Sequences (AI HL) [107 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.2

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

Markscheme

$(d =) -250$ ***A1***

[1 mark]

(b) Calculate the price of a ticket in the 16th row. [2]

Markscheme

$(u_{16} =) 6800 + (16 - 1)(-250)$ ***M1***

(¥)3050 ***A1***

[2 marks]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

Markscheme

$(S_{16} =) \left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)(-250)) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

OR

$(S_{16} =) \left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula. Award ***M1*** for multiplication by 2 seen.

(¥)158 000 (157 600) ***A1***

[3 marks]

2. [Maximum mark: 7] EXN.1.SL.TZ0.12

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.

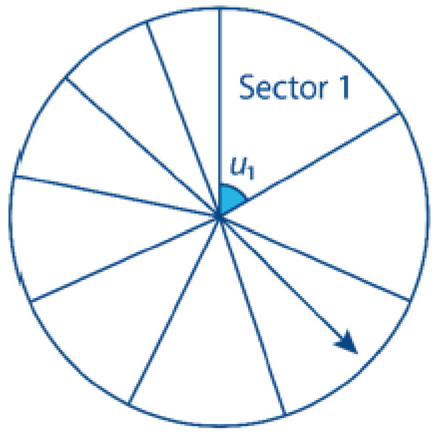


Diagram not to scale

(a) Write down the value of $\sum_{i=1}^{9} u_i$. [1]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

360° **A1**

[1 mark]

It is given that $u_9 = \frac{1}{3}u_1$.

(b) Find the value of u_1. [4]

Markscheme

EITHER

$360 = \frac{9}{2}(u_1 + u_9)$ **M1**

$360 = \frac{9}{2}\left(u_1 + \frac{1}{3}u_1\right) = 6u_1$ **M1A1**

OR

$360 = \frac{9}{2}(2u_1 + 8d)$ **M1**

$u_9 = \frac{1}{3}u_1 = u_1 + 8d \Rightarrow u_1 = -12d$ **M1**

Substitute this value

$360 = \frac{9}{2}\left(2u_1 - 8 \times \frac{u_1}{12}\right) \left(= \frac{9}{2} \times \frac{4}{3}u_1 = 6u_1\right)$ **A1**

THEN

$u_1 = 60^\circ$ **A1**

[4 marks]

(c) A game is played in which the arrow attached to the centre of the disc is spun and the sector in which the arrow stops is noted. If the arrow stops in sector 1 the player wins 10 points, otherwise they lose 2 points.

Let X be the number of points won

Find $\mathrm{E}(X)$. [2]

Markscheme

$\mathrm{E}(X) = 10 \times \frac{60}{360} - 2 \times \frac{300}{360} = 0$ **M1A1**

[2 marks]

3. [Maximum mark: 7] 23N.1.AHL.TZ0.1

The growth of a particular type of seashell is being studied by Manon. At the end of each month Manon records the increase in the width of a seashell since the end of the previous month.

She models the monthly increase in the width of the seashell by a geometric sequence with common ratio 0.8. In the first month, the width of the seashell increases by $4\ \mathrm{mm}$.

(a) Find by how much the width of the seashell will increase during the third month, according to her model. [2]

Markscheme

attempt to use the term formula for a geometric sequence ***(M1)***

$4\ \times\ (0.8)^2$

$=\ 2.56\,(\mathrm{mm})\ \ \left(\frac{64}{25}\right)$ ***A1***

[2 marks]

(b) Find the total increase in the width of the seashell, predicted by Manon's model, during the first year. [2]

Markscheme

attempt to use the sum formula for a geometric sequence ***(M1)***

$\frac{4\times\left(1-0.8^{12}\right)}{1-0.8}$

$=\ 18.6\,(\mathrm{mm})\ \ (18.6256\ldots)$ ***A1***

[2 marks]

Manon's seashell had a width of $25\ \mathrm{mm}$ at the beginning of the first month.

(c) Find the maximum possible width of the seashell, predicted by Manon's model. [3]

Markscheme

attempt to use infinite geometric sum *(M1)*

e.g. $\frac{4}{1-0.8}$ **OR** 20

adding 25 to their expression or value *(M1)*

(maximum width $= \ 25 \ + \ 20$)

$45\,(\mathrm{mm})$ *A1*

[3 marks]

4. [Maximum mark: 15] 23M.2.SL.TZ2.2

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12 th day. [3]

Markscheme

recognizing arithmetic sequence (may be seen in part (b)) ***(M1)***

$(u_{12} =)\ 10 + (12 - 1) \times 6$ ***(A1)***

76 ***A1***

[3 marks]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

(b) Calculate the total number of pendulums she would have available at the market. [2]

Markscheme

correct substitution into either arithmetic series formula ***(A1)***

$\left(S_{15} =\right)\ \frac{15}{2}\left(2 \times 10 + \left(15 - 1\right) \times 6\right)$ **OR**
$\left(S_{15} =\right)\ \frac{15}{2}\left(10 + 94\right)$

780 ***A1***

[2 marks]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Markscheme

attempt to use either arithmetic series formula equated to 1000 ***(M1)***

$\frac{15}{2}\left(2 \times 10 + \left(15 - 1\right) \times x\right) = 1000$ **OR** $\frac{15}{2}\left(10 + u_{15}\right) = 1000$

$x = 8.09523\ldots$ ***(A1)***

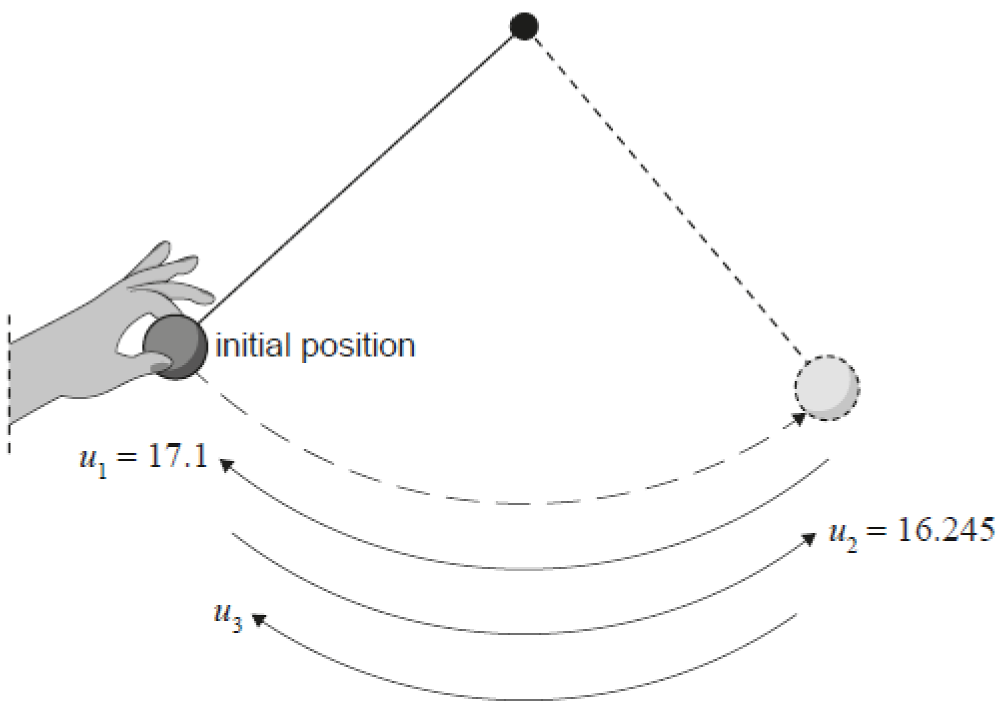
$x = 9$ ***A1***

Note: Follow through within question part for final ***A1*** for candidates correctly rounding their value of x up to the nearest integer. Award ***(M0)(A0)A0*** for a response of $x = 8$ with no working shown.

[3 marks]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

(d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]

Markscheme

recognizing geometric sequence (may be seen in part (e)) ***(M1)***

$17.1 \times 0.95^{5-1}$ ***(A1)***

$13.9\text{ (cm)}\ (13.9280\ldots)$ ***A1***

[3 marks]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

Markscheme

correct substitution into geometric series formula ***(A1)***

$\frac{17.1\left(1-0.95^{16}\right)}{1-0.95}$

$191\ (\text{cm})\ (191.476\ldots\ (\text{cm}))$ ***A1***

[2 marks]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

Markscheme

correct method to find u_0 ***(M1)***

$u_0 = 17.1 \times (0.95)^{0-1}$ **OR** $17.1 = 0.95x$ **OR** $\frac{17.1}{0.95}$ (seen)

Note: Award ***(M0)A0*** for any attempt to find answer using 0.05 or 1.05.

$18\ (\text{cm})$ ***A1***

[2 marks]

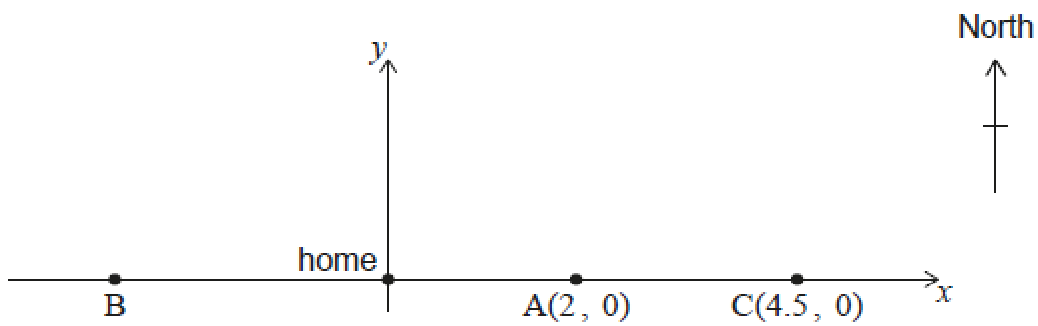
5. [Maximum mark: 7] 22N.1.SL.TZ0.11

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

Markscheme

$4.5 = 2(r)^{3-1}$ ***(M1)***

$r = \pm 1.5$, ***R1***

(Some x-values are negative or direction from house changes each day)

$r = -1.5$ ***AG***

Note: Award ***M0R0AG*** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

[2 marks]

On the 6th day, Kristi runs to point F.

(b) Find the location of point F. [2]

Markscheme

$2(-1.5)^{6-1}$ ***(M1)***

EITHER

$(-15.2,\ 0)$ $(-15.1875\ldots,\ 0)$ ***A1***

OR

$x = -15.2\,\text{km}$ ***A1***

OR

15.2 km west (of the origin) ***A1***

Note: Award ***(M1)A0*** for an answer of "$-15.2\,(\text{km})$" without indicating that it is the x-value.

[2 marks]

(c) Find the total distance Kristi runs during the first 7 days of training. [3]

Markscheme

choosing, $r = 1.5$ ***(A1)***

$\frac{2\left((1.5)^7-1\right)}{1.5-1}$ ***(M1)***

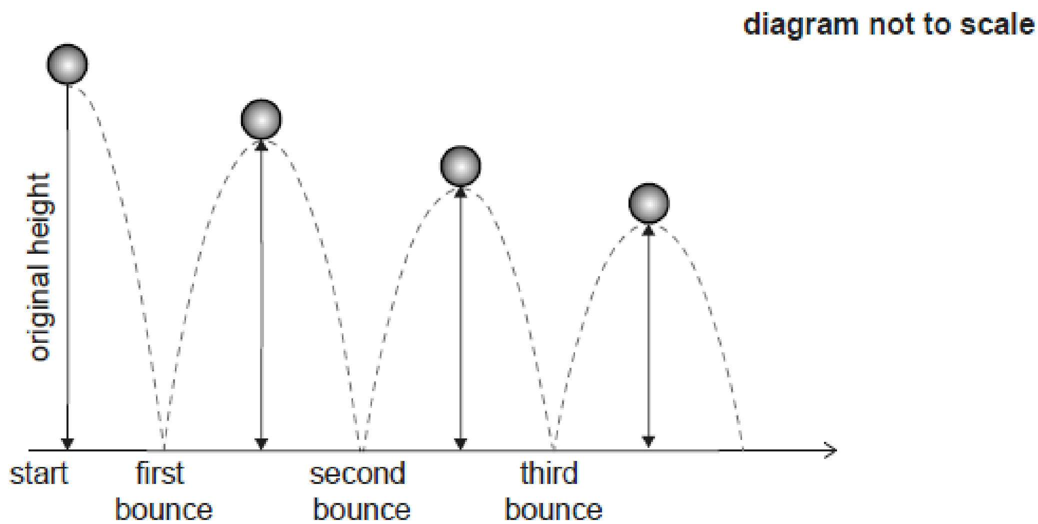
Note: Award ***M1*** for an attempt at a substituted GP formula with $n = 7$. Award ***A0M1A0*** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of $14.4687\ldots$).

$64.3\ldots \text{ km} \ \ (64.3437\ldots)$ ***A1***

[3 marks]

6. [Maximum mark: 7] 22M.1.SL.TZ1.13

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

Markscheme

use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68\,\text{m}$

$= 68\,\text{cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68\,\text{cm}$ ***AG***

[2 marks]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

Markscheme

EITHER

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If $1.8\,\text{m}$ (or $180\,\text{cm}$) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If $1.53\,\text{m}$ (or $153\,\text{cm}$) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114\,\text{m}$ and $(0.85)^{18}(1.8) = 0.0966\,\text{m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

Markscheme

EITHER

distance (in one direction) travelled between first and fourth bounce

$$= \frac{(1.8\times0.85)(1-0.85^3)}{1-0.85} \ (= 3.935925\ldots)$$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$$18 + 2(3.935925)$$

$$= 9.67\,\text{m} \ \ (9.67185\ldots\,\text{m})$$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$$= \frac{(1.8)(1-0.85^4)}{1-0.85} \ (= 5.735925\ldots)$$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$$2(5.735925) - 1.8$$

$$= 9.67\,\text{m} \ \ (9.67185\ldots\,\text{m})$$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8) \ \ (= 3.935925\ldots)$$

(A1)

recognizing distances are travelled twice except first distance ***(M1)***

$$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$$

$$= 9.67\,\mathrm{m} \ \ (9.67185\ldots\,\mathrm{m})$$ ***A1***

Note: Answers may be given in cm.

[3 marks]

7. [Maximum mark: 5] 22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer. [5]

Markscheme

METHOD 1

$\frac{u_1}{1-r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + (9 - 9r)r$ **OR** $\frac{4}{(1-r)^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1-r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$$\frac{u_1}{1-\left(\frac{u_1-4}{u_1}\right)}=9$$

$$\frac{u_1}{\left(\frac{4}{u_1}\right)}=9$$

$$(u_1)^2=36$$

$$u_1=\pm 6$$

attempting to solve both possible sequences

$6,\ 2,\ \ldots$ or $-6,\ -10\ \ldots$

$r=\frac{1}{3}$ or $r=\frac{5}{3}$

only $r=\frac{1}{3}$ is possible as the sum to infinity exists ***R1***

$u_3=6\times\left(\frac{1}{3}\right)^2=\frac{2}{3}$ ***A1***

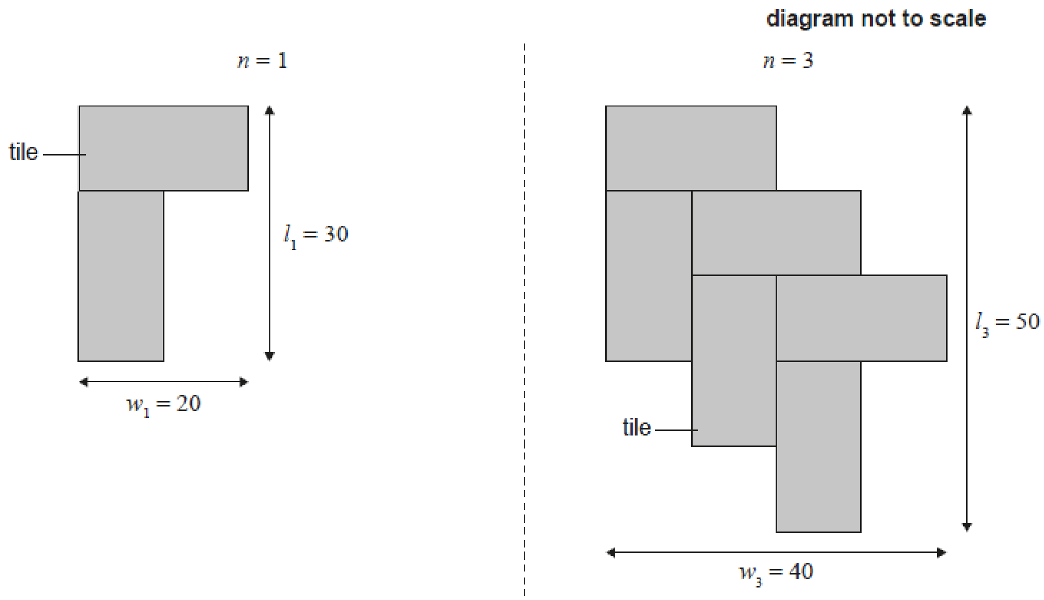
[5 marks]

8. [Maximum mark: 19] 22M.2.SL.TZ1.2

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

(a.i) a. [1]

Markscheme

30 ***A1***

[1 mark]

(a.ii) b. [1]

Markscheme

40 ***A1***

[1 mark]

Write down an expression in terms of n for

(b.i) w_n. [2]

Markscheme

arithmetic formula chosen ***(M1)***

$w_n = 20 + (n-1)10 \quad (= 10 + 10n)$ ***A1***

[2 marks]

(b.ii) l_n. [1]

Markscheme

arithmetic formula chosen

$l_n = 30 + (n-1)10 \quad (= 20 + 10n)$ ***A1***

[1 mark]

Eddie's lawn has a length $740\,\mathrm{cm}$.

(c.i) Show that Eddie needs 144 tiles. [2]

Markscheme

$740 = 30 + (n-1)10$ **OR** $740 = 20 - 10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

[2 marks]

(c.ii) Find the value of w_n for this path. [1]

Markscheme

$w_{72} = 730$ ***A1***

[1 mark]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

Markscheme

$(10 \times 20) \times 144$ ***(M1)***

$= 28800$ ***(A1)***

$2.88 \times 10^4 \text{ cm}^2$ ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

Markscheme

EITHER

1 square metre $= 100\,\text{cm} \times 100\,\text{cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10 \text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2\,\mathrm{m} \times 0.1\,\mathrm{m} \times 5 =)\ 0.1\,\mathrm{m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$\$2.45$ per pack (of 5 tiles) ***A1***

[3 marks]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

Markscheme

$\frac{1.08 \times 144}{5}$ $(= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

Markscheme

$35 + (32 \times 2.45)$ *(M1)*

$\$113 \ \ (113.4)$ *A1*

[2 marks]

9. [Maximum mark: 13] 22M.2.SL.TZ2.2

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

(a.i) fed to the dog per day. [3]

Markscheme

EITHER

$$115.5 = u_1 + (3-1) \times d \quad (115.5 = u_1 + 2d)$$

$$108 = u_1 + (8-1) \times d \quad (108 = u_1 + 7d) \qquad \textbf{\textit{(M1)(A1)}}$$

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =)\ \frac{115.5-108}{5}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =)\ 1.5$ (cups/day) ***A1***

[3 marks]

(a.ii) remaining in the bag at the end of the first day. [1]

Markscheme

$(u_1 =)\ 118.5$ (cups) ***A1***

[1 mark]

(b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

Markscheme

attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$$0 = 118.5 + (n-1) \times (-1.5)$$

$(n =)\ 80$ days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

(c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

Markscheme

$(t_5 =)\ 625 \times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution

$\$801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

(d.i) Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$. [1]

Markscheme

$(S_{10} =)\ (\$)\ 8390\ \ (8394.39\ldots)$ ***A1***

[1 mark]

(d.ii) Describe what the value in part (d)(i) represents in this context. [2]

Markscheme

EITHER

the total cost (of dog food) ***R1***

for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***

from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[2 marks]

(e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

Markscheme

EITHER
According to the model, the cost of dog food per year will eventually be too high to keep a dog.

OR
The model does not necessarily consider changes in inflation rate.

OR
The model is appropriate as long as inflation increases at a similar rate.

OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.

OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either "model" is mentioned specifically, or other mathematical terms such as "increasing" or "discrete quantities" are seen. Do not accept a contextual argument in isolation, e.g. "The dog will eventually die".

[1 mark]

10. [Maximum mark: 5] 21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10.$

(a) Find the common ratio, r, for the sequence. [2]

Markscheme

$10 = \frac{2}{1-r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

Markscheme

$2 \times (0.8)^{n-1} < 0.5$ **OR** $2 \times (0.8)^{n-1} = 0.5$ ***(M1)***

$(n >)\ 7.212\ldots$ ***(A1)***

$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value $7.212\ldots$ then award ***M1A1A0***.

[3 marks]

11. [Maximum mark: 16] 21N.2.SL.TZ0.2

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

Markscheme

$\frac{12\,669-12\,300}{12\,300} \times 100$ ***(M1)***

3% ***A1***

[2 marks]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b.i) Write down the common ratio of the sequence. [1]

Markscheme

1.03 ***A1***

Note: Follow through from part (a).

[1 mark]

(b.ii) Find an expression for u_n. [1]

Markscheme

$(u_n =)\ 12\,300 \times 1.03^{n-1}$ ***A1***

[1 mark]

(b.iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [2]

Markscheme

$(u_{11} =)\ 12\,300 \times 1.03^{10}$ ***(M1)***

16530 ***A1***

Note: Answer must be to the nearest integer. Do not accept 16500.

[2 marks]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

Markscheme

$(v_n =)\ 10380 + 600(n-1)$ **OR** $600n + 9780$ ***M1A1***

Note: Award ***M1*** for substituting into arithmetic sequence formula, ***A1*** for correct substitution.

[2 marks]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

Markscheme

$80 \times \frac{10}{2}(2(10380) + 9(600))$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 80 and ***(M1)*** for substitution into sum of arithmetic sequence formula.

$\$10\,500\,000 \quad (\$10\,464\,000)$ ***A1***

[3 marks]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

Markscheme

$12\,300 \times 1.03^{n-1} < 10\,380 + 600(n-1)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for equating their expressions from parts (b) and (c).

EITHER

graph showing $y = 12\,300 \times 1.03^{n-1}$ and $y = 10\,380 + 600(n-1)$ ***(M1)***

OR

graph showing $y = 12\,300 \times 1.03^{n-1} - (10\,380 + 600(n-1))$ ***(M1)***

OR

list of values including, $(u_{n=})$ 17537 and $(v_{n=})$ 17580 ***(M1)***

OR

$12.4953\ldots$ from graphical method or solving numerical equality ***(M1)***

Note: Award ***(M1)*** for a valid attempt to solve.

THEN

$(k =) 13$ ***A1***

[3 marks]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

Markscheme

this will not guarantee enough places. ***A1***

EITHER

A written statement that $u_n > v_n$, with range of n. ***R1***

Example: "when $n = 24$ (or greater), the number of applications will exceed the number of places again" ("$u_n > v_n,\ n \geq 24$").

OR

exponential growth will always exceed linear growth ***R1***

Note: Accept an equivalent sketch. Do not award ***A1R0***.

[2 marks]