### INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

# MAA

# EXERCISES [MAA 2.13-2.15] RATIONAL AND MODULUS FUNCTIONS – INEQUALITIES

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### O. Practice questions

1. [Maximum mark: 18] [without GDC]

For the following functions write down the roots, the y-intercept, the vertical and horizontal asymptotes (if there exist), the domain and the range.

	$f(x) = \frac{3x+8}{2x+7}$	$f(x) = \frac{8}{2x+7}$	$f(x) = \frac{3x + 8}{7}$
Roots			
y -intercept			
V.A.			
H.A.			
Domain			
Range			

**2.** [Maximum mark: 15] *[without GDC]* 

For the following functions write down the roots, the y-intercept, the vertical and horizontal asymptotes and the domain.

	$f(x) = \frac{(x-3)(x-4)}{(x+1)(x-2)}$	$f(x) = \frac{(2x-3)(x-4)}{(x+1)(x-2)}$	$f(x) = \frac{2x-3}{(x+1)(x-2)}$
Roots			
y -intercept			
V.A.			
H.A.			
Domain			

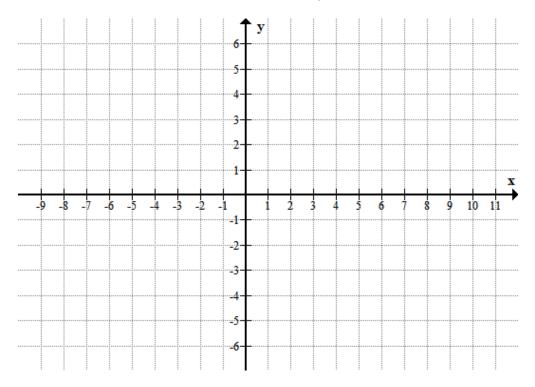
3.

	the following functions, find all the asymptotes (horizontal, vertical, or oblique).	
(a)	$f(x) = \frac{4x^2 + 4x + 1}{2x + 3}$	[4]
	$f(x) = \frac{2x^3 + 5x^2 + 4x + 1}{x^2 - x - 2}$	[6]

4. [Maximum mark: 10] [without GDC]

Let 
$$f(x) = \frac{(x-1)(x-6)}{(x+2)(x-3)}$$
.

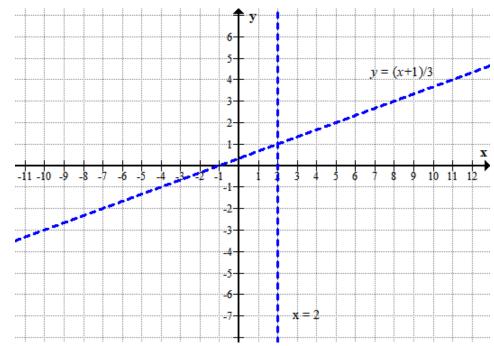
- (a) Solve the equation f(x) = 1. [3]
- (b) On the diagram below, sketch the graph of y = f(x). Indicate all the asymptotes, the x- and the y-intercepts, and the information found in (a). [5]
- (c) Hence, write down the domain and the range of f. [2]



5. [Maximum mark: 12] [without GDC]

Let 
$$f(x) = \frac{(x+2)(x-3)}{3x-6}$$

- (a)\_ Show that graph y = f(x) has oblique asymptote the line  $y = \frac{x+1}{3}$  [3]
- (b) Solve the equation f(x) = 1. [3]
- (c) On the diagram below, sketch the graph of y = f(x). The asymptotes are shown. Indicate the x- and the y-intercepts and the information found in (b). [4]
- (d) Hence, write down the domain and the range of f. [2]



6.	[Maximum	mark: 12	l [with	/ without	<b>GDC</b>
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Express the following rational functions in partial fractions.

(a) 
$$f(x) = \frac{6}{x^2 - x - 2}$$
 [4]

(b) 
$$f(x) = \frac{3x+6}{x^2-x-2}$$
 [4]

(c) 
$$f(x) = \frac{x}{2x^2 - 2x - 4}$$
 [4]

## 7. [Maximum mark: 10] [without GDC]

Consider the polynomial

$$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$$

(a) Complete the sign table below (Indicate + or - in each interval)

х	-∞	1 2	2 3	4	. :	5 +∞
f(x)						

- (b) Solve the inequality f(x) > 0. [2]
- (c) Solve the inequality  $f(x) \ge 0$ . [2]
- (d) Solve the inequality  $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)} > 0$ . [2]
- (e) Solve the inequality  $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)} \ge 0.$  [2]

### 8. [Maximum mark: 13] [without GDC]

Consider the following functions.

Solve the inequality g(x) > 0.

(b)

$$g(x) = (x-1)(x-2)(x-3)(x-4)^{2}(x-5)$$

(a) Complete the sign table below (Indicate + or – in each interval)

x	-∞	1	2	3	4	5	$+\infty$
g(x)							

[2]

[2]

- (c) Solve the inequality  $g(x) \ge 0$ . [2]
- (d) Solve the inequality g(x) < 0. [2]
- (e) Solve the inequality  $g(x) \le 0$ . [2]
- (f) Solve the inequality  $\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)^2} \ge 0$ . [3]

[without GDC]

9.

[Maximum mark: 8]

9	Solv	e the inequalities (r	notice	that they a	II involve	the same	e factors	).	
(	(i)	$\frac{5(x-1)(x-2)^2}{(x-3)^3} \ge 0$	0	(ii) $\frac{(x-1)}{5(x-1)}$	$\frac{1(x-3)^3}{(x-2)^2}$	≥ 0	(iii) -	$\frac{(x-2)^2(x-2)^2}{5(x-2)^2}$	$\frac{(-3)^3}{1)} \ge 0$
		••••••							
)*. [	Max	kimum mark: 6]	[with	/ without	GDC]				
5	Solv	e the inequality $x$	$+\frac{2}{x} \ge 3$	3					

	ximum mark: 6] <i>[witi</i>	-
(a)		x-5 =3
(b)	Solve the inequality	x-5  < 3
[Max	ximum mark: 8] <i>[witi</i>	h / without GDC]
[Max (a)		x-5  =  x-3
	Solve the equation	
(a)	Solve the equation	x-5  =  x-3
(a)	Solve the equation	x-5  =  x-3
(a)	Solve the equation Solve the inequality	x-5  =  x-3  $ x-5  <  x-3 $
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13*.	[Max	ximum mark: 10] <i>[with / without GDC]</i>	
	(a)	Solve the equation $ x-5  = x-3$	[5]
	(b)	Solve the inequality $ x-5  < x-3$	[5]
14*.	[Max	ximum mark: 5] <i>[with / without GDC]</i>	
		e the inequality $x^2 - 3 x  > 2$	

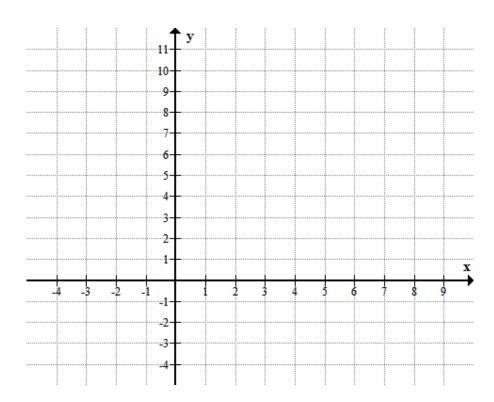
15*.		imum mark: 12] <i>[</i>					
	(a)	Solve the equation		x-5 = x-5	x -3		[6]
	(b)	Solve the inequality	y	x-5  <  x	x -3		[6]

- **16\*.** [Maximum mark: 8] [without GDC]
  - Draw the graph of f(x) = |x-5|-|x|+3

[4]

- Hence, solve the inequalities (b)
  - (i) f(x) > 0
- (ii) f(x) > 4 (iii) f(x) > -4 (iv) f(x) > 10

[4]




# A. Exam style questions (SHORT)

17.	[Maximum mark: 5]	[without GDC]
	Find all the asymptotes	is (horizontal, vertical, or oblique) of $f(x) = \frac{3x^2 - x + 1}{2x^2 - 14x + 24}$
18.	[Maximum mark: 5]	[without GDC]
	Find all the asymptotes	is (horizontal, vertical, or oblique) of $f(x) = \frac{3x^2 - x + 1}{x^2 - x + 1}$

19.	[Maximum mark: 6]	[without GDC]
	Find all the asymptote	s (horizontal, vertical, or oblique) of $f(x) = \frac{6x^2 + 5x + 1}{3x + 7}$
20.	[Maximum mark: 7]	[without GDC]
	Find all the asymptote	s (horizontal, vertical, or oblique) of $f(x) = \frac{6x^3 + 1}{2x^2 - 14x + 24}$
		2x - 14x + 24
		2x - 14x + 24
		2x -14x + 24

# [MAA 2.13-2.15] RATIONAL AND MODULUS FUNCTIONS - INEQUALITIES

	[Maximum mark: 7]	
	Express in partial frac	tions the function $f(x) = \frac{5}{2x^2 - 14x + 24}$
22.	[Maximum mark: 7]	[without GDC]
	Express in partial frac	tions the function $f(x) = \frac{5x+1}{2}$
	Express in partial frac	tions the function $f(x) = \frac{5x+1}{2x^2-14x+24}$
	Express in partial frac	tions the function $f(x) = \frac{5x+1}{2x^2-14x+24}$
	Express in partial frac	tions the function $f(x) = \frac{5x+1}{2x^2-14x+24}$
		tions the function $f(x) = \frac{5x+1}{2x^2 - 14x + 24}$

[without GDC]

23.

[Maximum mark: 8]

So	lve the inequalities	(notice that	they all invo	olve the sam	e factors).	
(i)	$(x-1)^2(x+1)(x-1)$	$(2) \ge 0$ (i	$\frac{(x-1)^2}{x-1}$	$\frac{(x+1)}{2} > 0$	(iii) $\frac{(x)^2}{(x)^2}$	$\frac{-1)^2(x+1)}{x-2} \ge 0$
			•••••			
			•••••			
[Ma	aximum mark: 5]	[without	GDC]			
So	lve the inequalities	(i) $\frac{(2x-1)^{-1}}{(2x-1)^{-1}}$	$\frac{-1)^2(x+1)}{2-x} \ge$	0 (ii)	$\frac{(2x-1)^2(x+1)^2}{2-x}$	$(+1) \leq 0$
			•••••			
				,		

<b>25</b> .	[Maximum mark: 8]	[with /	without	GDCI	
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Let 
$$f(x) = \frac{x+4}{x+1}$$
,  $x \ne -1$  and  $g(x) = \frac{x-2}{x-4}$ ,  $x \ne 4$ .

(	(a)	Solve the equation $f(x) = g(x)$	[3]

(α)	Solve the equation $f(x) - g(x)$	[0]
(b)	Find the set of values of $x$ such that $f(x) \le g(x)$ .	[5]

26.	[Maximum mark: 5]	
	Solve the inequality $x^2$	$x^2 - 4 + \frac{3}{x} < 0$ .
27*.	[Maximum mark: 8]	[without GDC]
		$x^2 - 4 + \frac{3}{x} < 0$ given that one of the zeroes of the function is $x = 1$ .

28.	[Maximum mark: 5] [With / Without GDC]
	Find the values of $x$ for which $ 5-3x  \le  x+1 $ .
29.	[Maximum mark: 5] [with / without GDC]
	Solve the inequality $ x-2  \ge  2x+1 $ .

30*.	[Maximum mark: 6]	[with / without GDC]
	Solve the inequality $\frac{x}{x}$	$\left \frac{1+9}{1-9}\right  \le 2.$
31.		[with / without GDC]
		values of $x$ such that the function $f$ given by $f(x) = \sqrt{\frac{8x-4}{x-3}}$
	takes real values.	

32.	[Maximum mark: 7]	[with / without	GDC]
	The functions $f$ and	$oldsymbol{g}$ are defined by	
	f(x)	)=2x-1,	$g(x) = \frac{x}{x+1}, \ x \neq -1$
	Find the values of $x$ for	or which $(f \circ g)(x)$	$0 \leq (g \circ f)(x)$ .

### B. Exam style questions (LONG)

33.	[Maximum mark: 24]	[with / without GDC]
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Let 
$$f(x) = \frac{6}{x^2 - 5x + 4}$$
,  $g(x) = \frac{x^2 - 5x + 10}{x^2 - 5x + 4}$ ,  $h(x) = \frac{x^3 - 5x^2 + 4x + 12}{x^2 - 5x + 4}$ 

(a) Find the equations of all the asymptotes for the graphs of

(i) 
$$y = f(x)$$
 (ii)  $y = g(x)$  (iii)  $y = h(x)$  [8]

(b) Express 
$$f(x)$$
 in partial fractions [5]

(c) Express 
$$g(x)$$
 in the form  $g(x) = q + \frac{p}{x^2 - 5x + 4}$ , where  $p, q \in \mathbb{Z}$  [2]

(d) Hence express

(e)

(i) 
$$g(x)$$
 in the form  $g(x) = C + \frac{A}{(x-a)} + \frac{B}{(x-b)}$ 

(ii) 
$$h(x)$$
 in the form  $h(x) = Ex + \frac{C}{(x-c)} + \frac{D}{(x-d)}$  [4]

Solve the inequality $5f(x) \le 3g(x)$	[5]
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[MAA 2.13-2.15] RATIONAL AND MODULUS FUNCTIONS - INEQUALITIES

34.	[Maximum	mark: 13	l [with]	/ without	<b>GDC</b>

Let 
$$f(x) = \frac{x^3 + 50}{x^2 - 6x + 8}$$
,  $x \ne 2, x \ne 4$ 

(a) Find the quotient and the remainder of the long division of 
$$x^3 + 50$$
 by  $x^2 - 6x + 8$  [3]

(b) **Hence,** express 
$$f(x)$$
 in the form  $f(x) = q(x) + \frac{ax+b}{x^2-6x+8}$  [2]

(c) Write down all the asymptotes of 
$$y = f(x)$$
 [3]

(d)	Express $f(x)$ in the form $f(x) = q(x) + \frac{A}{x-2} + \frac{B}{x-4}$	[5]