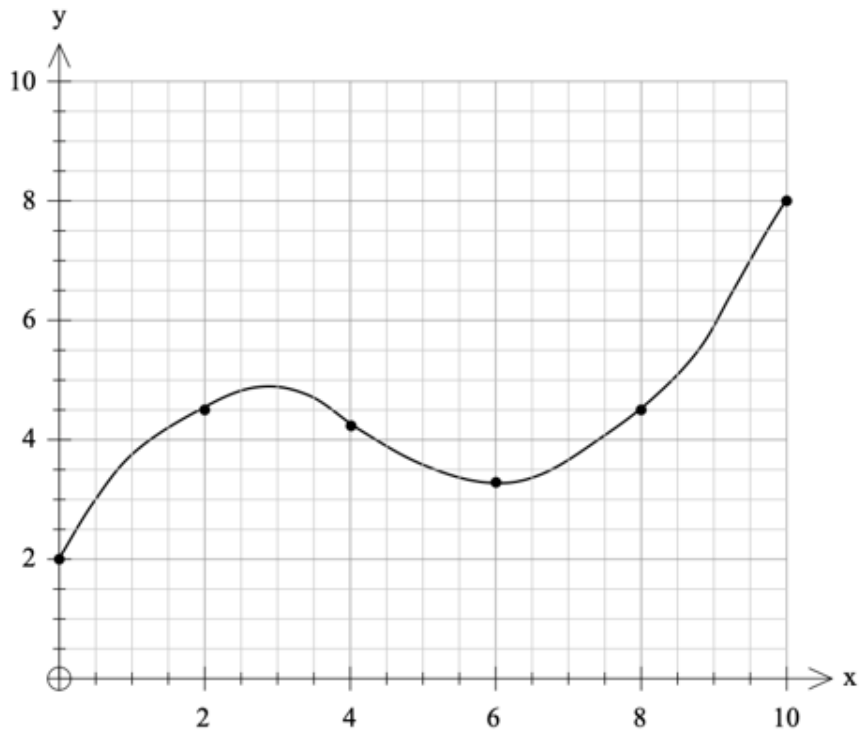
Area under graph [46 marks]

1. [Maximum mark: 10]

The curve $y = f(x)$ is shown in the graph, for $0 \leqslant x \leqslant 10$.



The curve $y = f(x)$ passes through the following points.

x	0	2	4	6	8	10
y	2	4.5	4.2	3.3	4.5	8

It is required to find the area bounded by the curve, the x-axis, the y-axis and the line $x = 10$.

(a) Use the trapezoidal rule to find an estimate for the area. [3]

One possible model for the curve $y = f(x)$ is a cubic function.

(b.i) Use all the coordinates in the table to find the equation of the least squares cubic regression curve. [3]

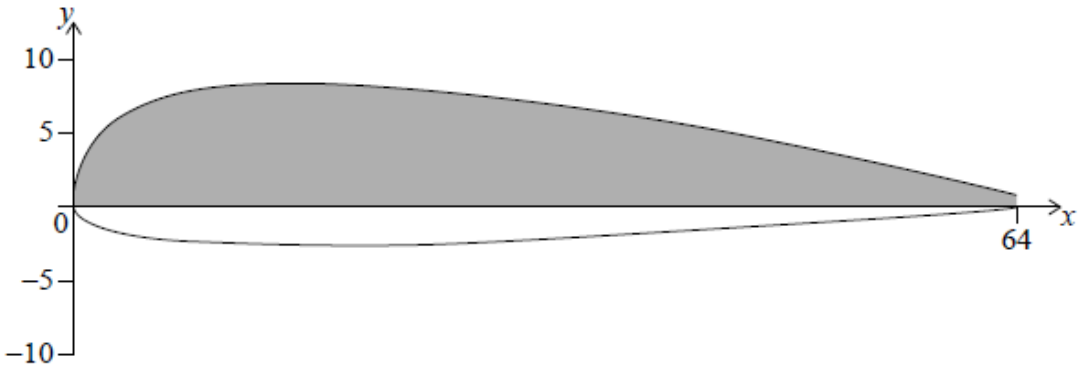
(b.ii) Write down the coefficient of determination. [1]

(c.i) Write down an expression for the area enclosed by the cubic regression curve, the x-axis, the y-axis and the line $x = 10$. [1]

(c.ii) Find the value of this area. [2]

2. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{for } 0 \leq x \leq 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table.

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

[3]

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

(b) Find Jan's estimate of the shaded area of the cross-section. [3]

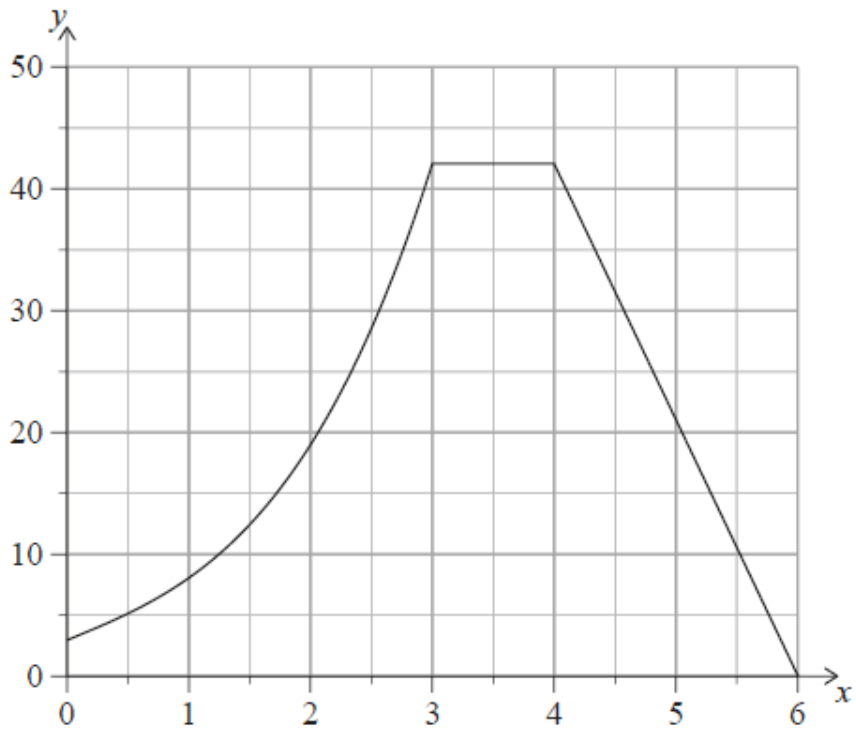
(c.i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section. [2]

(c.ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [2]

(d) Calculate the percentage error of Jan's estimate in part (b). [2]

3. [Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \leq x \leq 3$.

x	0	0.5	1	1.5	2	2.5	3

$y = f(x)$	3	5.13	8	12.4	19	28.6	42

(a) Calculate an estimate for the area in the interval $0 \leq x \leq 3$ by using the trapezoidal rule with three equal intervals. [2]

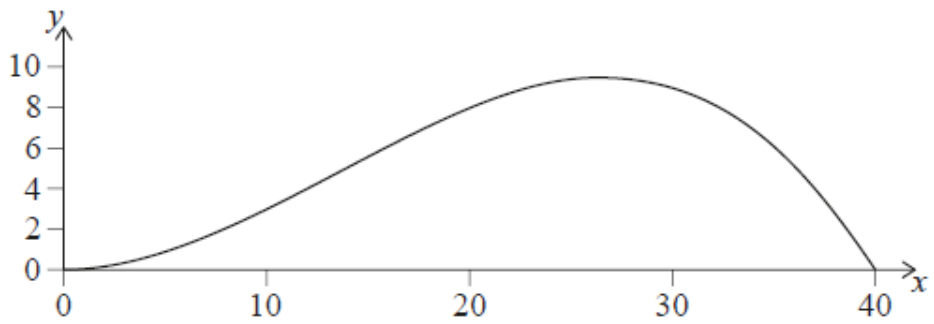
It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]

(c) **Hence** find the actual area of the **entire** cross-section. [3]

4. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is

$y = 0.04x^2 - 0.001x^3,\ 0 \leq x \leq 40.$

(b.i) Write down an integral to find the exact cross-sectional area. [2]

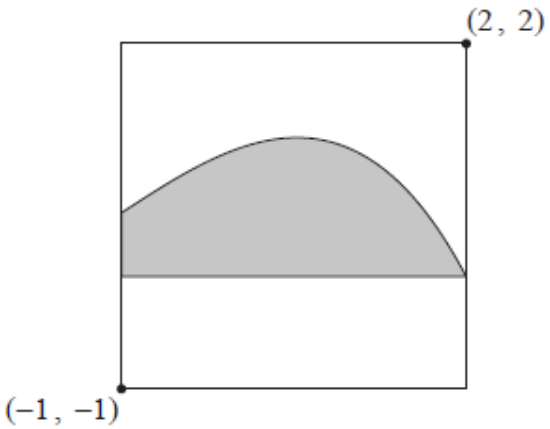
(b.ii) Calculate the value of the cross-sectional area to two decimal places. [2]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

5. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, -1)$ and the top right corner has coordinates $(2, 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region.

x	−1	0	1	2
y	0.6	1.2	1.2	0

[3]

The artist used the equation $y = \frac{-x^3-3x^2+4x+12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]