Practice questions

1 Two independent events A and B are given such that $P(A) = k$, $P(B) = k + 0.3$ and $P(A \cap B) = 0.18$

a) Find k.
b) Find $P(A \cup B)$.
c) Find $P(A' \mid B')$.

2 Many airport authorities test prospective employees for drug use, with the intent of improving efficiency and reducing accidents. This procedure has plenty of opponents who claim that it creates difficulties for some classes of people and that it prevents others from getting these jobs even if they were not drug users. The claim depends on the fact that these tests are not 100% accurate. To test this claim, let us assume that a test is 98% accurate in the sense that it identifies a person as a user or non-user 98% of the time. Each job applicant takes this test twice. The tests are done at separate times and are designed to be independent of each other. What is the probability that
a) a non-user fails both tests?
b) a drug user is detected (i.e. he/she fails at least one test)?
c) a drug user passes both tests?

3 Communications satellites are difficult to repair when something goes wrong. One satellite works on solar energy and has two systems that provide electricity: the main system with a probability of failure of 0.002, and a back-up system that works independently of the main one. It has a failure rate of 0.01. What is the probability that the systems do not fail at the same time?

4 In a group of 200 students taking the IB examination, 120 take Spanish, 60 take French and 10 take both.
a) If a student is selected at random, what is the probability that he/she
(i) takes either French or Spanish?
(ii) takes either French or Spanish but not both?
(iii) does not take any French or Spanish?
b) Given that a student takes the Spanish exam, what is the chance that he/she takes French?

5 In a factory producing disk drives for computers, there are three machines that work independently to produce one of the components. In any production process, machines are not 100% fault free. The production after one 'run' from these machines is listed below.

	Defective	Non-defective
Machine I	6	120
Machine II	4	80
Machine III	10	150

a) A component is chosen at random from the produced lot. Find the probability that the chosen component is
(i) from machine I
(ii) a defective component from machine II
(iii) non-defective or from machine I
(iv) from machine I given that it is defective.
b) Is the quality of the component dependent on the machine used?

6 At a school, the students are organizing a lottery to raise money for the needy in their community. The lottery tickets they have consist of small coloured envelopes inside which there is a small note. The note says: 'You won a prize!' or 'Sorry, try another ticket.' The envelopes have several colours. They have 70 red envelopes that contain two prizes, and the rest (130 tickets) contain four other prizes.

a) You want to help this class and you buy a ticket hoping that it does not have a prize. Additionally, you don't like the red colour. You pick your ticket at random by closing your eyes. What is the probability that your wish comes true?

b) You are surprised – you picked a red envelope. What is the probability that you did not win a prize?

7 You are given two events A and B with the following conditions:

$$P(A \mid B) = 0.30, \; P(B \mid A) = 0.60, \; P(A \cap B) = 0.18$$

a) Find $P(B)$.

b) Are A and B independent? Why?

c) Find $P(B \cap A')$.

8 In several ski resorts in Austria and Switzerland, the local sports authorities use high school students as 'ski instructors' to help deal with the surge in demand during vacations. However, to become an instructor, you have to pass a test and be a senior at your school. Here are the results of a survey of 120 students in a Swiss school who are training to become instructors. In this group, there are 70 boys and 50 girls. 74 students took the test, 32 boys and 16 girls passed the test, and the rest, including 12 girls, failed the test. 10 of the students, including 6 girls, were too young to take the ski test.

a) Copy and complete the table.

	Boys	Girls
Passed the ski test	32	16
Failed the ski test		12
Training, but did not take the test yet		
Too young to take the test		

b) Find the probability that

(i) a student chosen at random has taken the test

(ii) a girl chosen at random has taken the test

(iii) a randomly chosen boy and randomly chosen girl have both passed the ski test.

9 Two events A and B are such that $P(A) = \frac{9}{16}$, $P(B) = \frac{3}{8}$, and $P(A|B) = \frac{1}{4}$. Find the probability that

a) both events will happen

b) only one of the events will happen

c) neither event will happen.

10 Martina plays tennis. When she serves, she has a 60% chance of succeeding with her first serve and continuing the game. She has a 95% chance on the second serve. Of course if both serves are not successful, she loses the point.

a) Find the probability that she misses both serves.

If Martina succeeds with the first serve, her chances of gaining the point against Steffy is

75%. If she is only successful with the second serve, her chances against Steffy for that point go down to 50%.

b) Find the probability that Martina wins a point against Steffy.

11 For the events *A* and *B*, $P(A) = 0.6$, $P(B) = 0.8$ and $P(A \cup B) = 1$.

Find

a) $P(A \cap B)$

b) $P(A' \cup B')$.

12 In a survey, 100 students were asked, 'Do you prefer to watch television or play sport?' Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

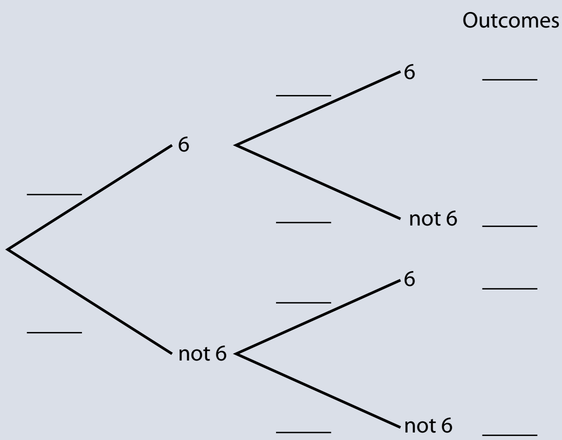
By completing this table or otherwise, find the probability that

a) a student selected at random prefers to watch television

b) a student prefers to watch television given that the student is a boy.

13 Two ordinary, six-sided dice are rolled and the total score is noted.

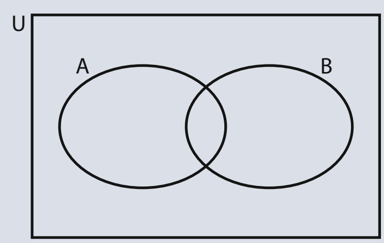
a) Complete the tree diagram by entering probabilities and listing outcomes.



b) Find the probability of getting one or more sixes.

14 The Venn diagram right shows a sample space *U* and events *A* and *B*.

$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.



a) On the diagram, shade the region $(A \cup B)'$.

b) Find

(i) $n(A \cap B)$

(ii) $P(A \cap B)$.

c) Explain why events *A* and *B* are not mutually exclusive.

15 In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

a) Using this information, complete the table below.

	Males	Females	Totals
Unemployed			
Employed			
Totals			200

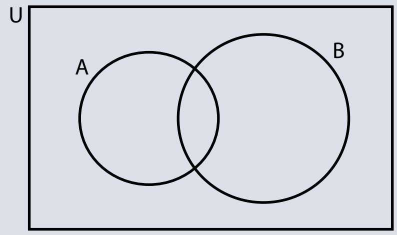
b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female

(ii) a male given that the person is employed.

16 A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

17 The Venn diagram right shows the universal set U and the sets A and B.



a) Shade the area in the diagram which represents the set $B \cap A'$.

$$n(U) = 100,\ n(A) = 30,\ n(B) = 50,\ n(A \cup B) = 65.$$

b) Find $n(B \cap A')$.

c) An element is selected at random from U. What is the probability that this element is in $B \cap A'$?

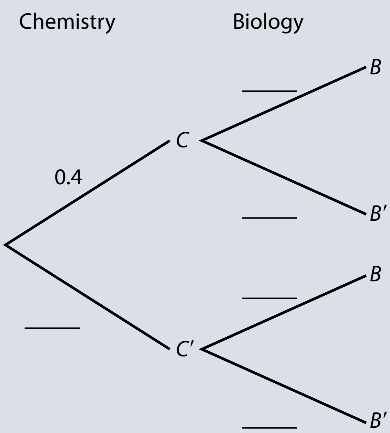
18 The events B and C are dependent, where C is the event 'a student takes chemistry', and B is the event 'a student takes biology'. It is known that

$$P(C) = 0.4,\ P(B|C) = 0.6,\ P(B|C') = 0.5.$$

a) Complete the following tree diagram.

b) Calculate the probability that a student takes biology.

c) Given that a student takes biology, what is the probability that the student takes chemistry?



19 Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is S. Find the probability that

a) S is less than 8

b) at least one die shows a 3

c) at least one die shows a 3 given that S is less than 8.

20 For events A and B, the probabilities are $P(A) = \frac{3}{11}$ and $P(B) = \frac{4}{11}$.
Calculate the value of $P(A \cap B)$ if

a) $P(A \cup B) = \frac{6}{11}$

b) events A and B are independent.

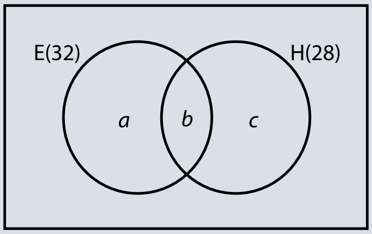
21 Consider events A and B such that $P(A) \neq 0$, $P(A) \neq 1$, $P(B) \neq 0$ and $P(B) \neq 1$.
In each of the situations **a)**, **b)**, **c)** below, state whether A and B are mutually exclusive (M), independent (I), or neither (N).

a) $P(A|B) = P(A)$

b) $P(A \cap B) = 0$

c) $P(A \cap B) = P(A)$

22 In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



a) Calculate the values a, b, c.

b) A student is selected at random.

(i) Calculate the probability that he studies both economics and history.

(ii) Given that he studies economics, calculate the probability that he does not study history.

c) A group of three students is selected at random from the school.

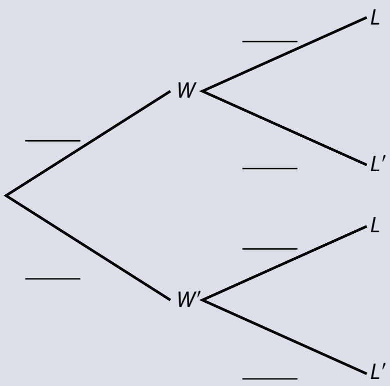
(i) Calculate the probability that none of these students studies economics.

(ii) Calculate the probability that at least one of these students studies economics.

23 A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.

24 Dumisani is a student at IB World College.
The probability that he will be woken by his alarm clock is $\frac{7}{8}$.
If he is woken by his alarm clock, the probability he will be late for school is $\frac{1}{4}$.
If he is not woken by his alarm clock, the probability he will be late for school is $\frac{3}{5}$.
Let W be the event 'Dumisani is woken by his alarm clock'.
Let L be the event 'Dumisani is late for school'.

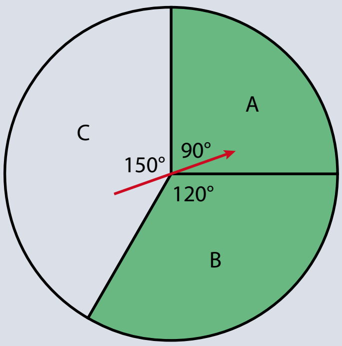
a) Copy and complete the tree diagram below.



b) Calculate the probability that Dumisani will be late for school.

c) Given that Dumisani is late for school, what is the probability that he was woken by his alarm clock?

25 The diagram shows a circle divided into three sectors A, B and C. The angles at the centre of the circle are 90°, 120° and 150°. Sectors A and B are shaded as shown.



The arrow is spun. It cannot land on the lines between the sectors. Let *A*, *B*, *C* and *S* be the events defined by

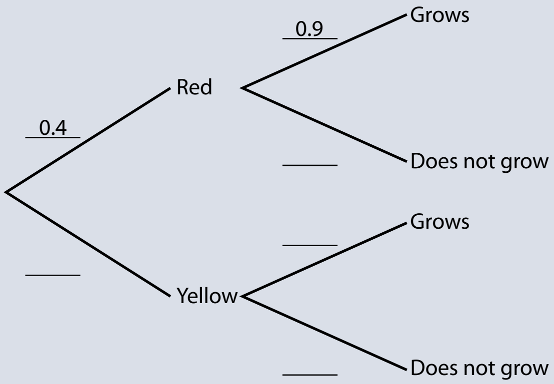
A : Arrow lands in sector A
B : Arrow lands in sector B
C : Arrow lands in sector C
S : Arrow lands in a shaded region.

Find

a) P(*B*) **b)** P(*S*) **c)** P(*A*|*S*).

26 A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

a) Complete the probability tree diagram below.



b) **(i)** Calculate the probability that the chosen seed is red and grows.
(ii) Calculate the probability that the chosen seed grows.
(iii) Given that the seed grows, calculate the probability that it is red.

27 Two unbiased six-sided dice are rolled, a red one and a black one. Let E and F be the events

E: the same number appears on both dice

F: the sum of the numbers is 10.

Find **a)** P(E)

b) P(F)

c) P($E \cup F$).

28 The table below shows the subjects studied by 210 students at a college.

	Year 1	Year 2	Totals
History	50	35	85
Science	15	30	45
Art	45	35	80
Totals	110	100	210

a) A student from the college is selected at random.

Let A be the event the student studies art.

Let B be the event the student is in year 2.

(i) Find P(A).

(ii) Find the probability that the student is a year 2 art student.

(iii) Are the events A and B independent? Justify your answer.

b) Given that a history student is selected at random, calculate the probability that the student is in year 1.

c) Two students are selected at random from the college. Calculate the probability that one student is in year 1 and the other in year 2.

29 A bag contains 2 red balls, 3 blue balls and 4 green balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one green ball and one blue ball in any order.

30 In a bilingual school there is a class of 21 pupils. In this class, 15 of the pupils speak Spanish as their first language and 12 of these 15 pupils are Argentinian. The other 6 pupils in the class speak English as their first language and 3 of these 6 pupils are Argentinian.

A pupil is selected at random from the class and is found to be Argentinian. Find the probability that the pupil speaks Spanish as his/her first language.

31 A new blood test has been shown to be effective in the early detection of a disease. The probability that the blood test correctly identifies someone with this disease is 0.99, and the probability that the blood test correctly identifies someone without that disease is 0.95. The incidence of this disease in the general population is 0.0001.

A doctor administered the blood test to a patient and the test result indicated that this patient had the disease. What is the probability that the patient has the disease?

32 The local Football Association consists of ten teams. Team A has a 40% chance of winning any game against a higher-ranked team, and a 75% chance of winning any game against a lower-ranked team. If A is currently in fourth position, find the probability that A wins its next game.

33 Given that events A and B are independent with $P(A \cap B) = 0.3$ and $P(A \cap B') = 0.3$, find $P(A \cup B)$.

34 A girl walks to school every day. If it is not raining, the probability that she is late is $\frac{1}{5}$. If it is raining, the probability that she is late is $\frac{2}{3}$. The probability that it rains on a particular day is $\frac{1}{4}$.

On one particular day the girl is late. Find the probability that it was raining on that day.

35 Given that $P(x) = \frac{2}{3}$, $P(y|x) = \frac{2}{5}$ and $P(y|x') = \frac{1}{4}$, find

a) $P(y')$ **b)** $P(x' \cup y')$.

36 The probability that a man leaves his umbrella in any shop he visits is $\frac{1}{3}$. After visiting two shops in succession, he finds he has left his umbrella in one of them. What is the probability that he left his umbrella in the second shop?

37 Two women, Ann and Bridget, play a game in which they take it in turns to throw an unbiased six-sided die. The first woman to throw a '6' wins the game. Ann is the first to throw.

a) Find the probability that

(i) Bridget wins on her first throw

(ii) Ann wins on her second throw

(iii) Ann wins on her nth throw.

b) Let p be the probability that Ann wins the game. Show that $p = \frac{1}{6} + \frac{25}{36}p$.

c) Find the probability that Bridget wins the game.

d) Suppose that the game is played six times. Find the probability that Ann wins more games than Bridget.

38 A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

a) The first two apples are green. What is the probability that the third apple is red?

b) What is the probability that exactly two of the three apples are red?

39 The probability that it rains during a summer's day in a certain town is 0.2. In this town, the probability that the daily maximum temperature exceeds 25 °C is 0.3 when it rains and 0.6 when it does not rain. Given that the maximum daily temperature exceeded 25 °C on a particular summer's day, find the probability that it rained on that day.

40 An integer is chosen at random from the first one thousand positive integers. Find the probability that the integer chosen is

a) a multiple of 4

b) a multiple of **both** 4 and 6.

41 The independent events A and B are such that $P(A) = 0.4$ and $P(A \cup B) = 0.88$. Find

a) $P(B)$

b) the probability that either A occurs or B occurs but **not both**.

42 Robert travels to work by train every weekday from Monday to Friday. The probability that he catches the 08:00 train on Monday is 0.66. The probability that he catches the 08:00 train on any other weekday is 0.75. A weekday is chosen at random.

a) Find the probability that he catches the train on that day.

b) Given that he catches the 08:00 train on that day, find the probability that the chosen day is Monday.

43 Jack and Jill play a game, by throwing a die in turn. If the die shows a 1, 2, 3 or 4, the player who threw the die wins the game. If the die shows a 5 or 6, the other player has the next throw. Jack plays first and the game continues until there is a winner.

a) Write down the probability that Jack wins on his first throw.

b) Calculate the probability that Jill wins on her first throw.

c) Calculate the probability that Jack wins the game.

44 Bag A contains 2 red and 3 green balls.

a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

Bag B contains 4 red and n green balls.

b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\frac{2}{15}$, show that $n = 6$.

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

c) Calculate the probability that two red balls are chosen.

d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

45 Given that $(A \cup B)' = \varnothing$, $\mathrm{P}(A'|B) = \frac{1}{3}$ and $\mathrm{P}(A) = \frac{6}{7}$, find $\mathrm{P}(B)$.

P = 0.069

b) $A \cup B = \{(1,12),...,(1,1),(2,12),...,(3,12),...,(4,11),...,(5,10),...\}$,
P = 0.778

c) list, P = 0.931 d) list, P = 0.222

e) same as c) f) same as d)

g) This is $(A \cup B) - (A \cap B)$; P = 0.709

20 Proof

21 a) $\frac{1}{36}$ b) $\frac{1}{28}$ c) $\frac{91}{216}$ d) 0.5

22 $\frac{2}{3}$

23 a) 0.103 b) 0.0887 c) 0.537

24 a) 0.10 b) 0.000 01

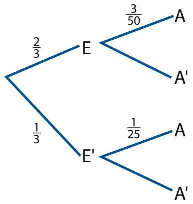
25 a) $\frac{21}{22} \approx 0.955$ b) $\frac{23}{66} \approx 0.348$ c) $\frac{13}{63} \approx 0.206$

26 a) 0.36 b) 0.64 c) 0.75 d) 0.17
e) 0.0455 f) 0.682

27 a) 0.8805 b) 0.0471

Exercise 12.5

1 a)



b) $\frac{4}{75}$ c) $\frac{3}{4}$

2 a) 0.060 1872 b) 0.003 37

3 a) 0.52 b) 0.692

4 a) $\frac{1}{3}$ b) $\frac{1}{2}$

5 a) 0.055 b) 0.444

6 0.875

7 a) 85.5% b) 10.5%

8 a) 0.64 b) 0.703

9 a) 0.7 b) 0.50

10 0.915

11 a) 3.6% b) 66.7% c) 5.37% d) 4.24%

12 0.382 **13** Antonio

14 a) 0.1445 b) 0.000 58

15 a) 0.425 b) 0.176

16 a) 0.93 b) 0.108

17 a) $P(F) = 0.4$, $P(F \cap T) = 0.224$, $P(F \cup A') = 0.944$,
$P(F'|A) = 0.35$

b) M is mutually exclusive with F. T is independent as $P(F \cap T) = P(T)\,P(F)$, …

c) (i) 0.716 (ii) 0.704

18 a) 0.012 b) 0.030 c) 0.40

Practice questions

1 a) 0.30 b) 0.72 c) 0.70

2 a) 0.0004 b) 0.9996 c) 0.0004

3 0.999 98

4 a) (i) 0.85 (ii) 0.80 (iii) 0.15 b) 0.083

5 a) (i) 0.3405 (ii) 0.0108 (iii) 0.9622 (iv) 0.30
b) Yes

6 a) 0.63 b) 0.971

7 a) 0.60 b) yes, $P(B|A) = P(B) = 0.60$ c) 0.42

8 a)

	Boys	Girls
Passed the ski test	32	16
Failed the ski test	14	12
Training, but did not take the test yet	20	16
Too young to take the test	4	6

b) (i) 0.6167 (ii) 0.56 (iii) 0.1463

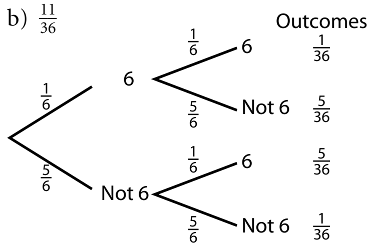
9 a) $\frac{3}{32}$ b) $\frac{3}{4}$ c) $\frac{5}{32}$

10 a) 0.02 b) 0.64

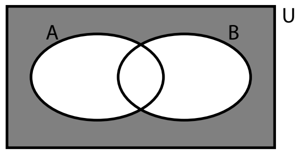
11 a) 0.4 b) 0.6

12 a) 0.38 b) 0.283

13 b) $\frac{11}{36}$



14 a)



b) (i) 2 (ii) $\frac{1}{18}$ c) $n(A \cap B) \neq 0$

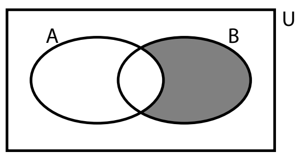
15 a)

	Male	Female	Total
Unemployed	20	40	60
Employed	90	50	140
Total	110	90	200

b) (i) $\frac{1}{5}$ (ii) $\frac{9}{14}$

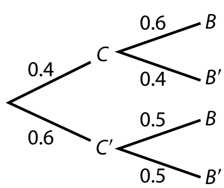
16 $\frac{44}{65}$

17 a)



b) 35 c) 0.35

18 a)



b) 0.54 c) 0.444

19 a) $\frac{7}{12}$ b) $\frac{11}{36}$ c) $\frac{1}{3}$

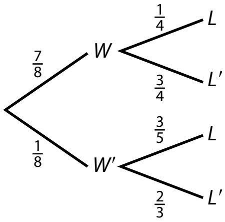
20 a) $\frac{1}{11}$ b) $\frac{12}{121}$

21 a) Independent b) M c) N

22 a) $a = 21, b = 11, c = 17$
b) (i) $\frac{1}{8}$ (ii) $\frac{21}{32}$
c) (i) 0.253 (ii) 0.747

23 $\frac{31}{66}$

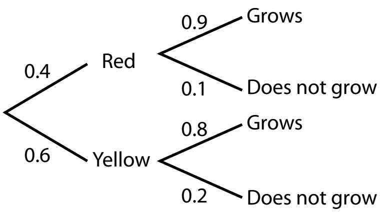
24 a)



b) $\frac{47}{160}$ c) $\frac{35}{47}$

25 a) $\frac{1}{3}$ b) $\frac{7}{12}$ c) $\frac{3}{7}$

26 a)



b) (i) 0.36 (ii) 0.84 (iii) 0.429

27 a) $\frac{1}{6}$ b) $\frac{1}{12}$ c) $\frac{2}{9}$

28 a) (i) $\frac{8}{21}$ (ii) $\frac{1}{6}$ (iii) no, $P(A \cap B) \neq P(A)P(B)$

b) $\frac{10}{17}$ c) $\frac{200}{399}$

29 $\frac{1}{3}$ **30** $\frac{4}{5}$ **31** 0.001 98 **32** $\frac{19}{30}$

33 0.80 **34** $\frac{10}{19}$ **35** a) $\frac{13}{20}$ b) $\frac{11}{15}$

36 $\frac{2}{5}$

37 a) (i) $\frac{5}{36}$ (ii) $\frac{25}{216}$ (iii) $\frac{1}{6}\left(\frac{5}{6}\right)^{2n-2}$

b) No answer required – proof

c) $\frac{5}{11}$ d) 0.432

38 a) 0.957 b) 0.301

39 $\frac{1}{9}$

40 a) 0.25 b) 0.083

41 a) 0.80 b) 0.56

42 a) 0.732 b) $\frac{11}{61}$

43 a) $\frac{2}{3}$ b) $\frac{2}{9}$ c) $\frac{3}{4}$

44 a) $\frac{1}{10}$ b) proof

c) $\frac{11}{90}$ d) $\frac{3}{11}$

45 $\frac{3}{7}$

Chapter 13

Exercise 13.1

1 4 **2** $3x^2$ **3** $2x$ **4** 6

5 0 **6** $\frac{5}{2}$ **7** d.n.e. (increases without bound)

8 $\frac{1}{8}$ **9** $\frac{3}{2}$ **10** $\frac{\sqrt{2}}{4}$ **11** $\frac{1}{4}$

12 1 **13** 3 **14** $\frac{1}{e}$

15 $\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$

16 As $x \to a$, $g(x) \to +\infty$

17 a) Horizontal: $y = 3$; vertical: $x = -1$

b) Horizontal: $y = 0$; vertical: $x = 2$

c) Horizontal: $y = b$; vertical: $x = a$

d) Horizontal: $y = 2$; vertical: $x = \pm 3$

e) Horizontal: $y = 0$; vertical: $x = 0$, $x = 5$

f) Horizontal: none; vertical: $x = 4$

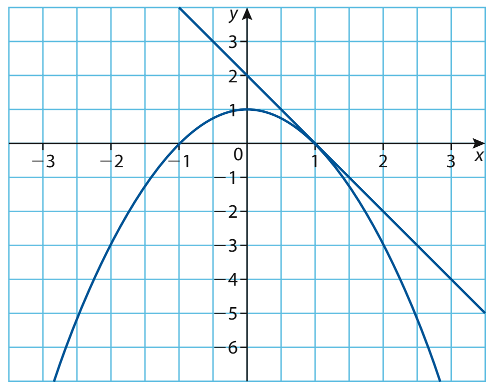
18 $\frac{1}{3}$ **19** 4

Exercise 13.2

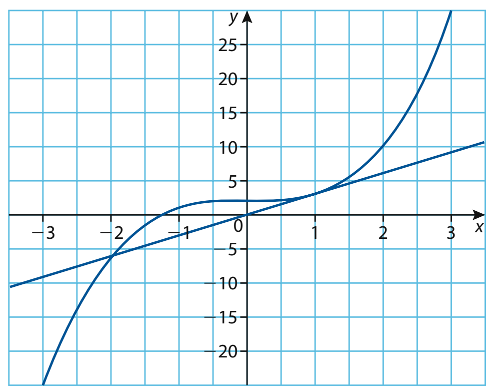
1 $f'(x) = -2x$ **2** $g'(x) = 3x^2$

3 $h'(x) = \frac{1}{2\sqrt{x}}$ **4** $r'(x) = -\frac{2}{x^3}$

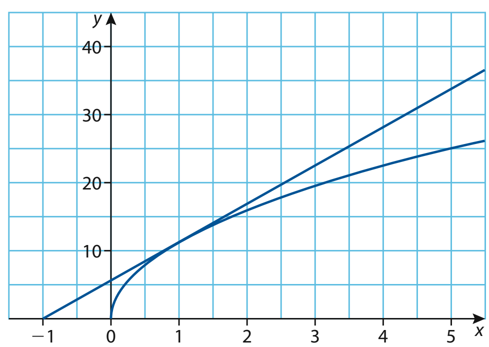
5 (i)



(ii)



(iii)



(iv)