Sequences (AI HL) [107 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.2

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

2. [Maximum mark: 7] EXN.1.SL.TZ0.12

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.

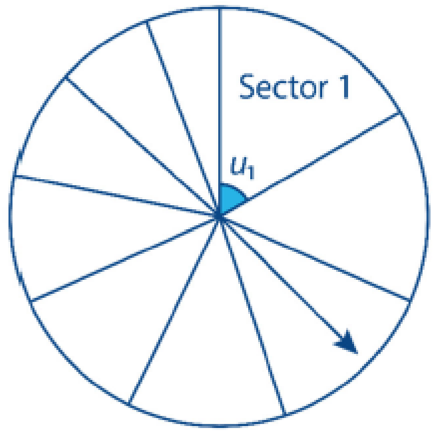


Diagram not to scale

(a) Write down the value of $\sum_{i=1}^{9} u_i$. [1]

It is given that $u_9 = \frac{1}{3}u_1$.

(b) Find the value of u_1. [4]

(c) A game is played in which the arrow attached to the centre of the disc is spun and the sector in which the arrow stops is noted. If the arrow stops in sector 1 the player wins 10 points, otherwise they lose 2 points.

Let X be the number of points won

Find $\mathrm{E}(X)$. [2]

3. [Maximum mark: 7] 23N.1.AHL.TZ0.1

The growth of a particular type of seashell is being studied by Manon. At the end of each month Manon records the increase in the width of a seashell since the end of the previous month.

She models the monthly increase in the width of the seashell by a geometric sequence with common ratio 0.8. In the first month, the width of the seashell increases by 4 mm.

(a) Find by how much the width of the seashell will increase during the third month, according to her model. [2]

(b) Find the total increase in the width of the seashell, predicted by Manon's model, during the first year. [2]

Manon's seashell had a width of 25 mm at the beginning of the first month.

(c) Find the maximum possible width of the seashell, predicted by Manon's model. [3]

4. [Maximum mark: 15] 23M.2.SL.TZ2.2

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12th day. [3]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

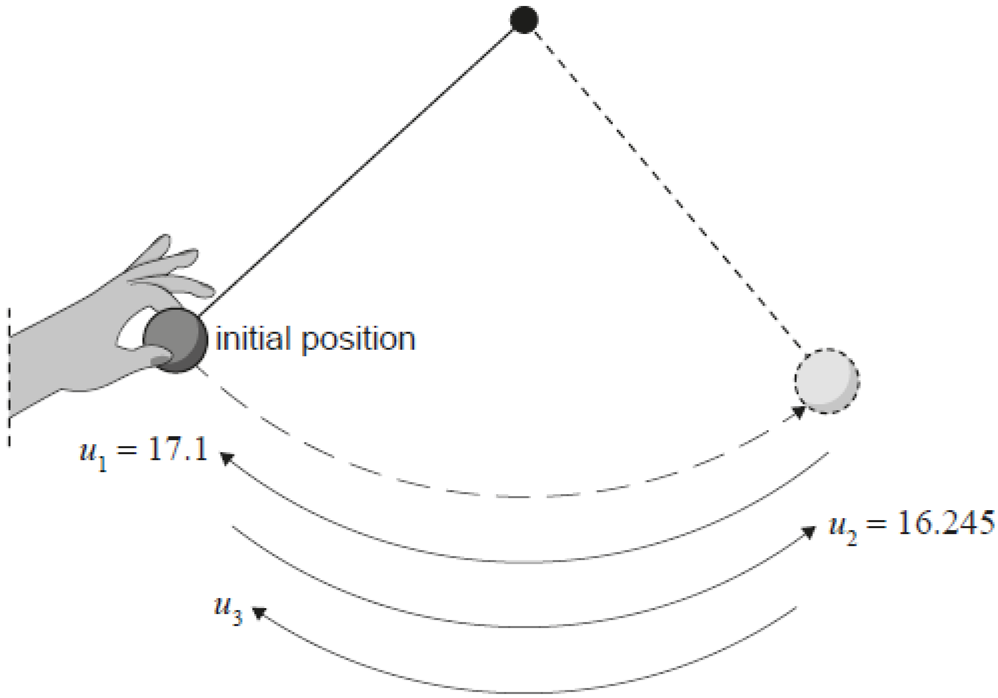
(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of $17.1\ \mathrm{cm}$. During the second swing that she recorded, it travelled a distance of $16.245\ \mathrm{cm}$.

(d) Calculate the distance that the ball travelled during the $5\mathrm{th}$ recorded swing. [3]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

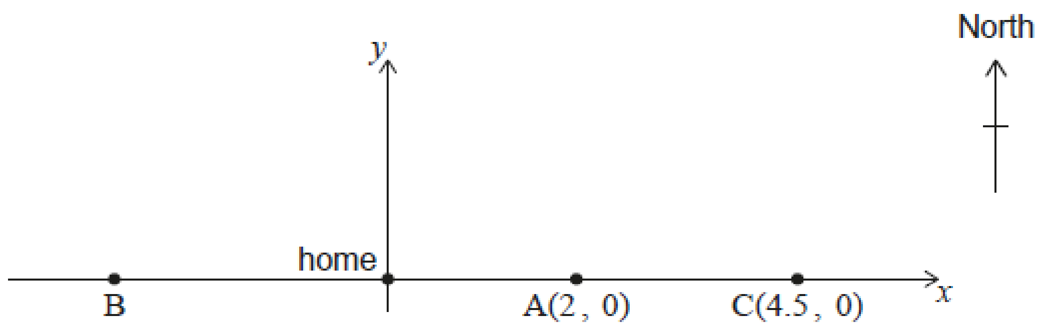
5. [Maximum mark: 7] 22N.1.SL.TZ0.11

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0,\ 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2,\ 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5,\ 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

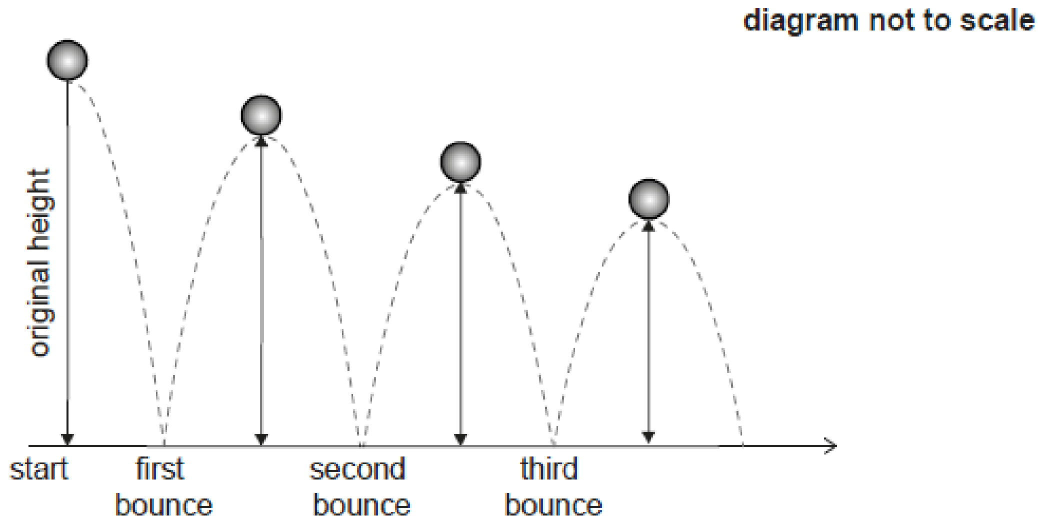
On the $6\mathrm{th}$ day, Kristi runs to point F.

(b) Find the location of point F. [2]

(c) Find the total distance Kristi runs during the first 7 days of training. [3]

6. [Maximum mark: 7] 22M.1.SL.TZ1.13

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\mathrm{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\mathrm{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

7. [Maximum mark: 5] 22M.1.AHL.TZ2.7

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

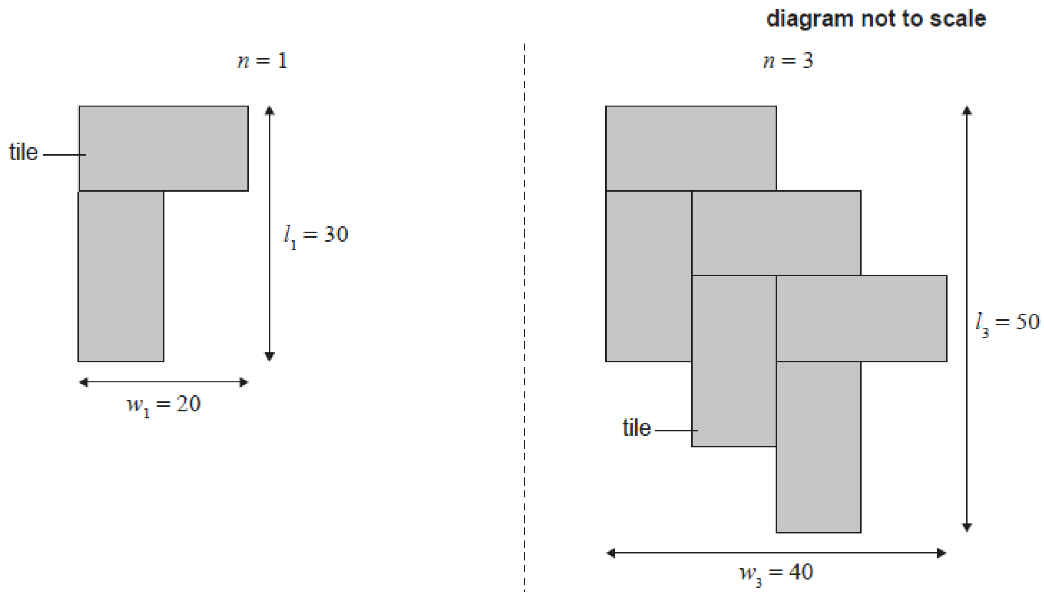
Find the third term. Justify your answer. [5]

8. [Maximum mark: 19] 22M.2.SL.TZ1.2

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

Find the value of

(a.i) a. [1]

(a.ii) b. [1]

Write down an expression in terms of n for

(b.i) w_n. [2]

(b.ii) l_n. [1]

Eddie's lawn has a length $740\,\text{cm}$.

(c.i) Show that Eddie needs 144 tiles. [2]

(c.ii) Find the value of w_n for this path. [1]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

9. [Maximum mark: 13] 22M.2.SL.TZ2.2

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

Find the number of cups of dog food

(a.i) fed to the dog per day. [3]

(a.ii) remaining in the bag at the end of the first day. [1]

(b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

(c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

(d.i) Calculate the value of $\sum_{n=1}^{10}\left(625 \times 1.064^{(n-1)}\right)$. [1]

(d.ii) Describe what the value in part (d)(i) represents in this context. [2]

(e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

10. [Maximum mark: 5] 21N.1.AHL.TZ0.6

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

11. [Maximum mark: 16] 21N.2.SL.TZ0.2

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b.i) Write down the common ratio of the sequence. [1]

(b.ii) Find an expression for u_n. [1]

(b.iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [2]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n . [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]