Disguised quadratic equations

Introduction

Before you start make sure you are comfortable with solving quadratic equations using factorization, completing the square or quadratic formula and that you are able recognize when a quadratic equation has no solutions.

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This may look like a complicated equation, but in fact it can be easily reduced to a quadratic, which we can solve in few seconds.

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$$t^2 - 3t - 10 = 0$$

This can be easily solved using factorization:

$$(t-5)(t+2) = 0$$

$$t - 5 = 0 \qquad \text{or} \qquad t + 2 = 0$$

$$t = 5 \qquad \text{or} \qquad t = -2$$

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$$\frac{1}{x+1} = 5 \quad \text{or} \quad \frac{1}{x+1} = -2$$

$$1 = 5x + 5 \quad \text{or} \quad 1 = -2x - 2$$

$$x = -\frac{4}{5} \quad \text{or} \quad x = -\frac{3}{2}$$

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$$1 = 5x + 5 \quad \text{or} \quad 1 = -2x - 2$$
$$x = -\frac{4}{5} \quad \text{or} \quad x = -\frac{3}{2}$$

And these are our final two solutions.

What we need to practice now is the ability to recognize when a seemingly complicated equation can be reduced to a quadratic by introducing a new variable.

Example 1

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$$\begin{aligned} (t-8)(t-2) &= 0 \\ t-8=0 \quad &\text{or} \quad t-2=0 \\ t=8 \quad &\text{or} \quad t=2 \end{aligned}$$

Going back to x we get:

$$\begin{aligned} x^3 = 8 \quad &\text{or} \quad x^3 = 2 \\ x = 2 \quad &\text{or} \quad x = \sqrt[3]{2} \end{aligned}$$

And these are our solutions to the original equation.

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$$x - \sqrt{x} - 6 = 0$$

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This can be easily solved using factorization:

$$(t-3)(t+2) = 0$$

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$$\sqrt{x} = 3 \qquad \text{or} \qquad \sqrt{x} = -2$$
$$x = 9 \qquad \text{or} \qquad \text{no solution}$$

So in the end we only have one solution $x = 9$.

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We factorize and get:

$$(2t+1)(t-3) = 0$$

$$2t + 1 = 0 \qquad \text{or} \qquad t - 3 = 0$$

$$t = -\frac{1}{2} \qquad \text{or} \qquad t = 3$$

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$$x^2+1=-\frac{1}{2} \quad \text{or} \quad x^2+1=3$$
$$x^2=-\frac{3}{2} \quad \text{or} \quad x^2=2$$
$$\text{no real solutions} \quad \text{or} \quad x=\pm\sqrt{2}$$

So we have two real solution $x=\sqrt{2}$ or $x=-\sqrt{2}$.

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Now we have:

$$t = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$t = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

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We have only one solution $x = 6 + 2\sqrt{5}$.

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Important point here.

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Consider the two equations:

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In the fist equation we're looking for a number whose square root is -3. Clearly there is no such number. Note that $(-3)^2 = 9$, but $\sqrt{9} \neq -3$, we have $\sqrt{9} = 3$.

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In the second equation we're looking for a number whose cube root is -3. Now we know that $(-3)^3 = -27$ and we have $\sqrt[3]{-27} = -3$. So the equation has a solution and it's $x = -27$.

Now we go back to disguised quadratics. We will now try different examples.

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Solve:

$$(x^2 + 2x)^2 + 2(x^2 + 2x) - 15 = 0$$

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In the end we have two real solutions: $x = -3$ or $x = 1$.

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Since we assumed that $x \neq 0$ we can multiply both sides of both equations $2x$ and x respectively and get:

$$2x^2 + 2 = 5x \qquad \text{or} \qquad x^2 + 1 = -2x$$

$$2x^2 - 5x + 2 = 0 \qquad \text{or} \qquad x^2 + 2x + 1 = 0$$

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Each equation can be solve by factorization:

$$(2x - 1)(x - 2) = 0 \qquad \text{or} \qquad (x + 1)^2 = 0$$
$$x = \frac{1}{2} \quad \text{or} \quad x = 2 \qquad \text{or} \qquad x = -1$$

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$$2x^2 + 2 = 5x \qquad \text{or} \qquad x^2 + 1 = -2x$$
$$2x^2 - 5x + 2 = 0 \qquad \text{or} \qquad x^2 + 2x + 1 = 0$$

Each equation can be solve by factorization:

$$(2x - 1)(x - 2) = 0 \qquad \text{or} \qquad (x + 1)^2 = 0$$
$$x = \frac{1}{2} \quad \text{or} \quad x = 2 \qquad \text{or} \qquad x = -1$$

We end up with three solution $x = \frac{1}{2}$ or $x = 2$ or $x = -1$.

If you have any questions you can contact via MS Teams or Librus.