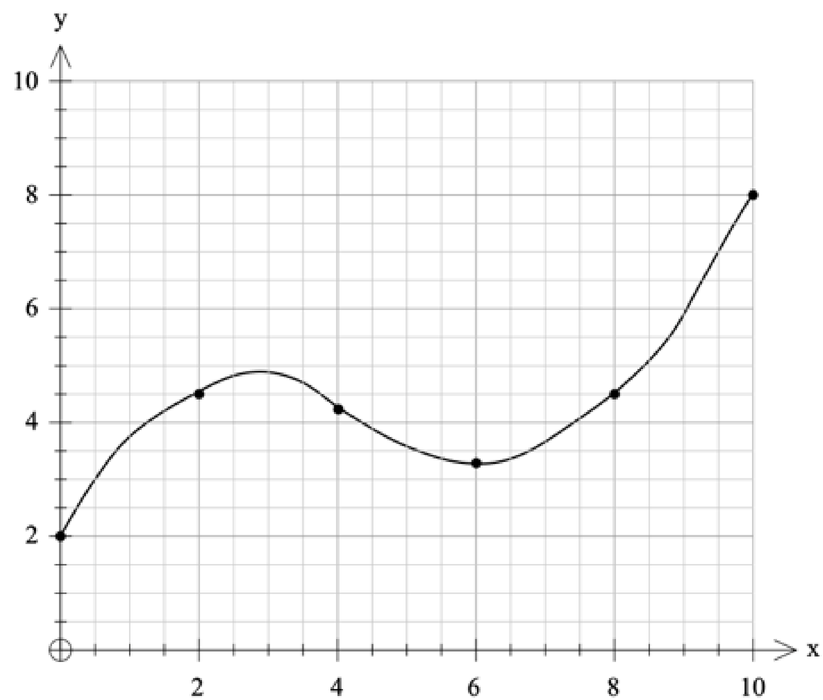
Area under graph [46 marks]

1. [Maximum mark: 10]

The curve $y = f(x)$ is shown in the graph, for $0 \leqslant x \leqslant 10$.



The curve $y = f(x)$ passes through the following points.

x	0	2	4	6	8	10
y	2	4.5	4.2	3.3	4.5	8

It is required to find the area bounded by the curve, the x-axis, the y-axis and the line $x = 10$.

(a) Use the trapezoidal rule to find an estimate for the area. [3]

Markscheme

Area = $\frac{2}{2}(2 + 2(4.5 + 4.2 + 3.3 + 4.5) + 8)$ ***M1A1***

Area = 43 ***A1***

[3 marks]

One possible model for the curve $y = f(x)$ is a cubic function.

(b.i) Use all the coordinates in the table to find the equation of the least squares cubic regression curve. [3]

Markscheme

$y = 0.0389x^3 - 0.534x^2 + 2.06x + 2.06$ ***M1A2***

[3 marks]

(b.ii) Write down the coefficient of determination. [1]

Markscheme

$R^2 = 0.991$ ***A1***

[1 mark]

(c.i) Write down an expression for the area enclosed by the cubic regression curve, the x-axis, the y-axis and the line $x = 10$. [1]

Markscheme

Area = $\int_0^{10} y\,dx$ ***A1***

[1 mark]

(c.ii) Find the value of this area. [2]

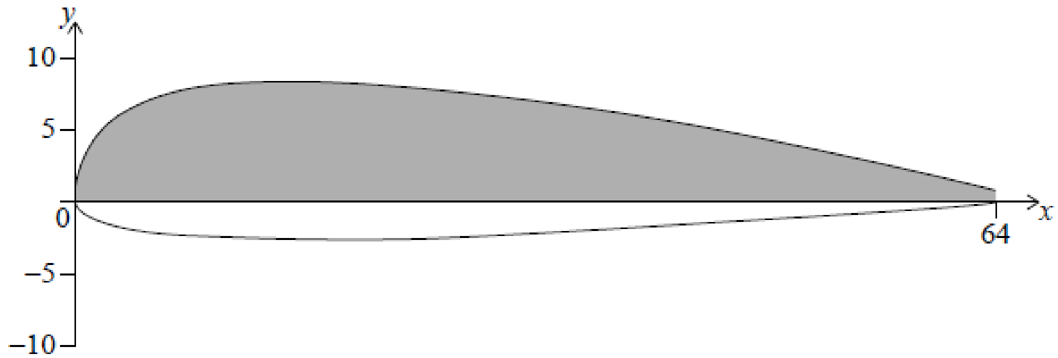
Markscheme

42.5 ***A2***

[2 marks]

2. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$y = 4\sqrt{x} - \frac{x}{2} + 1$, for $0 \leq x \leq 64$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table.

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

[3]

Markscheme

$(a =)\ 9$ *A1*

$(b =)\ 7.63\ \ (7.62741\ldots)$ *A1*

$(c =)\ 4.71\ \ (4.71281\ldots)$ *A1*

[3 marks]

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

(b) Find Jan’s estimate of the shaded area of the cross-section. [3]

Markscheme

attempt to use the trapezoidal rule ***(M1)***

$(\text{area } =)\ \frac{1}{2}(16)((1 + 1) + 2(9 + 7.62741\ldots + 4.71281\ldots))$ ***(A1)***

$(\text{area } =)\ 357\left(\text{mm}^2\right)\ (357.443\ldots)$ ***A1***

[3 marks]

(c.i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section. [2]

Markscheme

$\int_0^{64}\left(4\sqrt{x} - \frac{x}{2} + 1\right)\,\mathrm{d}\,x$ ***A1A1***

Note: Award ***A1*** for correct function seen within the integral and ***A1*** for correct limits in the correct location and the inclusion of the $\mathrm{d}\,x$.

[2 marks]

(c.ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [2]

Markscheme

$405.3\left(\text{mm}^2\right)$ ***A2***

[2 marks]

(d) Calculate the percentage error of Jan’s estimate in part (b). [2]

Markscheme

attempt to substitute **their** area values into the percentage error formula ***(M1)***

$\left|\frac{357.443...-405.3}{405.3}\right| \times 100$

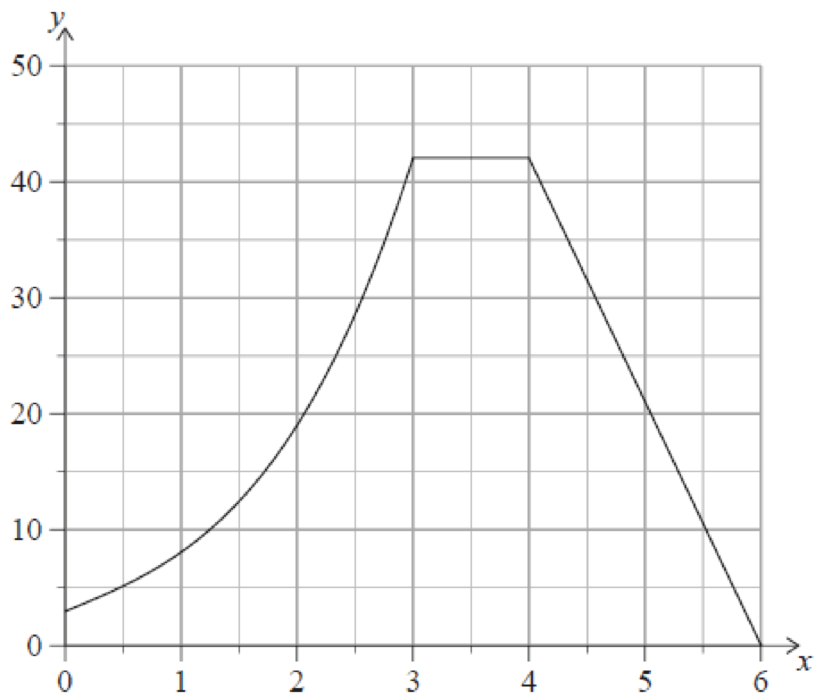
$11.8(\%)$ $(11.8076...)$ ***A1***

Note: Accept an answer of 11.9 from use of 357 from part (b).

[2 marks]

3. [Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

(a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals. [2]

Markscheme

attempt at using the trapezoidal rule ***(M1)***

area $= \frac{1}{2}\left(3 + 2\left(8 + 19\right) + 42\right)$

$= 49.5\ \left(\mathrm{m}^2\right)$ ***A1***

[2 marks]

It is known that $f\prime\left(x\right) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f\left(x\right)$, in the domain $0 < x < 3$. [4]

Markscheme

recognition of need to integrate (*e.g.* reverse power rule or integral symbol) ***(M1)***

$\int 3x^2 + 4\,\mathrm{d}x = x^3 + 4x + c$ ***(A1)(A1)***

Note: Award ***A1*** for each correct term.

$f\left(x\right) = x^3 + 4x + 3$ ***A1***

Note: Award ***A1*** for simplified correct answer including the value of c. Accept a value of c of 3.005 or 3.025 or 2.975 for using the non-integer x-values and their corresponding y-values.

[4 marks]

(c) **Hence** find the actual area of the **entire** cross-section. [3]

Markscheme

METHOD 1

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle ***(M1)***

$\int_0^3 x^3 + 4x + 3\,\mathrm{d}x\left(= 47.25\right) + 42 \times 1 + \frac{1}{2} \times 2 \times 42\left(= 84\right)$ ***(A1)***

$= 131\ \left(\mathrm{m}^2\right)\ \left(131.25\right)$ ***A1***

METHOD 2

forming expression for sum of integral and trapezoid ***(M1)***

$\int_0^3 x^3 + 4x + 3\,\mathrm{d}x\left(= 47.25\right) + \frac{1}{2} \times 4 \times 42\left(= 84\right)$ ***(A1)***

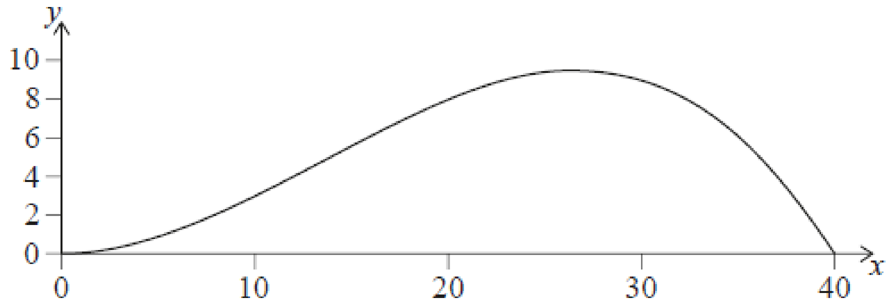
$= 131\ \left(\mathrm{m}^2\right)\ \left(131.25\right)$ ***A1***

Note: Award ***(A1)*** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]

4. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

Markscheme

attempt to substitute $h = 10$ and at least two different values of γ into the trapezoidal rule *(M1)*

$\frac{10}{2}\left((0+0)+2(3+8+9)\right)$

$= 200\ \left(\text{cm}^2\right)$ ***A1***

[2 marks]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \leq x \leq 40$.

(b.i) Write down an integral to find the exact cross-sectional area. [2]

Markscheme

$\int_0^{40} 0.04x^2 - 0.001x^3 \mathrm{d}x$ **OR** $\int_0^{40} y \mathrm{d}x$ ***A1A1***

Note: Award **A1** for a correct integral (including $\mathrm{d}x$), ***A1*** for correct limits in the correct location.

[2 marks]

(b.ii) Calculate the value of the cross-sectional area to two decimal places. [2]

Markscheme

$213.33\ (\mathrm{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\mathrm{cm}^2)$.

[2 marks]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

Markscheme

attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$\left|\frac{213.333...-200}{213.333...}\right| \times 100$

$= 6.25(\%)\ \ (6.23999\ldots(\%))$ ***A1***

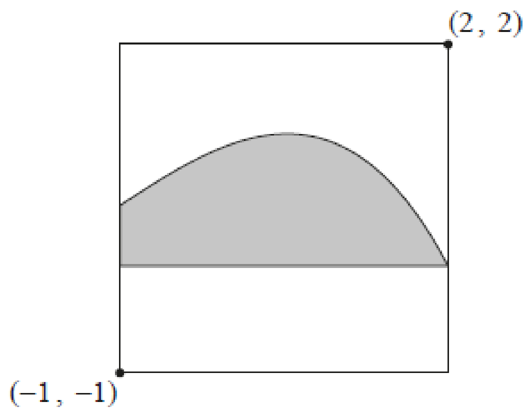
Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

5. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1,\ -1)$ and the top right corner has coordinates $(2,\ 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region.

x	–1	0	1	2
y	0.6	1.2	1.2	0

[3]

Markscheme

$\frac{1}{2}(0.6 + 0 + 2(1.2 + 1.2))$ ***(A1)(M1)***

Note: Award ***A1*** for evidence of $h = 1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

$2.7\ \mathrm{m}^2$ ***A1***

[3 marks]

The artist used the equation $y = \frac{-x^3-3x^2+4x+12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

Markscheme

$\int_{-1}^{2} \frac{-x^3-3x^2+4x+12}{10}\, \mathrm{d}\, x$ **OR** $\int_{-1}^{2} f(x)\, \mathrm{d}\, x$ ***(M1)***

Note: Award ***M1*** for using definite integration with correct limits.

$2.925\ \mathrm{m}^2$ ***A1***

Note: Question requires exact answer, do not award final ***A1*** for 2.93.

[2 marks]

(c) Find the area of the unshaded region in the painting. [2]

Markscheme

$9 - 2.925$ ***(M1)***

Note: Award ***M1*** for 9 seen as part of a subtraction.

$= 6.08\ \mathrm{m}^2 \quad (6.075)$ ***A1***

[2 marks]