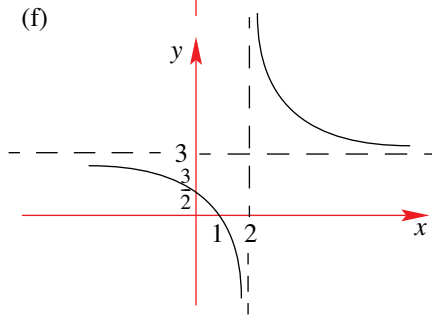
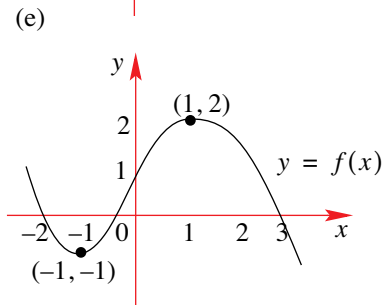
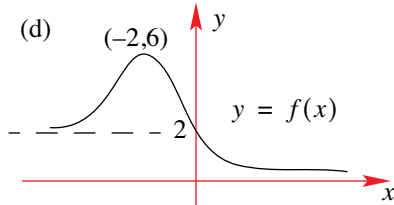
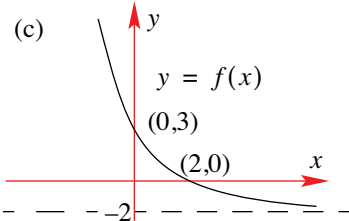
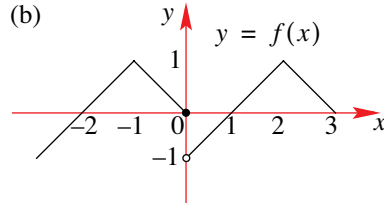
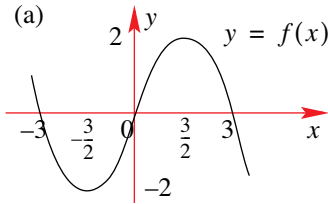


EXERCISES 6.4

1. Sketch the graphs of $y = \frac{1}{f(x)}$ for each of the following



2. On the same set of axes sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ where

- | | | |
|---|----------------------------------|-------------------------------|
| (a) $f(x) = \sqrt{x} - 2$ | (b) $f(x) = x^2 + 4x$ | (c) $f(x) = x^3 + 2$ |
| (d) $f(x) = e^x - 1$ | (e) $f(x) = \ln x$ | (f) $f(x) = x - 1$ |
| (g) $f(x) = (2 - x)^3$ | (h) $f(x) = 2 - 2^x$ | (i) $f(x) = \frac{x+1}{x}$ |
| (j) $f(x) = \left \frac{1}{x} - 2 \right $ | (k) $f(x) = \frac{1}{ x^2 - 4 }$ | (l) $f(x) = \log_{10}(3 - x)$ |

3. (a) Sketch the graph of $f(x) = x^2 - 4x + 3$.

(b) Using your graph in part (a), sketch the graph of $y = \frac{1}{x-3} - \frac{1}{x-1}$

4. Sketch the graph of $f(x) = 2\left(\frac{1}{x-5} - \frac{1}{x-3}\right)$.

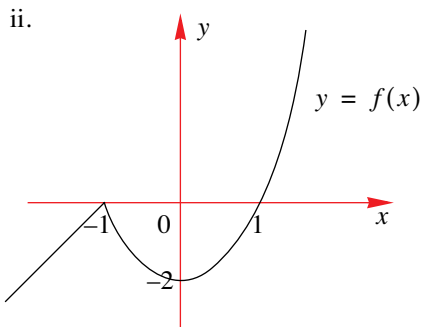
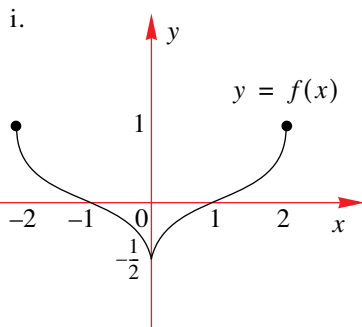
EXERCISES 6.5 - MISCELLANEOUS QUESTIONS

1. For the functions shown below sketch the graphs of

(a) $y = \frac{1}{f(x-1)}$

(b) $y = \frac{2}{f(x)}$

(c) $y = \frac{2}{f(2x)}$



2. On the same set of axes sketch the graphs of

(a) $f(x) = x^2 - 6x + 8$ and $y = \frac{1}{f(x)}$.

(b) $f(x) = x^2 - 6x + 9$ and $y = \frac{1}{f(x)}$.

(c) $f(x) = x^2 - 6x + 10$ and $y = \frac{1}{f(x)}$.

(d) $f(x) = x^2 - 6x + 11$ and $y = \frac{2}{f(x)}$.

3. Consider the functions $f(x) = a^x, a > 1$ and $g(x) = a^{-x}, a > 1$.

(a) On the same set of axes sketch the graphs of $y = f(x)$ and $y = g(x)$.

(b) i. Sketch the graph of $y = f(x) + g(x)$.

ii. Sketch the graph of $y = f(x) - g(x)$.

Let $u(x) = a^x + a^{-x}, a > 1$ and $v(x) = a^x - a^{-x}, a > 1$.

(c) Sketch the graph of

i. $y = \frac{2}{u(x)}$

ii. $y = \frac{1}{v(x)}$

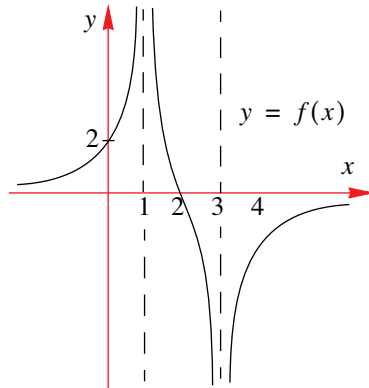
iii. $y = \frac{2}{u(|x|)}$

iv. $y = \frac{1}{v(|x|)}$

v. $y = \left| \frac{2}{u(x)} \right|$

vi. $y = \left| \frac{1}{v(x)} \right|$

4. Consider the graph shown below



Sketch the graph of

i. $y = \frac{4}{f(x)}$ ii. $y = \frac{1}{f(|x|)}$ iii. $y = \frac{2}{|f(x)|}$

5. Consider the function $u(x) = \begin{cases} x^2 - ax & \text{if } x \geq 0 \\ a - ae^x & \text{if } x < 0 \end{cases}$, where $a > 0$.

- (a) Sketch the graph of $y = u(x)$.
 (b) Sketch the graph of $y = \frac{a}{u(x)}$ for
 (c) If $v(x) = |x|$ sketch the graph of
 i. $y = (u \circ v)(x)$ ii. $y = -(v \circ u)(x)$.

6. (a) Given that the curve $y = x^3 - 12x$ has turning points at $(2, -16)$ and $(-2, 16)$ sketch its graph.

- (b) Hence, sketch the graph of
 i. $y = \frac{12}{x^3 - 12x}$.
 ii. $y^2 = x^3 - 12x$.

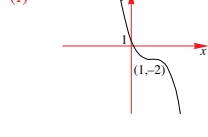
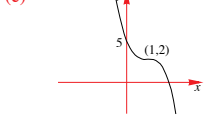
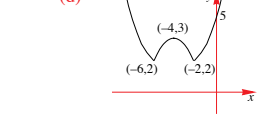
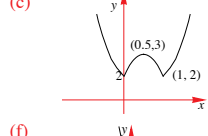
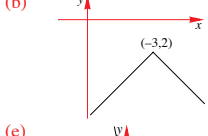
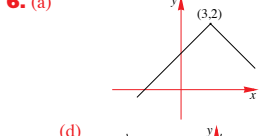
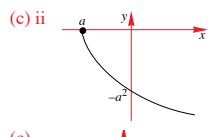
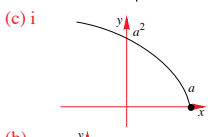
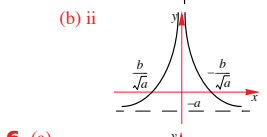
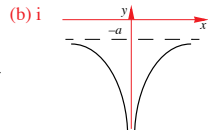
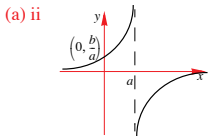
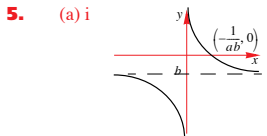
7. On the same set of axes sketch the graphs of $f(x) = \frac{1}{a} \log_a(x - a)$, $a > 1$ and $y = \frac{1}{f(x)}$.

8. (a) On the same set of axes sketch the graphs of $f(x) = \frac{1}{a}x$ and $g(x) = \frac{a}{x}$, $a > 0$.

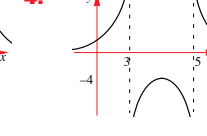
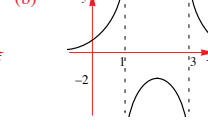
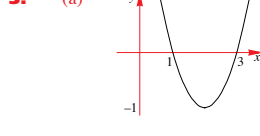
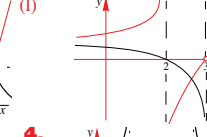
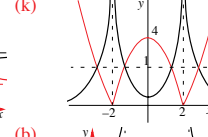
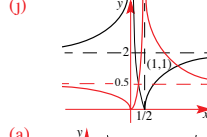
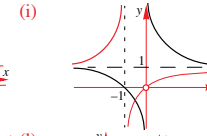
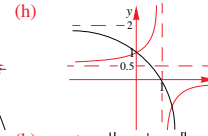
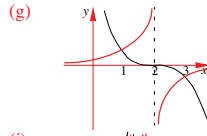
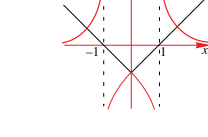
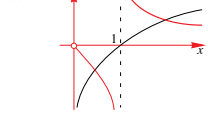
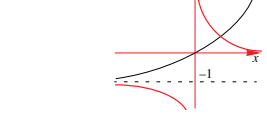
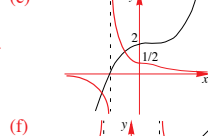
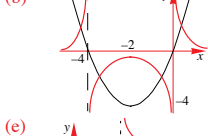
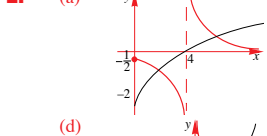
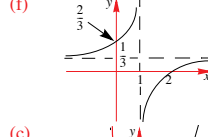
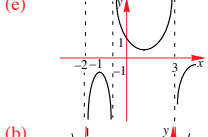
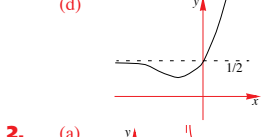
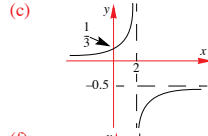
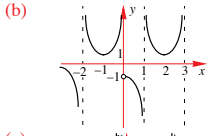
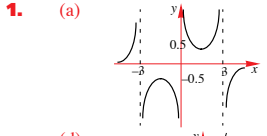
Hence sketch the graph of $y = \frac{x^2 + a^2}{ax}$, $a > 0, x \neq 0$.

- (b) Using your results from (a), sketch the graph of $h(x) = \frac{1}{f(x) + g(x)}$.

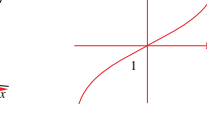
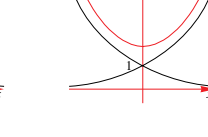
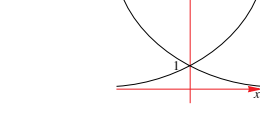
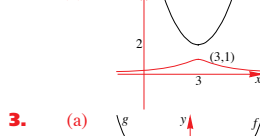
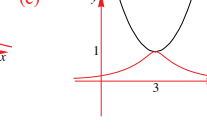
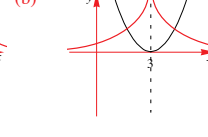
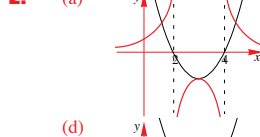
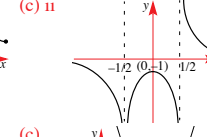
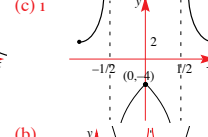
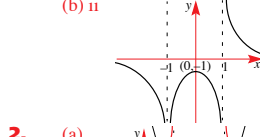
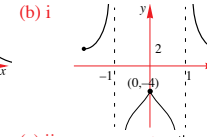
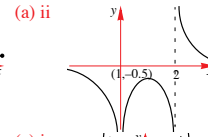
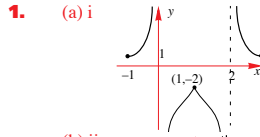
- (c) Hence, show that $-\frac{1}{2} \leq \frac{ax}{x^2 + a^2} \leq \frac{1}{2}$.

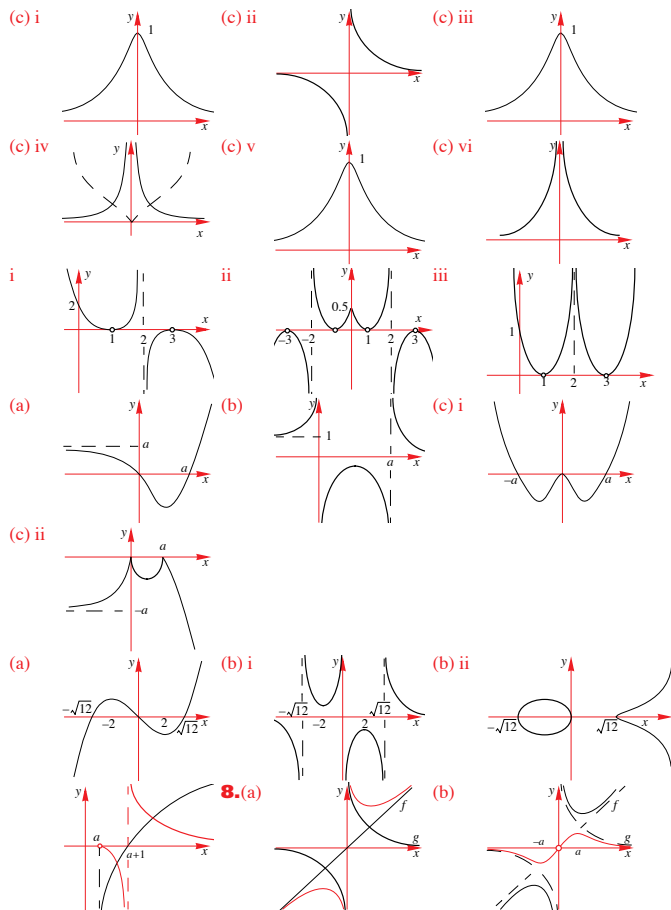


EXERCISE 6.4



EXERCISE 6.5





EXERCISE 7.1.1

1. (a) $\frac{27y^{15}}{8x^3}$ (b) $\frac{91}{216a^6}$ (c) $2^n + 2$ (d) $\frac{8x^{11}}{27y^2}$ (e) $\frac{3x^2y^2}{8}$ (f) $3^{n+1} + 3$ (g) $4^{n+1} - 4$
 (h) $2(4^{n+1} - 4)$ (i) $\frac{1-b^6}{16b^4}$ 2. (a) 64 (b) $(\frac{2}{3})^x$ (c) $22y+1$ (d) $\frac{1}{b^{2x}}$ (e) $(\frac{y}{2})^6$ (f) $(\frac{9}{2})^{n+2}$ 3. (a) $\frac{z^2}{xy}$
 (b) $37n-2$ (c) $5n+1$ (d) 9 (e) $26n+1$ (f) $21-3n$ (g) x^2+4n-n^2 (h) x^{3n^2+n+1} (i) 27 4. $\frac{y^{2m-2}}{x^m}$
 5. (a) -81 (b) $\frac{9x^8}{8y^4}$ (c) $y-x$ (d) $\frac{2x+1}{x+1}$ (e) -1 (f) -b 6. (a) $\frac{1}{x^2y^2}$ (b) $\frac{1}{x^4}$ (c) $-\frac{1}{x(x+h)}$

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- (d) $\frac{1}{x-1}$ (e) $\frac{1}{(x+1)(x-1)^5}$ (f) $\frac{1}{x^2}$ 7. (a) $118 \times 5^{n-2}$ (b) 1 (c) $\frac{b^7}{a^4}$ (d) a^{mn} (e) $\frac{p+q}{pq}$ (f) $\frac{2\sqrt{a}}{a-1}$
 (g) $\frac{7}{8}$ (h) $a^{7/8}$ 8. (a) $x^{11/12}$ (b) $2a^{3n-2}b^{2n-2}$ (c) 2^n (d) $-\frac{7^{m-n}}{8}$ (e) $\frac{6 \times 5^n}{5^n+5}$ (f) $x+1$

EXERCISE 7.1.2

1. (a) 2 (b) -2 (c) $\frac{2}{3}$ (d) 5 (e) 6 (f) -2.5 (g) 2 (h) 1.25 (i) $\frac{1}{3}$ 2. (a) -6 (b) $-\frac{2}{3}$ (c) -3 (d) 1.5 (e) 0.25
 (f) 0.25 (g) $-\frac{1}{8}$ (h) $-\frac{11}{4}$ (i) -1.25

EXERCISE 7.1.3

1. (a) 3.5 (b) 3.5 (c) -3 (d) 1.5 (e) 3.5 (f) 1.5 (g) 1.8 (h) $-\frac{4}{7}$ (i) 0 2. (a) -0.75 (b) -1.4 (c) 0, 1
 (d) 3, 4 (e) -1, 4 (f) 0, 2 3. (a) -1, 1, 2 (b) -3, 1, 3, 4 (c) $\frac{4}{3}, \frac{5}{3}, 2$ (d) -1, 1, 2 (e) 3, 7, $-\frac{1 \pm \sqrt{233}}{2}, \frac{1}{3}$

EXERCISE 7.1.4

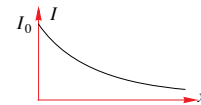
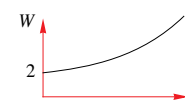
1. (a) i. 5.32 ii. 9.99 iii. 2.58 (b) i. 2.26 ii. 3.99 iii. 5.66 (c) i. 3.32 ii. -4.32 iii. -6.32 (d) i. -1.43
 ii. 1.68 iii. -2.86 2. (a) 0 (b) 0.54 (c) -0.21 (d) -0.75, 0 (e) 1.13 (f) 0, 0.16

EXERCISE 7.1.5

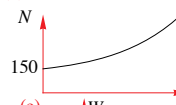
1. (a) 2 (b) -1 (c) 0.5 (d) 0.5 2. (a) 1 (b) 0.6 (c) 0 3. (a) 0 (b) $\frac{2}{3}$ 4. (a) -1, 2 (b) -2, 3 (c) -1
 (d) -6, 1 (e) 0, 1 (f) 1 5. (a) 1.3863 (b) 2.1972 (c) 3.2189 (d) \emptyset 6. (a) 0.4236 (b) 0.4055
 (c) 0.3054 (d) -0.4176 7. (a) 0 (b) -0.6733 (c) 0 9. 36 10. $a = \sqrt{2}e, k = \ln(\sqrt{2})$

EXERCISE 7.2

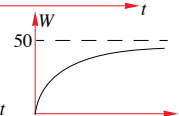
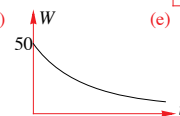
1. (a) 1000 (b) 1516 (c) 2000 (d) 10 days 2. (a) 0.0013 (b) 2.061 kg (c) 231.56 yrs
 (d) 3. (a) 0.01398 (b) 52.53% (c) 51.53m (d) 21.53m



4. (a) i. 157 ii. 165 iii. 191 (b) 14.2 yrs (c) 20.1 yrs (d)



5. (a) 50 (b) 0.0222 (c) 17.99 kg (d)



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