shape of the plotted data as well as when applying to the context.

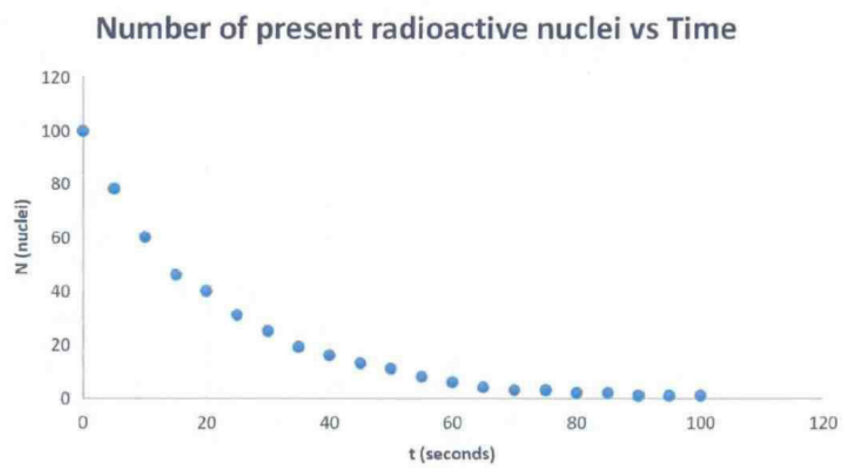

Figure 1. Scatter diagram of number of present radioactive nuclei vs Time

c) Analysis of the graph

The shape of the graph:

- Has 1 horizontal asymptote y = 0 because N will never reach 0: linear, polynomial, power and sinusoidal functions are not suitable because they do not have horizontal asymptotes; logistic function is not suitable because it has 2 different horizontal asymptotes.
- Does not have a vertical asymptote as t (x-coordinate) does reach 0: inverse variation function is not suitable
- The domain and range are all real numbers
- As x increases, y decreases (a decreasing graph)
- The graph is continuous and smooth

⇒ An exponential decay relationship (ae^{bx}) where a > 0 and 0 > b > 1, fits with both the shape of the graph and the contextual situation.

It is worth to note that there are more mathematical models to be considered, but they are not taken into account because they are beyond the level of this course.

The equation of the exponential regression line and the coefficient of determination (r^2) were generated (see Figure 2.)

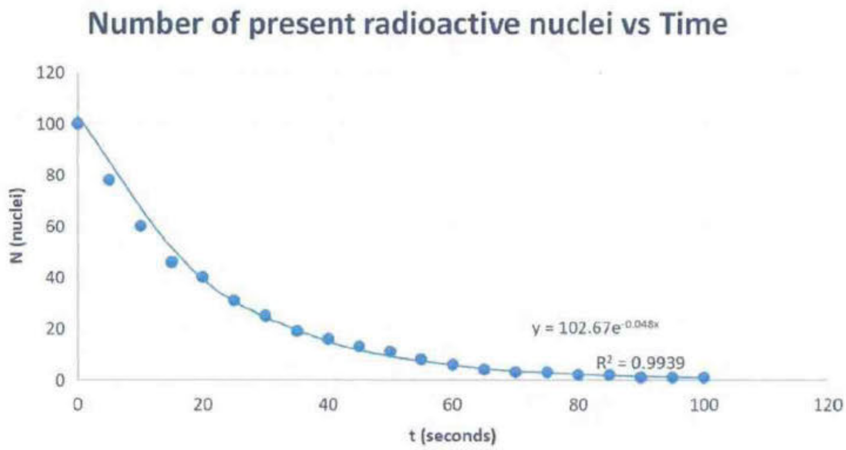

Figure 2. The relationship between the number of present radioactive nuclei and time

The value of r^2 is very high (= 0.9939) which means that 99.39% of the total variance in number of nuclei can be explained by the exponential relationship with the variance in time. Therefore it shows that exponential regression line can represent the data very well.

d) Finding parameters

In an exponential decay relationship $f(x) = ae^{bx}$: $\mathbf{f(0) = \mathit{a}}$

In this situation, that is the initial amount of the radioactive sample (N_o). However, the value of ***a*** in the regression line generated is not 100% true in this situation $(102.67 \neq 100)$

The percentage error is: $\left|\frac{102.67-100}{100}\right| \times 100\% = 2.67\%$

The error could be due to the randomness of radioactive decay process as well as the uncertainty of the nuclei count of the simulation. Also, the number of nuclei was not recorded exactly every 5s but instead 5s + reaction time, therefore the variation of reaction time in each record could decrease the precision of the result.

The general mathematical model for radioactive decay is therefore:

$N(t) = N_o e^{bt}$ where N_o is the initial number of nuclei, b is a constant.

2. Algebraic approach

To confirm the result from the graphing method, a further investigation was conducted to find the relationship between them again but this time, algebraically. From laboratory experiments, lots of scientists have found the relationship between the rate of decay and number of present nuclei to be: $\frac{dN}{dt} \propto N$ (Choppin, Liljenzin, Rydberg, & Ekberg, 2014)

Since different isotopes have different rates of decay, let λ be a decay constant unique to an isotope, we have $\frac{dN}{dt} = \lambda N$ **(1)**

(1) is a separable differential equation. N and t must be separated so that N is on one side and t is on the other side of the equation.

$$\frac{dN}{N} = \lambda \times dt$$

To take out dN and dt, both sides are integrated:

$\int \frac{1}{N} \times dN = \int \lambda \times dt$

$= \ln(N) = \lambda t + c$ (c = constant of $\int \lambda dt$ - constant of $\int \frac{1}{N} dN$)

To find the relationship between N and t, both sides are taken to be the exponent of the base e:

$e^{\ln(N)} = N = e^{\lambda t + c}$

Finally, t and λ must be eliminated to find c, that is when t = 0, $N(0) = e^{\lambda \times 0} e^{c} = e^{c}$

Applying to the context, N (0) is the initial amount of radioactive nuclei (N_o) and the radioactive decay equation is therefore:

$$N(t) = N_o \times e^{\lambda t} \quad \textbf{(2)},$$

which is an exponential equation.

In conclusion, both methods confirm that radioactive decay can be modelled using an exponential function $f(x) = ae^{bx}$ with $a = N_o$, x is time and b is the decay constant.

III. Half life and Decay constant

In nuclear physics, different radioactive isotopes are usually distinguished based on their half lives and since the radioactive decay equation does not have that variable, the relationship between half life and one of the variable in the equation, in this case is λ must be found

1. Finding the relationship

Half life ($t_{1/2}$) is the time it takes for half of the radioactive samples to decay

$\therefore N(t_{1/2}) = \frac{1}{2}N_{\circ}$ **(3)**

From **(2)** and **(3)**, $e^{\lambda t_{1/2}} = \frac{1}{2}$

$t_{1/2}$ and λ must be separated so that $t_{1/2}$ is on one side and λ is on the other side of the equation. Take the natural logarithm of both sides:

$\ln e^{\lambda t_{1/2}} = \ln\frac{1}{2}$ **(4)** and using the logarithm power rule: $\ln e^{\lambda t_{1/2}} = \lambda t_{1/2}\ln e = \lambda t_{1/2}$ **(5)**

From **(4)** and **(5)**: $\ln\frac{1}{2} = \lambda t_{1/2}$, rearranging that to have $t_{1/2} = \frac{\ln\frac{1}{2}}{\lambda}$

Using negative exponent rule, $\frac{1}{2} = 2^{-1}$

$$\therefore t_{1/2} = \frac{\ln\frac{1}{2}}{\lambda} = \frac{\ln 2^{-1}}{\lambda}$$

Using the logarithm power rule, $t_{1/2} = \frac{\ln 2^{-1}}{\lambda} = \frac{-1\ln 2}{\lambda}$

$$\boxed{\therefore t_{1/2} = -\frac{\ln 2}{\lambda}} \quad \textbf{(6)}$$

2. Applying to the experiment

Relationship **(6)** is applied to the experiment in the first place to test for its validity.
From the generated regression line, λ = -0.048

Substitute λ into **(6)**, $t_{1/2} = -\frac{\ln 2}{-0.048} = 14.4$

To confirm this result, the regression line was graphed on *Desmos* and interpolate to find the half life (Desmos, 2015) (see Figure 3. and 4.)

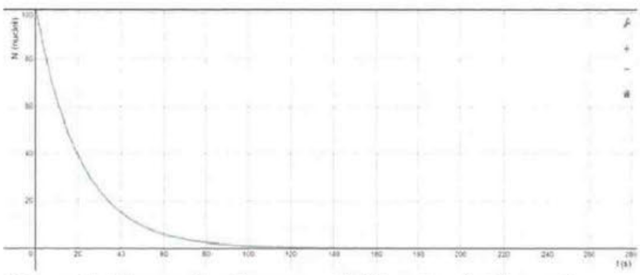

Figure 3. The graph of exponential function in Figure 2.

Figure 4. Finding the half life by interpolation

When N = 50, t = 15 (see Figure 4.) and the percentage error is:

$$\left|\frac{14.4-15}{15}\right|\times 100\% = 4\%$$

The difference could be due to level of accuracy of generated decay constant in the regression line, interpolation as well as the uncertainty of the experiment as mentioned earlier. The issue of level of accuracy means that λ = -0.048 is not the exact value but was already rounded. Also, because the half life from the graph was achieved by interpolation, there must be some degree of uncertainty in it.

IV. Activity equation

In nuclear physics, the activity of a radioactive sample is not usually shown as the number of nuclei (N) but actually "the average number of disintegrations per second" or the rate of decay. This can be expressed as $\frac{dN}{dt}$.

Let the $\frac{dN}{dt}$ be A, from **(1)** and **(2)**: $A = \lambda N = \lambda N_o \times e^{\lambda t}$ **(7)**

From **(7)**: A (0) = A_o (the initial activity) = λN_o

$$\therefore A(t) = A_o \times e^{\lambda t} \quad \textbf{(8)}$$

The unit of activity is becquerel (Bq). One becquerel means there is one decay per second.

V. Applying the model to the accident

1. <u>Information about the radioactive materials released</u>

There were 3 main radioactive materials released into the air and ocean: Iodine-131 (I^{131}); Caesium-134 (Cs^{134}) and Caesium-137 (Cs^{137}). Below is the information table about the them including their half life; decay constants (calculated using **(6)**); dose coefficients for ingestion and inhalation (ICRP, 2012); concentrations in air on 19th March, 2011 (IAEA, 2015) and in water on 28th March, 2011 (TEPCO, 2011):

	I^{131}	Cs^{134}	Cs^{137}
$t_{1/2}$ (year)	2.20×10^{-2}	2.06×10^{0}	3.00×10^{1}
λ (y^{-1})	-3.15×10^{1}	-3.36×10^{-1}	-2.31×10^{-2}
Concentration in water on 28th March, 2011 (Bq/dm³)	1.10×10^{4}	1.90×10^{3}	1.90×10^{3}
Ingestion dose coefficient for adults (Sv/Bq)	2.20×10^{-8}	1.90×10^{-8}	1.30×10^{-8}
Ingestion dose coefficient for infants (Sv/Bq)	4.80×10^{-7}	2.60×10^{-8}	1.10×10^{-8}
Concentration in air on 19th March, 2011 (Bq/m³)	1.00×10^{4}	3.00×10^{1}	3.00×10^{1}
Inhalation dose coefficient for adults (Sv/Bq)	2.40×10^{-9}	9.10×10^{-9}	9.70×10^{-9}
Inhalation dose coefficient for infants (Sv/Bq)	2.20×10^{-8}	3.20×10^{-8}	3.60×10^{-8}

Table 2. Information about radioactive materials released

It should be noted that there were a lot radioactive materials released in the accident, however, this exploration only investigates the 3 main ones, which can significantly decrease the accuracy of the calculated activity, dose as well as the final result. Also, all information in Table 2. has some level of uncertainty, especially the concentration in air and water. For water, it was assumed that the concentrations in Table 2., which were calculated based on a 500ml sample of seawater around the south of Fukushima Daiichi Nuclear Power Station, were the same for all areas, which is not realistic. This is also the case for the concentrations in air because they were calculated right at the nuclear power plant and were assumed to be the same for all areas in Fukushima.

2. Radiation dose limits and physiological data for different age groups and genders

a) Radiation dose limits

The safety radiation limit for human is counted in Sievert (Sv), that is "the SI unit for ionising radiation dose, measuring the amount of energy absorbed in a human's body per unit mass" (Donev, Stenhouse, Hanania, & Campbell, 2018). The safety doses published on the MIT's newspaper (1994) is in millirems (an alternative unit for radiation dose), therefore they need to be converted into Sv (1 millirem = 0.00001 Sv).

Age group	Radiation dose limit (millirems/year)	Radiation dose limit (Sv/year)
Adults	5000	0.05
Infants	500	0.005

Table 3. Radiation dose limit of two age groups

From Table 3., it means that in order for an adult to live safely in Fukushima, the annual ionising radiation dose must be ≤ 0.05 Sv and for an infant it must be ≤ 0.005 Sv.

b) Physiological data for calculation of dose

The data of yearly water consumption and air intake are necessary for the calculation of annual radiation dose, however, according the ICRP (2002) and WHO (2004), only the data of daily water consumption and air intake are presented, therefore they are multiplied by 365 (1 year = 365 days) and the results were as follows:

	Male adult	**Female adult**	**Infant**
Annual water consumption (dm^3/year)	949	715.4	255.5
Annual air intake (m^3/year)	8103	6643	1022

Table 4. Yearly water consumption and air intake

It should be noted that Table 3. and 4. only present an the average value for that age group or gender. In reality, these values vary with an individual's environment, body mass and activity, therefore it decreases the accuracy when generalising and applying the results to the whole population.

3. <u>Dose via ingestion of water</u>

a) Equation for the dose by drinking 1L of water

For radioactive materials in water, the dose is calculated by assuming that human absorb them via ingestion. To calculate the annual dose via ingestion of a radioactive isotope in water, firstly we need to find the dose by drinking a unit of water, in this case is 1 litre (= 1 dm^3).

$$C_{x_w} \times d.c_{x_{ingest}} = d_{x_{ingest}} \quad \textbf{(9a)}$$

C_{x_w} *is the **concentration** of radioactive isotope* x *in **water** (Bq/dm³)*

$d.c_{x_{ingest}}$ *is the **dose coefficient** for **ingestion** of radioactive isotope* x *(Sv/Bq)*

$d_{x_{ingest}}$ *is the **dose** from radioactive isotope* x *via **ingestion** of water (Sv/dm³)*

b) Equation for the dose per year by drinking water

After having the dose by drinking 1L of water, we multiple that to the average annual water consumption of an individual to yield the annual dose via ingestion of water:

$$d_{x_{ingest}} \times WC = D_{x_{ingest}} \quad \textbf{(9b)}$$

*WC is the **water consumption** per year (L/y)*

$D_{x_{ingest}}$ *is the **dose** from radioactive isotope* x *via **ingestion** of water per year (Sv/y)*

c) Total dose per year by drinking water

Let I^{131}, Cs^{134}, Cs^{137} be x, y, z respectively.

$D_{x_{ingest}} + D_{y_{ingest}} + D_{z_{ingest}} = D_{total_{ingest}}$

$D_{total_{ingest}}$ *is the **total dose** (from 3 radioactive materials) per year by drinking **water** (Sv/y)*

Using **(9a)** and **(9b):**

$$D_{total_{ingest}} = C_{x_w} \times d.c_{x_{ingest}} \times WC + C_{y_w} \times d.c_{y_{ingest}} \times WC + C_{z_w} \times d.c_{z_{ingest}} \times WC$$

Factorising the above,

$$(C_{x_w} \times d.c_{x_{ingest}} + C_{y_w} \times d.c_{y_{ingest}} + C_{z_w} \times d.c_{z_{ingest}})WC = D_{total_{ingest}} \quad \textbf{(9c)}$$

d) The concentration in water over time

In equation **(9c)**, $d.c_{x_{ingest}}; d.c_{y_{ingest}}; d.c_{z_{ingest}}; YWC$ are constants, $C_{x_w}; C_{y_w}; C_{z_w}$ vary over time.

Using equation **(8)** to have:

$$C_{x_w}(t) = \frac{A_{o_x} \times e^{\lambda_x t}}{\text{Volume of water}} = \frac{A_{o_x}}{\text{Volume of water}} e^{\lambda_x t} = C^w_{o_x} \times e^{\lambda_x t} \quad \textbf{(9d)}$$

Apply **(9d)** for y and z to have:

$$C_{y_w}(t) = C^w_{o_y} \times e^{\lambda_y t}$$
$$C_{z_w}(t) = C^w_{o_z} \times e^{\lambda_z t} \quad \textbf{(9e)}$$

$C^w_{o_x}; C^w_{o_y}; C^w_{oz}$ *are the **initial concentration** of radioactive isotope* x, y, z *in **water** respectively.*

$\lambda_x; \lambda_y; \lambda_z$ *are the **decay constant** of radioactive isotope* x, y, z *respectively*

However, since the samples used to calculate the concentrations of the radioactive materials were collected several days after the accident occurred, they were not the initial concentrations (C_o). To find C_o:

$C(t) = C_o \times e^{\lambda t}$

Rearranging to have $\frac{C(t)}{C_o} = e^{\lambda t}$

Taking the natural logarithm of both sides: $\ln\left(\frac{C(t)}{C_o}\right) = \ln e^{\lambda t}$

Using logarithm power rule: $\ln e^{\lambda t} = \lambda t \ln e = \lambda t$

Using logarithm quotient rule: $\ln\left(\frac{C(t)}{C_o}\right) = \ln C(t) - \ln C_o$

$\therefore \ln C(t) - \ln C_o = \lambda t$

$\ln C_o = \ln C(t) - \lambda t$

$$e^{\ln C(t) - \lambda t} = C_o \quad \textbf{(9f)}$$

The information from Table 2. were substituted in equation **(9f)**, with λ being the decay constant, C(t) being concentrations of radioactive materials in water on 28th March, 2011 and $t = \frac{17}{365}$ year (17 days after the accident). The results were as follows.

	I^{131}	Cs^{134}	Cs^{137}
Initial concentration in water (Bq/dm³)	4.77×10^4	1.93×10^3	1.90×10^3 1.90×10^3

Table 5. Initial concentration of radioactive materials in water

Because the initial concentrations from Table 5. were achieved by extrapolating backwards, therefore this can significantly decrease the accuracy of the calculated activity and final result

e) Substituting data into equation

Substitute **(9d)** and **(9e)** into equation **(9c)**:

$$(C_{o_x}^{w} \times e^{\lambda_x t} \times d.c_{x_{ingest}} + C_{o_y}^{w} \times e^{\lambda_y t} \times d.c_{y_{ingest}} + C_{o_z}^{w} \times e^{\lambda_z t} \times d.c_{z_{ingest}})WC = D_{total_{ingest}} \quad \textbf{(9)}$$

Substitute the information in Table 2., Table 4., Table 5. into equation **(9)**:

i. Male adult

$$D_{total_{ingest}} = (4.77 \times 10^4 \times e^{-3.15\times10^1 t} \times 2.20 \times 10^{-8} + 1.93 \times 10^3 \times e^{-3.36\times10^{-1} t} \times 1.90 \times 10^{-8}$$
$$+ 1.90 \times 10^3 \times e^{-2.31\times10^{-2} t} \times 1.30 \times 10^{-8}) \times 949$$

$$= 9.95 \times 10^{-1} \times e^{-3.15\times10^1 t} + 3.48 \times 10^{-2} \times e^{-3.36\times10^{-1} t} + 2.35 \times 10^{-2} \times e^{-2.31\times10^{-2} t}$$

ii. Female adult

$$D_{total_{ingest}} = 7.50 \times 10^{-1} \times e^{-3.15\times10^1 t} + 2.62 \times 10^{-2} \times e^{-3.36\times10^{-1} t} + 1.77 \times 10^{-2} \times e^{-2.31\times10^{-2} t}$$

iii. Infant

$$D_{total_{ingest}} = 5.85 \times 10^{0} \times e^{-3.15\times10^1 t} + 1.28 \times 10^{-2} \times e^{-3.36\times10^{-1} t} + 5.35 \times 10^{-3} \times e^{-2.31\times10^{-2} t}$$

4. <u>Dose via inhalation of air</u>

For radioactive materials in air, the dose is calculated by assuming human absorb them via inhalation. The similar process done to calculate he dose via ingestion of water (see Section 3.) was used.

a) Equation for dose by inhaling 1 m³ of air

$$C_{x_a} \times d.c_{x_{inhale}} = d_{x_{inhale}} \quad \textbf{(10a)}$$

C_{x_a} *is the* ***concentration*** *of radioactive isotope* x *in* ***air*** *(Bq/m³)*

$d.c_{x_{inhale}}$ *is the* ***dose coefficient*** *for* ***inhalation*** *of radioactive isotope* x *(Sv/Bq)*

$d_{x_{inhale}}$ *is the* ***dose*** *from radioactive isotope* x *via* ***inhalation*** *of air (Sv/dm³)*

b) Equation for the dose per year by inhaling air

$$d_{x_{inhale}} \times AI = D_{x_{inhale}} \quad \textbf{(10b)}$$

*AI is the **air intake** per year (m^3/y)*

*$D_{x_{inhale}}$ is the **dose** from radioactive isotope x pear year via **inhalation** of air (Sv/y)*

c) Total annual dose by inhaling air

Use equation **(10a)** and **(10b)** to achieve:

$$(C_{x_a} \times d.c_{x_{inhale}} + C_{y_a} \times d.c_{y_{inhale}} + C_{z_a} \times d.c_{z_{inhale}})AI = D_{total_{inhale}} \quad \textbf{(10c)}$$

*$D_{total_{inhale}}$ is the **total dose** (from 3 radioactive materials) per year by **inhaling** (Sv/y)*

d) Concentration in air over time

For radioactive isotope y: $C_{x_a}(t) = C^a_{o_x} \times e^{\lambda_x t}$ **(10d)**

For radioactive isotope y: $C_{y_a}(t) = C^a_{o_y} \times e^{\lambda_y t}$ **(10e)**

For radioactive isotope z: $C_{z_a}(t) = C^a_{o_z} \times e^{\lambda_z t}$ **(10f)**

Use equation **(9f)** to achieve the following table:

	I^{131}	Cs^{134}	Cs^{137}
Initial concentration in air (Bq/m^3)	1.99×10^4	3.02×10^1	3.00×10^1

Table 6. Initial concentrations of radioactive materials in air

e) Substituting data into equations

Substitute equation **(10d)**, **(10e)**, **(10f)** into equation **(10c)** to have:

$$(C^a_{o_x} \times e^{\lambda_x t} \times d.c_{x_{inhale}} + C^a_{o_y} \times e^{\lambda_y t} \times d.c_{y_{inhale}} + C^a_{o_z} \times e^{\lambda_z t} \times d.c_{z_{inhale}})AI = D_{total_{inhale}} \quad \textbf{(10)}$$

Substitute the information in Table 2., Table 4, Table 6. into equation **(10)**:

i. Male adult

$$D_{total_{inhale}} = 3.87 \times 10^{-1} \times e^{-3.15\times 10^1 t} + 2.23 \times 10^{-3} \times e^{-3.36\times 10^{-1} t} + 2.36 \times 10^{-3} \times e^{-2.31\times 10^{-2} t}$$

ii. Female adult

$$D_{total_{inhale}} = 3.18 \times 10^{-1} \times e^{-3.15\times 10^1 t} + 1.83 \times 10^{-3} \times e^{-3.36\times 10^{-1} t} + 1.93 \times 10^{-3} \times e^{-2.31\times 10^{-2} t}$$

iii. Infant

$$D_{total_{inhale}} = 4.48 \times 10^{-1} \times e^{-3.15\times 10^1 t} + 9.88 \times 10^{-4} \times e^{-3.36\times 10^{-1} t} + 1.10 \times 10^{-3} \times e^{-2.31\times 10^{-2} t}$$

5. <u>Total annual dose:</u>

$$D_{total} = D_{total_{ingest}} + D_{total_{inhale}} \quad \textbf{(11)}$$

D_{total} *is the* ***total dose*** *of a person (from both drinking water and inhaling air)* ***per year*** *(Sv/y)*

Using equations from Section 3e, Section 4e and equation **(11)** to have:

a) Male adult

$$D_{total} = 1.38 \times e^{-3.15\times10^{1}t} + 0.0370 \times e^{-3.36\times10^{-1}t} + 0.0258 \times e^{-2.31\times10^{-2}t}$$

Figure 5. The radiation dose of a male adult over time

In Figure 5., $y = D_{total}$ and $x = t$. As the concentrations of radioactive materials change with time, the radiation dose will also change with time, in this case it is decreasing, as seen from Figure 5. Therefore, to find the actual dose in a certain period of time, D_{total} is integrated with respect to t to find the area between the curve, the x-axis and the line $x = t$ and $x = t+1$.

The area (radiation dose) must be lower than 0.05 (see Table 3.):

$$\left(\int_{t}^{t+1} 1.38 \times e^{-3.15\times10^{1}t} + 0.0370 \times e^{-3.36\times10^{-1}t} + 0.0258 \times e^{-2.31\times10^{-2}t} dt\right) < 0.05$$

$$= \left[-0.0439 \times e^{-3.15\times10^{1}t} - 0.110 \times e^{-3.36\times10^{-1}t} - 1.12 \times e^{-2.31\times10^{-2}t}\right]_{t}^{t+1} < 0.05$$

$t \approx 0.697$ year ≈ 254 days

254 days from the accident is on **20th November, 2011**

b) Female adult

$$D_{total} = 1.07 \times e^{-3.15\times10^{1}t} + 0.0281 \times e^{-3.36\times10^{-1}t} + 0.0196 \times e^{-2.31\times10^{-2}t}$$

Figure 6. The radiation dose of a female adult over time

$$\left(\int_{t}^{t+1} 1.07\times e^{-3.15\times10^{1}t}+0.0281\times e^{-3.36\times10^{-1}t}+0.0196\times e^{-2.31\times10^{-2}t}dt\right)<0.05$$

$$=\left[-0.0339\times e^{-3.15\times10^{1}t}-0.0835\times e^{-3.36\times10^{-1}t}-0.849\times e^{-2.31\times10^{-2}t}\right]_{t}^{t+1}<0.05$$

$t\approx 0.0492$ year ≈ 18 days

18 days from the accident is on **29th March, 2011**

c) Infant

$$D_{total}=6.29\times e^{-3.15\times10^{1}t}+0.0138\times e^{-3.36\times10^{-1}t}+0.00645\times e^{-2.31\times10^{-2}}$$

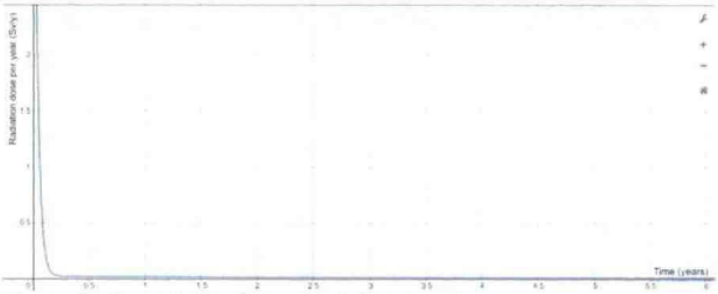

Figure 7. The radiation dose of an infant over time

The area must be lower than 0.005 (see Table 3.):

$$\left(\int_{t}^{t+1} 6.29\times e^{-3.15\times10^{1}t}+0.0138\times e^{-3.36\times10^{-1}t}+0.00645\times e^{-2.31\times10^{-2}}dt\right)<0.005$$

$$=\left[-0.200\times e^{-3.15\times10^{1}t}-0.0411\times e^{-3.36\times10^{-1}t}-0.279\times e^{-2.31\times10^{-2}}\right]_{t}^{t+1}<0.005$$

$t\approx 12.2$ years

12.2 years from the accident is around **June, 2023**

VI. Conclusion

On the way to investigate the question, *When is it safe for adults and infants to live normally in Fukushima again?*, a lot of useful models and relationships were found such as the exponential model for radioactive decay $N(t)=N_o\times e^{\lambda t}$ with $N(t)$ being the number of radioactive nuclei left at time t, N_o is the initial number of radioactive nuclei present and λ is the decay constant of that radioactive isotope. This exponential model is confirmed using both graphing and algebraic method as well as widely accepted by scientists, such as seen in the book *Nuclear and Particle Physics* (Amsler, 2015) as well as many other scientific works. This relationship leads to a more useful equation that can be well applied to real life problems, that is the activity equation $A(t)=A_o\times e^{\lambda t}$ with $A(t)$ being the activity of the isotope at time t and A_o is the initial activity. The relationship between half life and decay constant was also found to be $t_{1/2} = -\frac{\ln 2}{\lambda}$ with $t_{1/2}$ being the half life and this relationship is also very useful when applying to real life problems.

The models and relationships found were applied to the accident and it was found that female adults can start to live there the soonest, about 18 days from the accident which means on or after 29th March, 2011. Male adults can start to live there after about 254 days, which is on or after 20th November, 2011. For infants, it is a lot further away, 12.2 years from the accident which means around June, 2023. The results for adults were very surprising and not expected, especially for female because 18 days seem to be contradicted with the government's order of evacuation and also, too soon for a nuclear accident of that scale. This result means that adults can already live in Fukushima by now, however they cannot bring their young children, which can explain why the government had evacuated the residents that long. This makes me wonder are there any infants in those evacuees that have came back to Fukushima or not because based on the result, living there at this time would have very serious effect on their health. This result could also be used to discuss about economics impacts on the city because since the accident occurred, most of businesses in the city were shut down and the economics here had dropped significantly. With this result, it seems like a lot of families cannot go back until 2023, which implies that the economics of the city cannot fully recover until that time.

VII. Evaluation and Extensions

In regards to some unexpectedness about the result, the limitations of this exploration should be taken into account.

When modelling the radioactive decay:

1. Collecting data from a virtual simulation can make them quite artificial and not realistic. This can be improved by conducting real experiment in the lab, however, this is not accessible at high school level.

2. The experiment was only conducted once. In the future, there should be multiple trials which can decrease the effect of random errors and thus, increase the precision and accuracy of the data collected.

When applying to the accident:

3. The major limitation was only investigating 3 radioactive materials released in the accident and that could have a significant effect on the final result. In further exploration, other radioactive materials released should be taken into consideration to achieve a more accurate and realistic result.

4. A lot of assumptions were made such as: the concentrations in air and water; average dose limit and physiological data; human absorbed radiation via ingestion of water and inhalation of air but neglected direct exposure. Therefore this could decrease the accuracy of the results as well as the ability to apply them to different people. If this exploration were to be extended, the researchers should research and use concentrations for different areas; calculate and use the radiation dose limit for

different human body mass and age; use physiological data for different age, body mass and activity. These improvements can help to provide a more accurate and thorough answer for the question.
5. 'Background radiation' - the radiation occurs naturally from soil, food, buildings, etc., is neglected. If this is taken into account, it should also be counted in the radiation dose and therefore, the time at which human can live in Fukushima again will be further away.
6. Everything was rounded to 3 significant figures and this could decrease the accuracy of the final result. Higher level of accuracy can be achieved by rounding to more significant figures.

This exploration however, does have the strength that was using different methods to test the validity of the equation or relationship found.

Extension:
1. If this exploration were to be extended, especially at higher education, more complex analysis and mathematical models should be taken into consideration.
2. Similar investigation can also be applied to other nuclear accidents such as Chernobyl and the decay chain can be explored. This means that after a radioactive nuclei decays, it produces another 'daughter' nuclei that is also radioactive but has different half life. The 3 radioactive materials investigated in this exploration produce their 'daughter' nuclei, however since they're all stable (meaning that they will not decay and emit radiation) therefore the decay chain cannot be explored.

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