Probability - basics 2 (preDP2) [26 marks]

1. [Maximum mark: 6] 22M.2.SL.TZ1.6

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

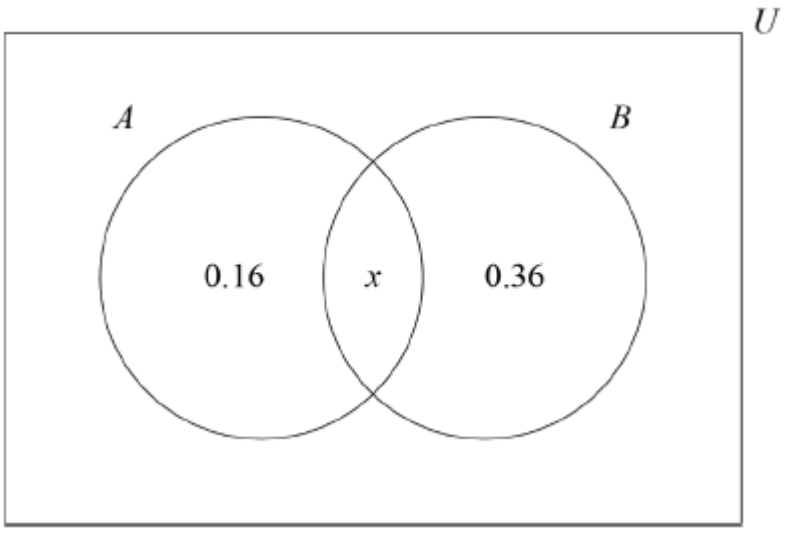
Markscheme

METHOD 1

EITHER

one of $\mathrm{P}(A) = x + 0.16$ OR $\mathrm{P}(B) = x + 0.36$ **A1**

OR



A1

THEN

attempt to equate their $\mathrm{P}(A \cap B)$ with their expression for $\mathrm{P}(A) \times \mathrm{P}(B)$ **M1**

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$ **A1**

$x = 0.24$ **A1**

METHOD 2

attempt to form at least one equation in $\mathrm{P}(A)$ and $\mathrm{P}(B)$ using independence **M1**

$(\mathrm{P}(A \cap B') = \mathrm{P}(A) \times P(B') \Rightarrow)\ \mathrm{P}(A) \times (1 - \mathrm{P}(B)) = 0.16$

OR

$(\mathrm{P}(A' \cap B) = \mathrm{P}(A') \times P(B) \Rightarrow)\ (1 - \mathrm{P}(A)) \times \mathrm{P}(B) = 0.36$

$\mathrm{P}(A) = 0.4$ and $\mathrm{P}(B) = 0.6$ **A1**

$\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B) = 0.4 \times 0.6$ **(A1)**

$x = 0.24$ **A1**

[4 marks]

(b) Find $\mathrm{P}(A' | B')$. [2]

Markscheme

METHOD 1

recognising $\mathrm{P}(A' | B') = \mathrm{P}(A')$ **(M1)**

$= 1 - 0.16 - 0.24$

$= 0.6$ **A1**

METHOD 2

$\mathrm{P}(B) = 0.36 + 0.24 (= 0.6)$

$\mathrm{P}(A' \mid B') = \frac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')} \quad \left(= \frac{0.24}{0.4}\right)$ *(A1)*

$= 0.6$ *A1*

[2 marks]

2. [Maximum mark: 6] 22M.2.SL.TZ2.4

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$. [6]

Markscheme

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = 0.68$

substitution of $\mathrm{P}(A) \cdot \mathrm{P}(B)$ for $\mathrm{P}(A \cap B)$ in $\mathrm{P}(A \cup B)$ ***(M1)***

$\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B) \ \ (= 0.68)$

substitution of $3\mathrm{P}(B)$ for $\mathrm{P}(A)$ ***(M1)***

$3\mathrm{P}(B) + \mathrm{P}(B) - 3\mathrm{P}(B)\mathrm{P}(B) = 0.68$ (or equivalent) ***(A1)***

Note: The first two ***M*** marks are independent of each other.

attempts to solve their quadratic equation ***(M1)***

$\mathrm{P}(B) = 0.2,\ 1.133\ldots\ \left(\frac{1}{5},\ \frac{17}{15}\right)$

$\mathrm{P}(B) = 0.2\ \left(= \frac{1}{5}\right)$ ***A2***

Note: Award ***A1*** if both answers are given as final answers for $\mathrm{P}(B)$.

[6 marks]

3. [Maximum mark: 8] 21M.2.SL.TZ2.4

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

Markscheme

EITHER

$\mathrm{P}(S) + \mathrm{P}(T) + \mathrm{P}(S' \cap T') - \mathrm{P}(S \cap T) = 1$ OR
$\mathrm{P}(S \cup T) = \mathrm{P}((S' \cap T')')$ ***(M1)***

$0.7 + 0.2 + 0.18 - \mathrm{P}(S \cap T) = 1$ OR
$\mathrm{P}(S \cup T) = 1 - 0.18$

OR

a clearly labelled Venn diagram ***(M1)***

THEN

$P(S \cap T) = 0.08$ (accept 8%) ***A1***

Note: To obtain the ***M1*** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

Markscheme

EITHER

$\mathrm{P}(T \cap S\prime) = \mathrm{P}(T) - \mathrm{P}(T \cap S)(= 0.2 - 0.08)$ OR

$\mathrm{P}(T \cap S\prime) = \mathrm{P}(T \cup S) - \mathrm{P}(S)(= 0.82 - 0.7)$ ***(M1)***

OR

a clearly labelled Venn diagram including $\mathrm{P}(S)$, $\mathrm{P}(T)$ and $\mathrm{P}(S \cap T)$ ***(M1)***

THEN

$= 0.12$ (accept 12%) ***A1***

[2 marks]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

Markscheme

$\mathrm{P}(G \cap T) = \mathrm{P}(T/G)\mathrm{P}(G)$ (0.25×0.48) ***(M1)***

$= 0.12$ **A1**

[2 marks]

(d) Determine if the events G and T are independent. Justify your answer. [2]

Markscheme

METHOD 1

$\mathrm{P}(G) \times \mathrm{P}(T) (= 0.48 \times 0.2) = 0.096$ **A1**

$\mathrm{P}(G) \times \mathrm{P}(T) \neq \mathrm{P}(G \cap T) \Rightarrow G$ and T are not independent **R1**

METHOD 2

$\mathrm{P}(T \mid G) = 0.25$ **A1**

$\mathrm{P}(T \mid G) \neq \mathrm{P}(T) \Rightarrow G$ and T are not independent **R1**

Note: Do not award ***A0R1***.

[2 marks]

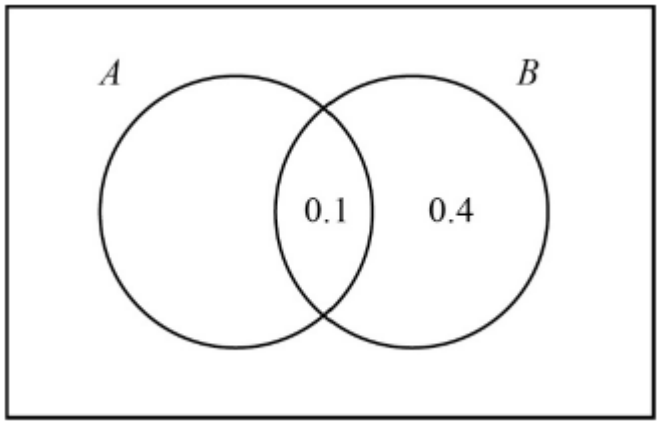
4. [Maximum mark: 6] 18N.1.AHL.TZ0.H_1

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



(M1)

Note: Award *M1* for a Venn diagram with at least one probability in the correct region.

EITHER

$\mathrm{P}(A \cap B') = 0.3$ *(A1)*

$\mathrm{P}(A \cup B) = 0.3 + 0.4 + 0.1 = 0.8$ *A1*

OR

$\mathrm{P}(B) = 0.5$ *(A1)*

$\mathrm{P}(A \cup B) = 0.5 + 0.4 - 0.1 = 0.8$ *A1*

[3 marks]

(b) Show that the events A and B are not independent. [3]

Markscheme

METHOD 1

$\mathrm{P}(A)\mathrm{P}(B) = 0.4 \times 0.5$ ***(M1)***

= 0.2 ***A1***

statement that their $\mathrm{P}(A)\mathrm{P}(B) \neq \mathrm{P}(A \cap B)$ ***R1***

Note: Award ***R1*** for correct reasoning from their value.

$\Rightarrow A, B$ not independent ***AG***

METHOD 2

$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \frac{0.1}{0.5}$ ***(M1)***

= 0.2 ***A1***

statement that their $\mathrm{P}(A|B) \neq \mathrm{P}(A)$ ***R1***

Note: Award ***R1*** for correct reasoning from their value.

$\Rightarrow A, B$ not independent ***AG***

Note: Accept equivalent argument using $\mathrm{P}(B|A) = 0.25$.

[3 marks]