EXERCISES [MAA 2.16]

SYMMETRIES OF FUNCTIONS – MORE TRANSFORMATIONS

Compiled by Christos Nikolaidis

O.   Practice questions

1.   [Maximum mark: 9]   ***[without GDC]***

Determine whether the following function are **even**, **odd** or **neither**. Prove your claim.

(a) $f(x) = 3x^4 - 5x^2 + 1$ [2]

(b) $f(x) = 3x^5 - 5x^3 + 1$ [2]

(c) $f(x) = 3x^5 - 5x^3 + 7x$ [2]

(d) $f(x) = \dfrac{5x^6 + 3|x| - 1}{x^3 + x}$ [3]

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2. [Maximum mark: 36] *[without GDC]*

The diagrams below show the graph of $y = f(x)$ which passes through the point $A(0,2)$.

On the same diagrams, sketch the graphs of the following transformations and for each transformation state the image A′ of point A.
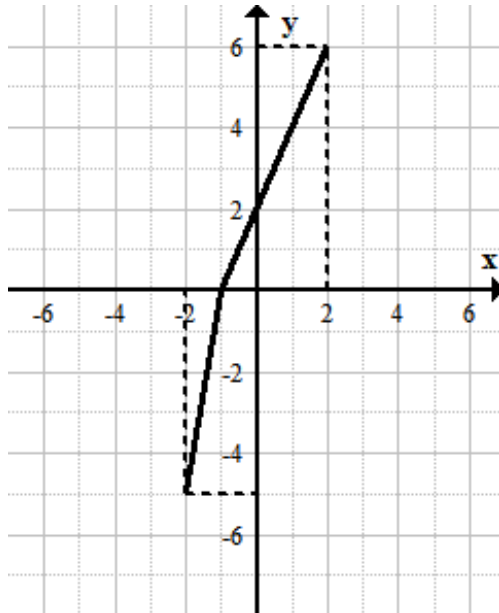
(a) $y = f(x) - 2$
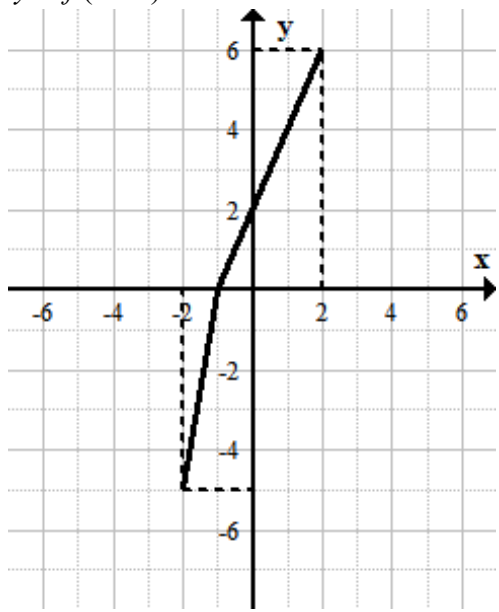

Image of $A(0,2)$: _____

[3]

(b) $y = f(x-2)$
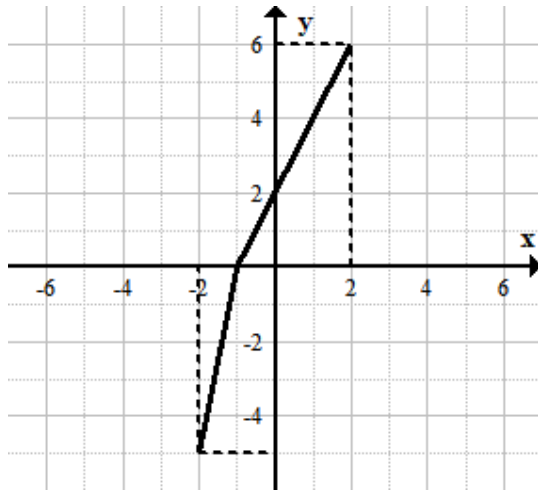

Image of $A(0,2)$: _____

[3]

(c) $y = \dfrac{f(x)}{2}$

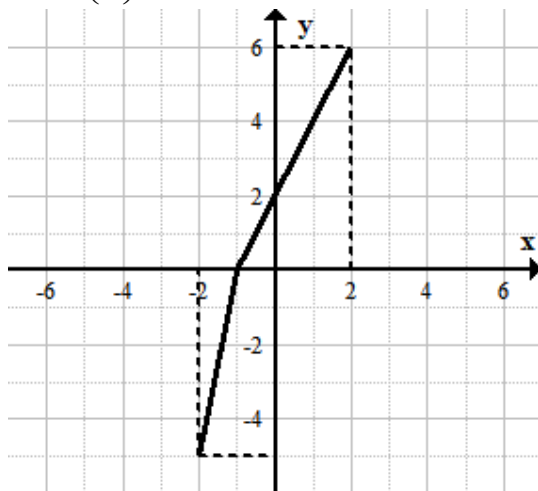Image of $A(0,2)$: _____

[3]

(d) $y = f\left(\dfrac{x}{2}\right)$

Image of $A(0,2)$: _____

[3]

(e) $y = f(2x)$
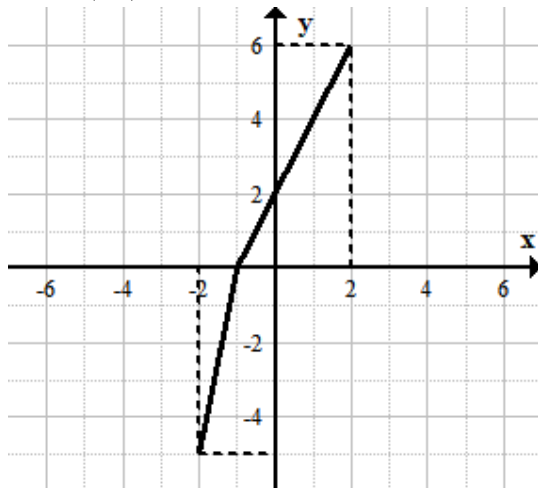
Image of $A(0,2)$: _____

[3]

(f) $y = -f(x)$
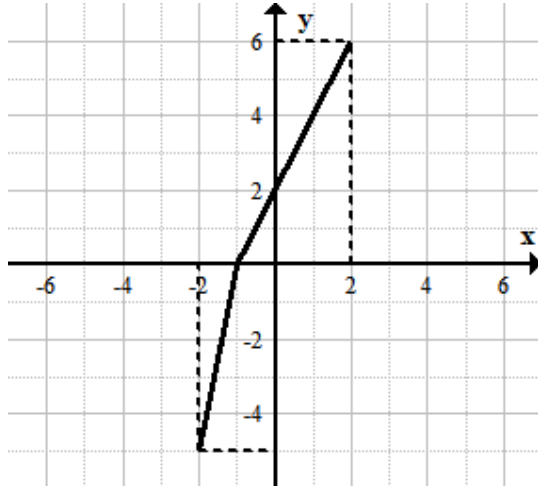
Image of A(0,2): _____
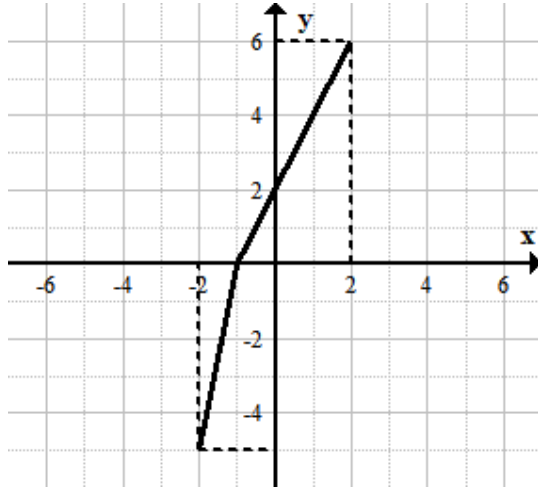
[3]

(g) $y = f(-x)$

Image of A(0,2): _____
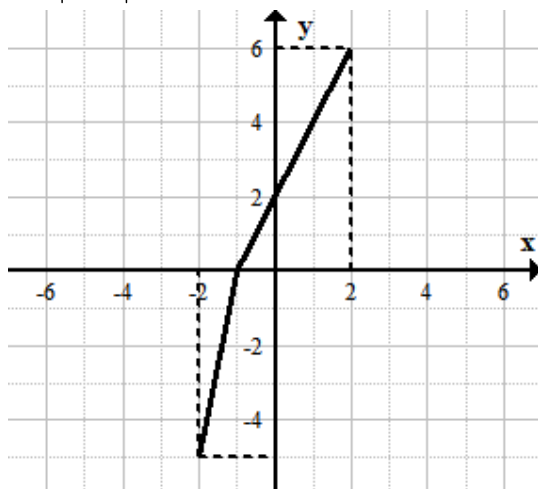
[3]

(h) $y = |f(x)|$

Image of A(0,2): _____

[3]

(i) $y = f(|x|)$

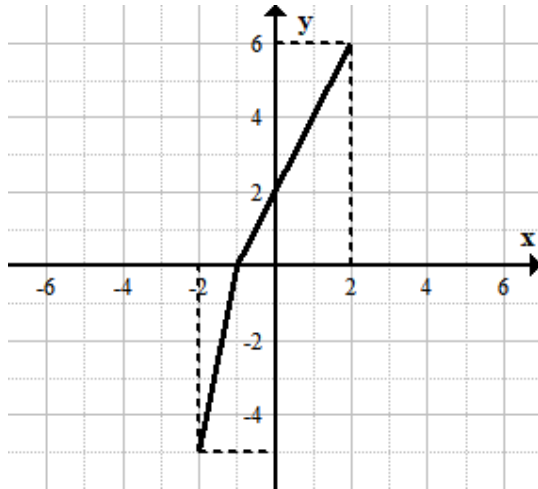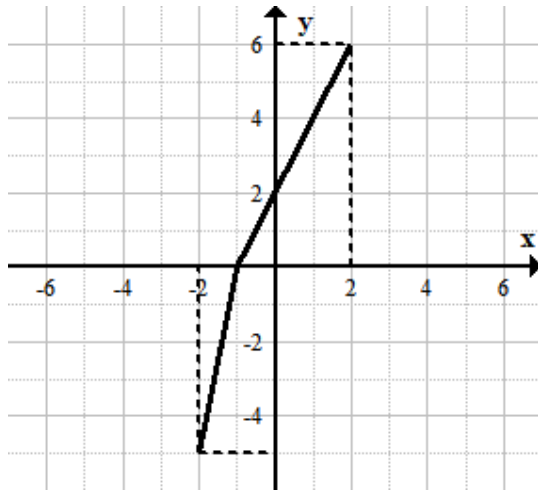
Image of A(0,2): _____

[3]

(j) $y = f(|x| - 1)$

Images of A(0,2): _____

[5]

(k) $y = f(|x - 1|)$
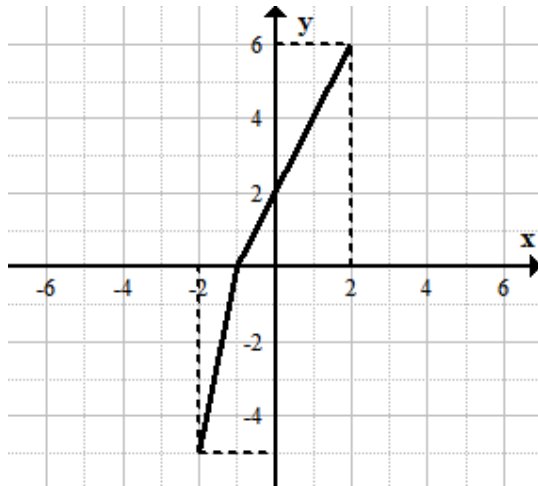

Image of A(0,2): _____

[4]

3.   [Maximum mark: 21]   *[without GDC]*

Let $f(x) = \dfrac{2x-4}{x+2}$

(a)   Complete the following table   [6]

Function	$y = f(x)$	$y = \dfrac{1}{f(x)}$	$y = f^{-1}(x)$
Domain			
Range			

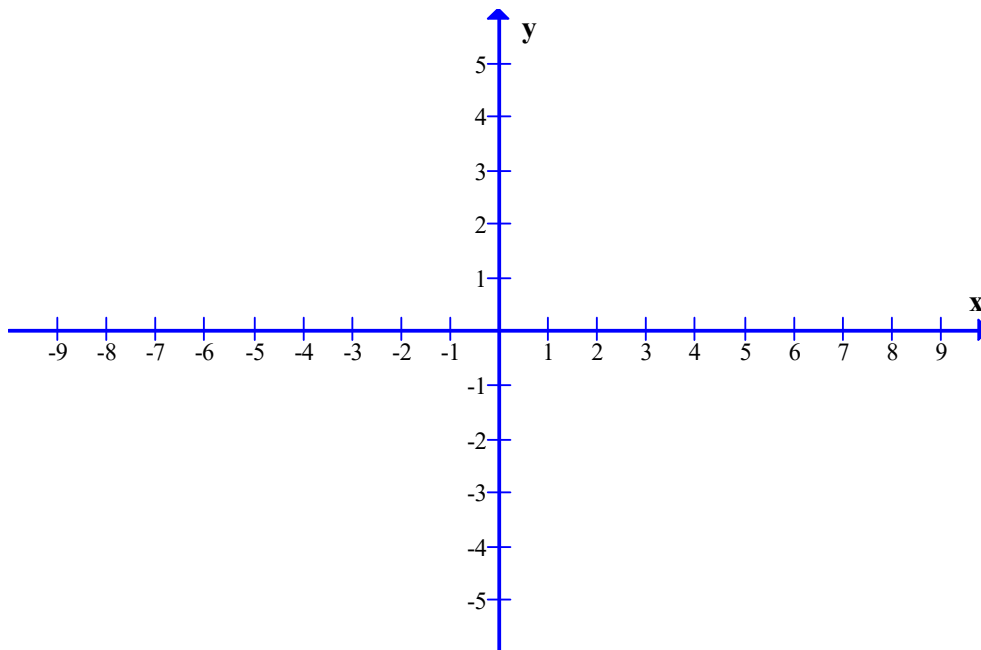(b)   Find the image of the horizontal asymptote $y = 2$ of $f(x)$, under the following

transformations: [4]

Transformation	$2f(x)$	$f(x)+2$	$f(x-7)$	$-f(x)$	$\dfrac{1}{f(x)}$
Horizontal asymptote					

(c)   The point A(3, 0.4) lies on the graph of $f(x)$. Find the image of the point A under

the transformation $y = 2f(3x)+5$.   [2]

. . . .

(d)   Sketch the graph of $f(x)$ by indicating any asymptotes and intersections with

x- and y-axes.   [3]

(e)   Sketch the graph of $\dfrac{1}{f(x)}$ by indicating any asymptotes and intersections with

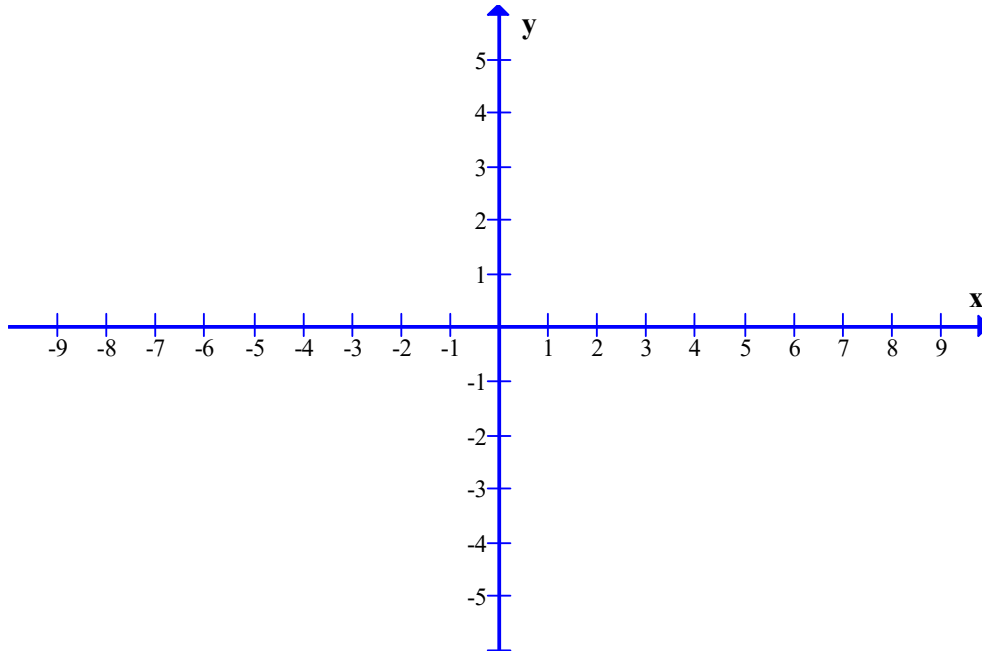 x- and y-axes.   [3]

(f)   Sketch the graph of $f^{-1}(x)$ by indicating any asymptotes and intersections with

 x- and y-axes.   [3]
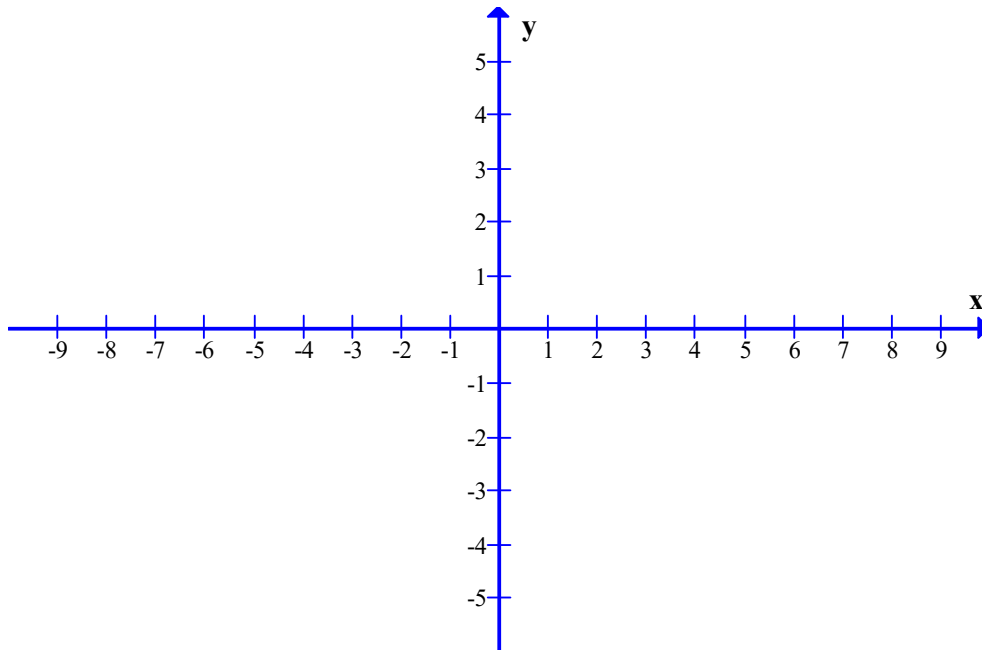
4.   [Maximum mark: 5]   *[without GDC]*

The graph of $y = f(x)$ is shown below.

$y = 1$

$x = -1$ $x = 4$
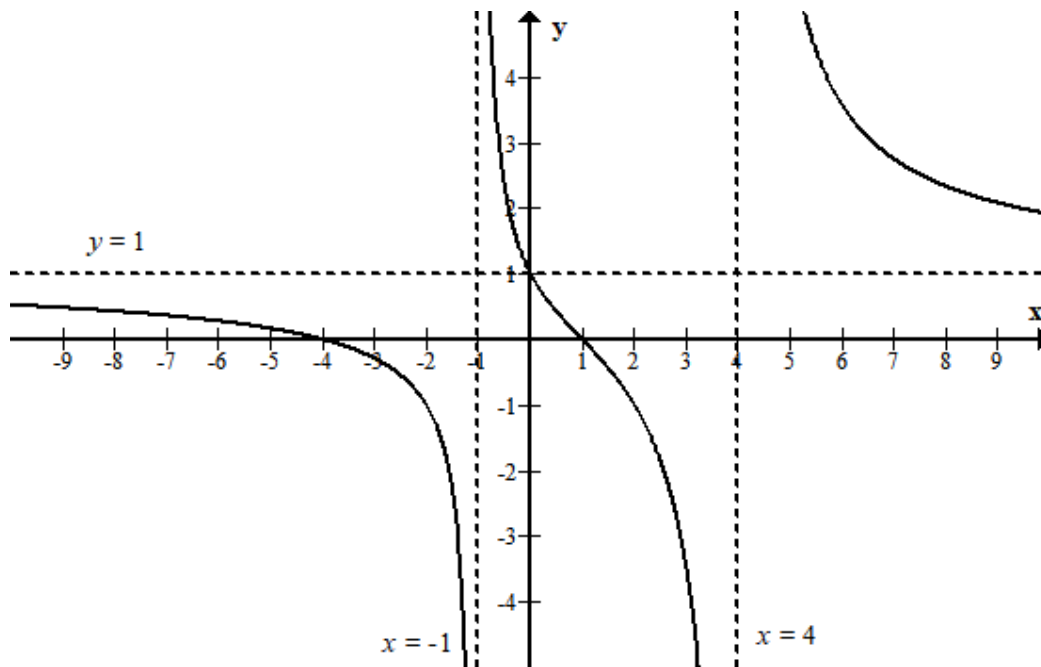
On a new diagram sketch the graph of $y = \dfrac{1}{f(x)}$. Indicate any asymptotes.
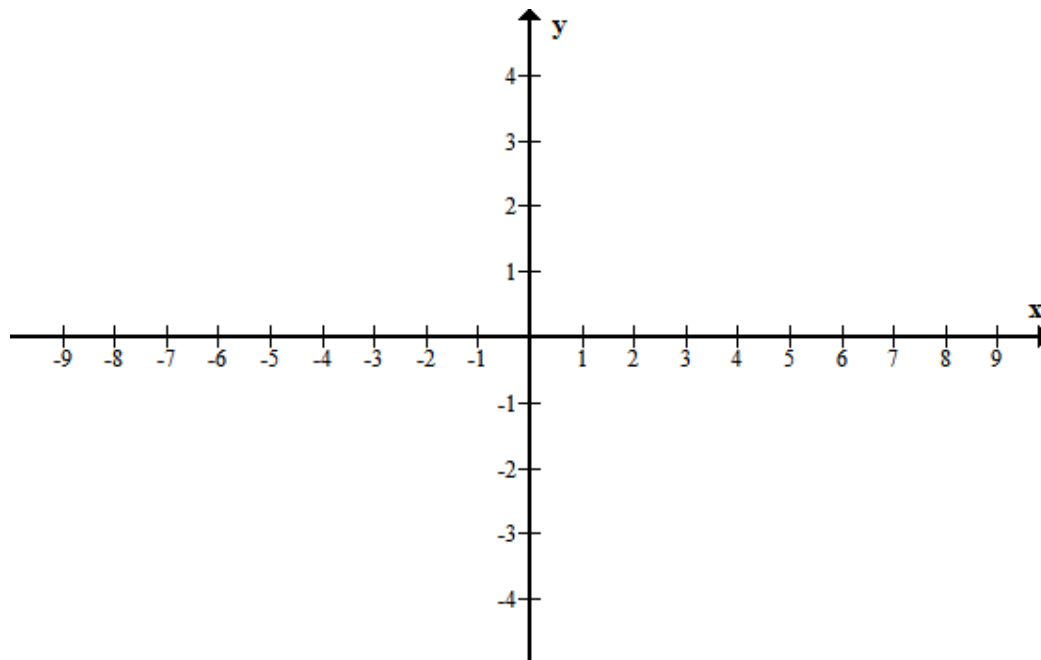
5. [Maximum mark: 5]   *[without GDC]*

The graph of $y = f(x)$ is shown below.
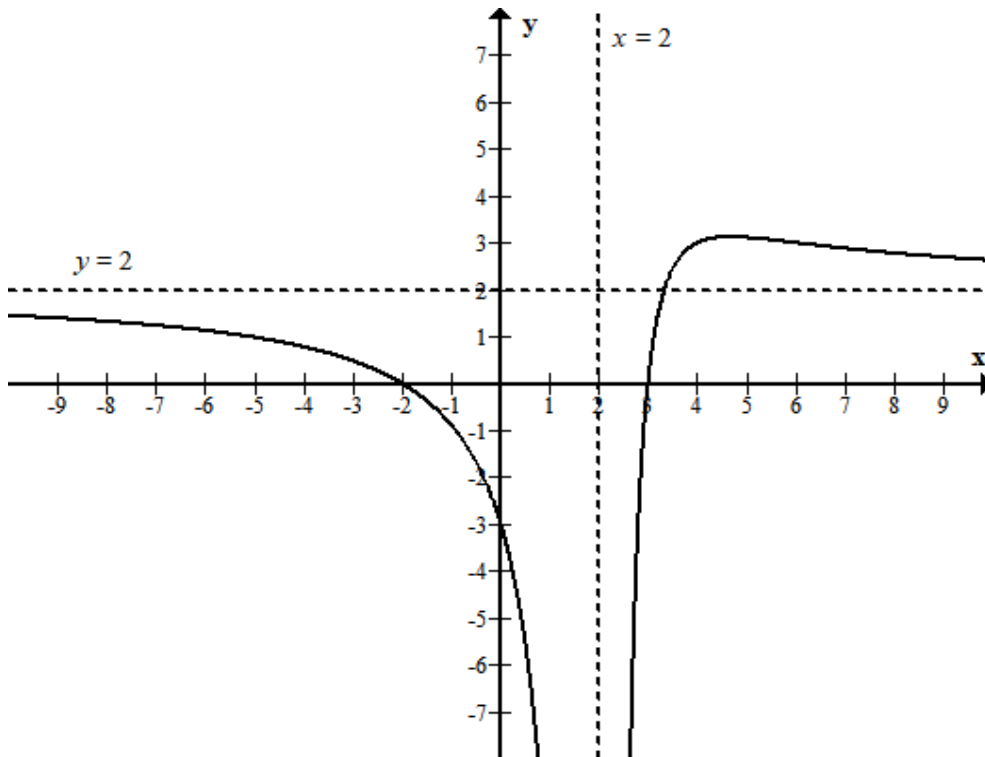
On a new diagram sketch the graph of $y = \dfrac{6}{f(x)}$. Indicate any asymptotes.
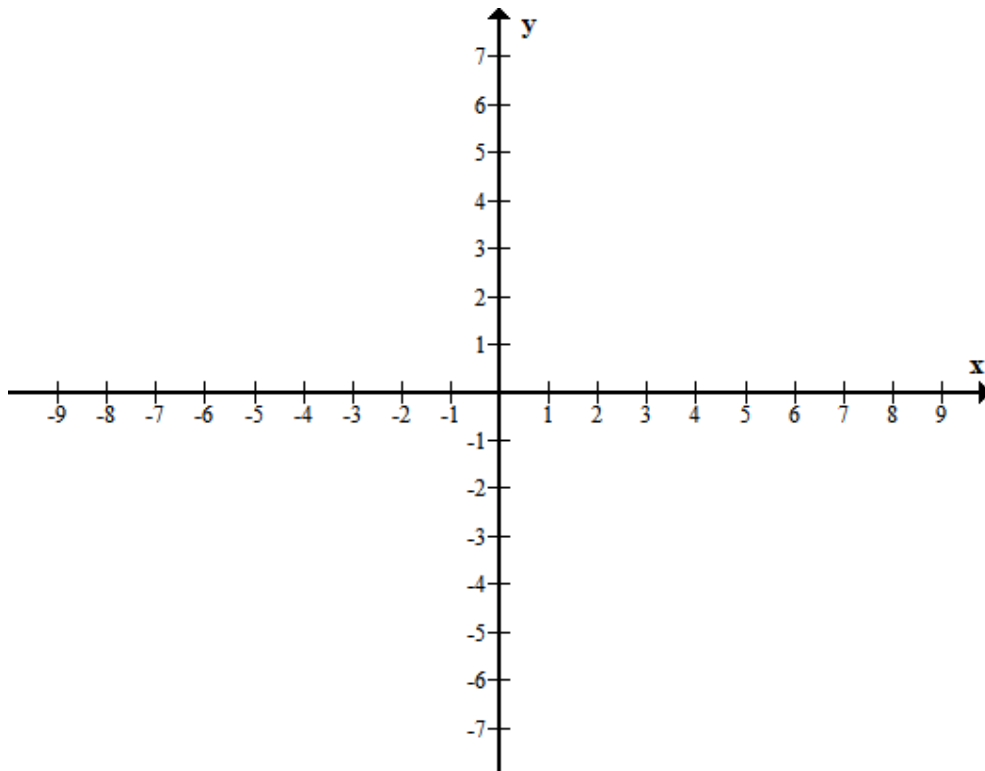
6. [Maximum mark: 5] *[without GDC]*

The graph of $y = f(x)$ is shown below. On the same diagram, sketch the graph of

$y = f(x)^2$

A.   Exam style questions (SHORT)

7.   [Maximum mark: 8]   *[without GDC]*

Determine whether the following function are **even**, **odd** or **neither**. Prove your claim.

(a) $f(x) = |x| - x$ [2]

(b) $f(x) = |x| - 3$ [2]

(c) $f(x) = |x - 3|$ [2]

(d) $f(x) = 3x|x| + \dfrac{1}{x}$ [2]

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8.   [Maximum mark: 6]   *[without GDC]*

Let $f(x) = \dfrac{k}{x-k}$, $x \neq k$, $k > 0$

(a)   On the diagram below, sketch the graph of f Label clearly any points of

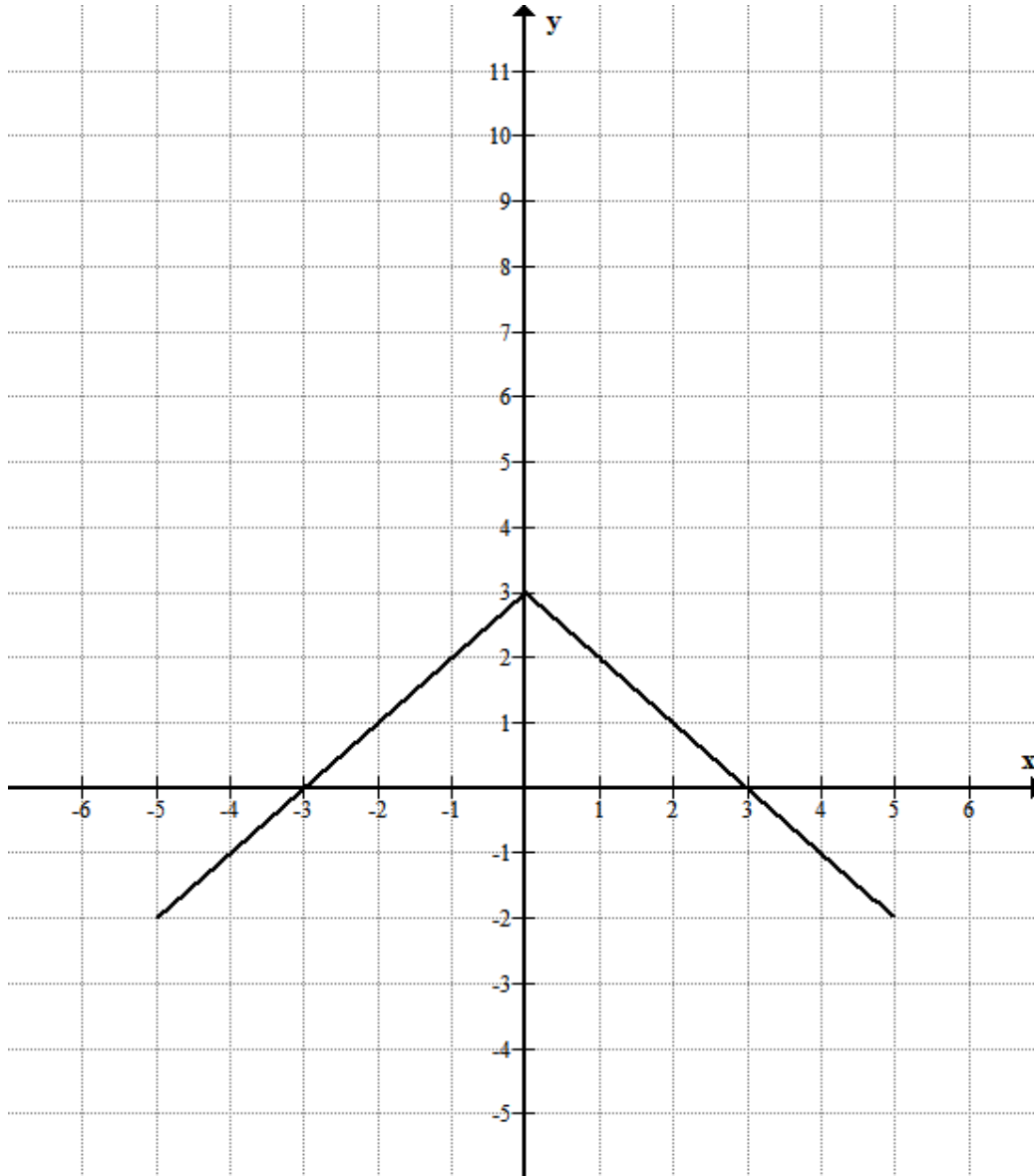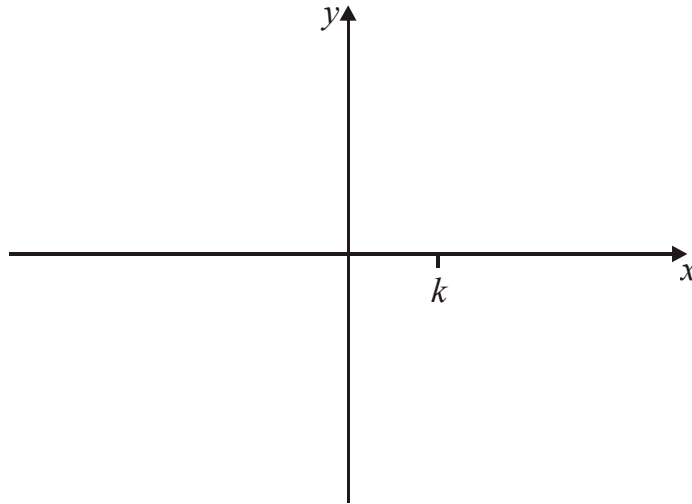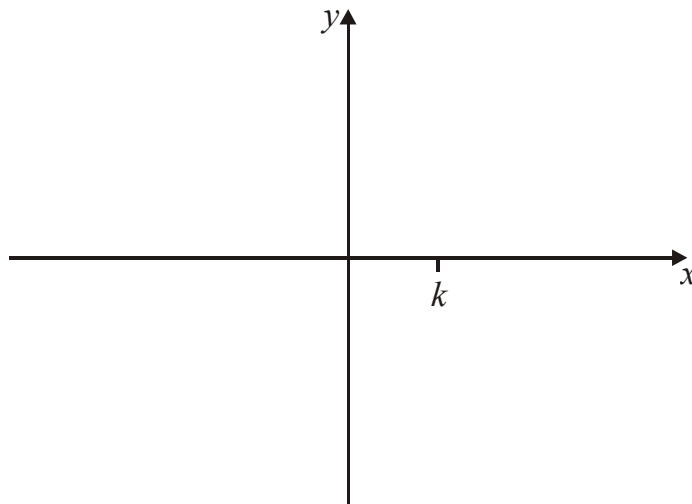intersection with the axes, and any asymptotes.

[3]

(b)   On the diagram below, sketch the graph of $\dfrac{1}{f}$. Label clearly any points of

intersection with the axes.

[3]

9.   [Maximum mark: 6]   *[without GDC]*

Each of the diagrams below shows the graph of a function f. Sketch on the given axes the graph of
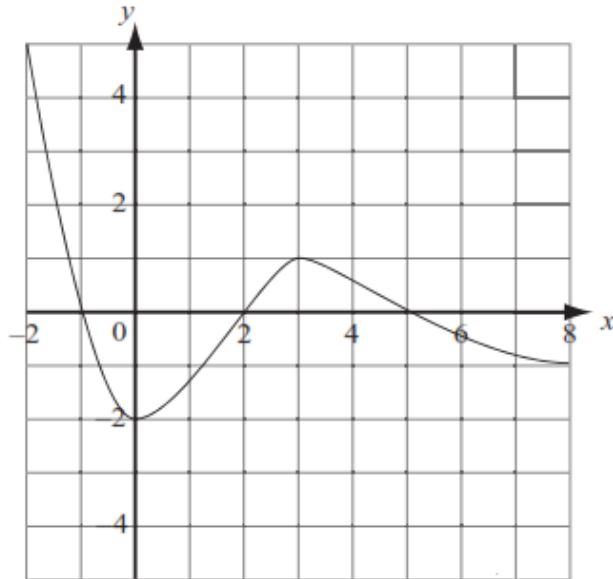
(a) $|f(-x)|$;
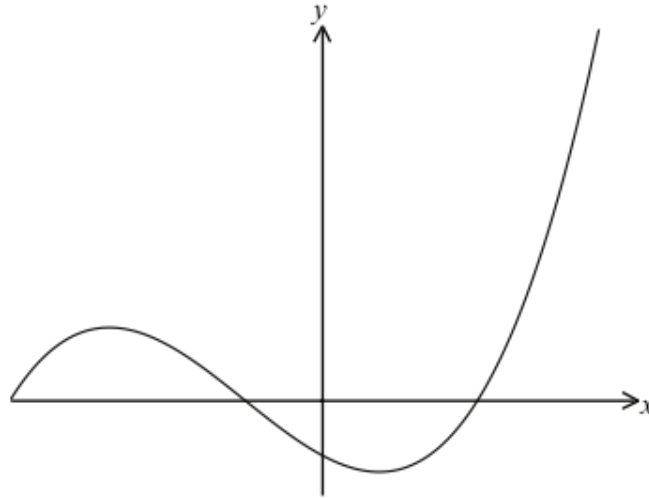
(b) $\dfrac{1}{f(x)}$;
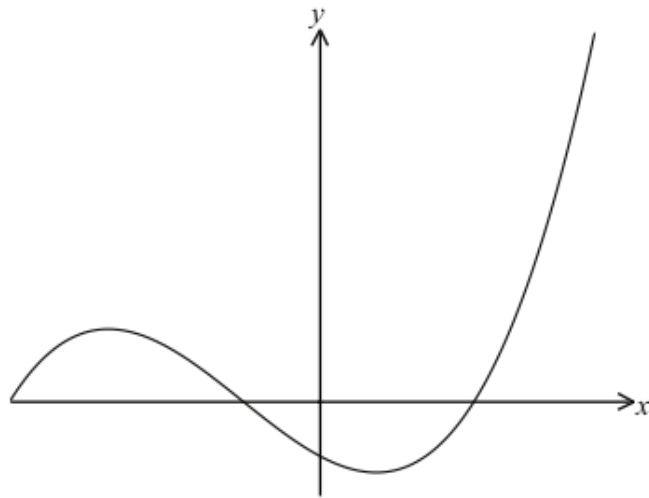
10. [Maximum mark: 6]   *[without GDC]*

The graph of $y = f(x)$ for $-2 \le x \le 8$ is shown.

On the set of axes provided, sketch the graph of $y = \dfrac{1}{f(x)}$, clearly showing any

asymptotes and indicating the coordinates of any local maxima or minima.
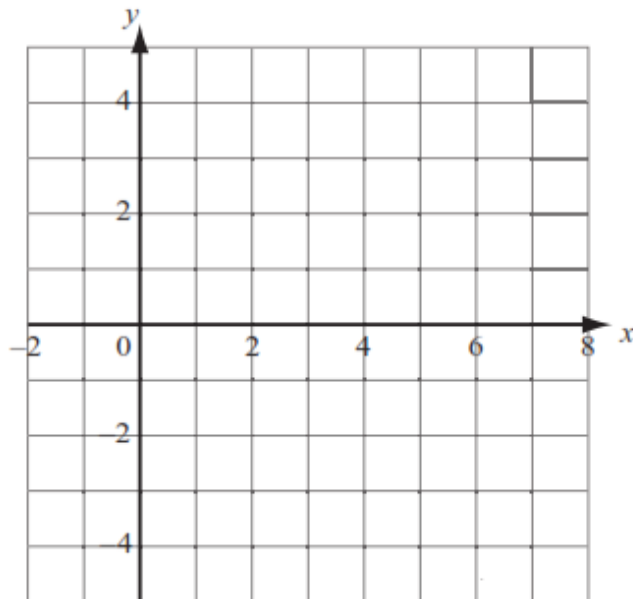
11. [Maximum mark: 8]   *[without GDC]*

The diagram shows the graph of $f(x)$

(a) On the same diagram, sketch the graph of $\dfrac{1}{f(x)}$, indicating clearly any

asymptotes. [3]
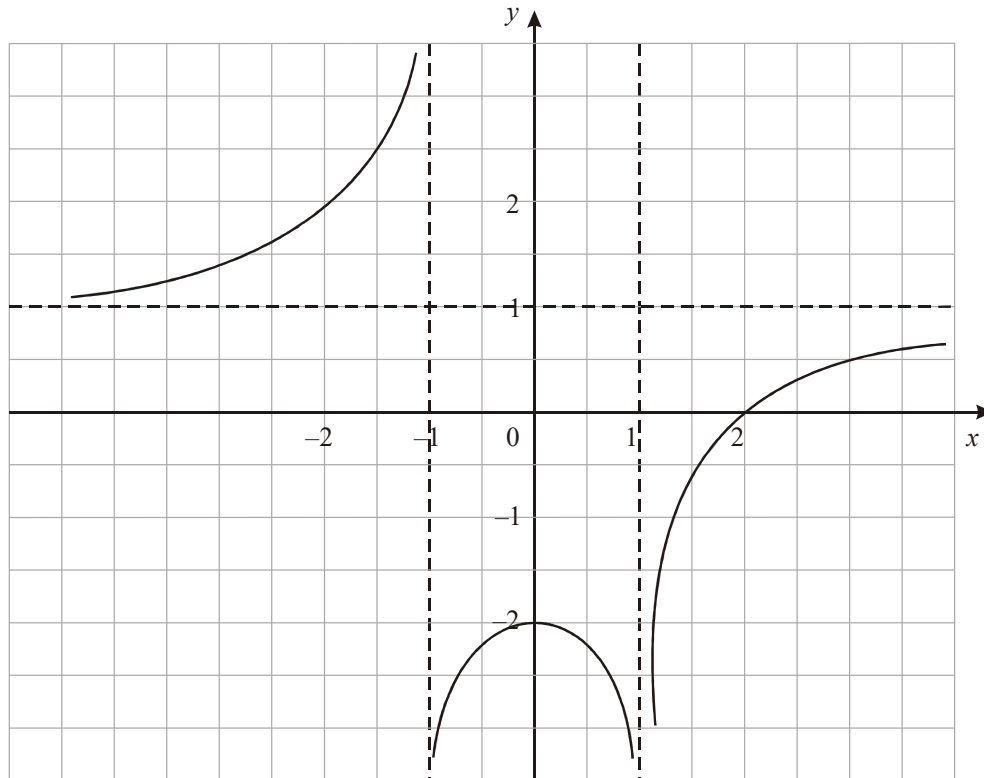
(b) On the diagram write down the coordinates of the local maximum point, the local minimum point, the x-intercepts and the y-intercept of $\dfrac{1}{f(x)}$. [3]

(c) The equation $f(x) = k$ has exactly one solution. Write down the possible values of k. [2]

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12. [Maximum mark: 6]   *[without GDC]*

The diagram shows the graph of $f(x)$.

(a)   On the same diagram, sketch the graph of $y = \dfrac{8}{f(x)}$ indicating any asymptotes.
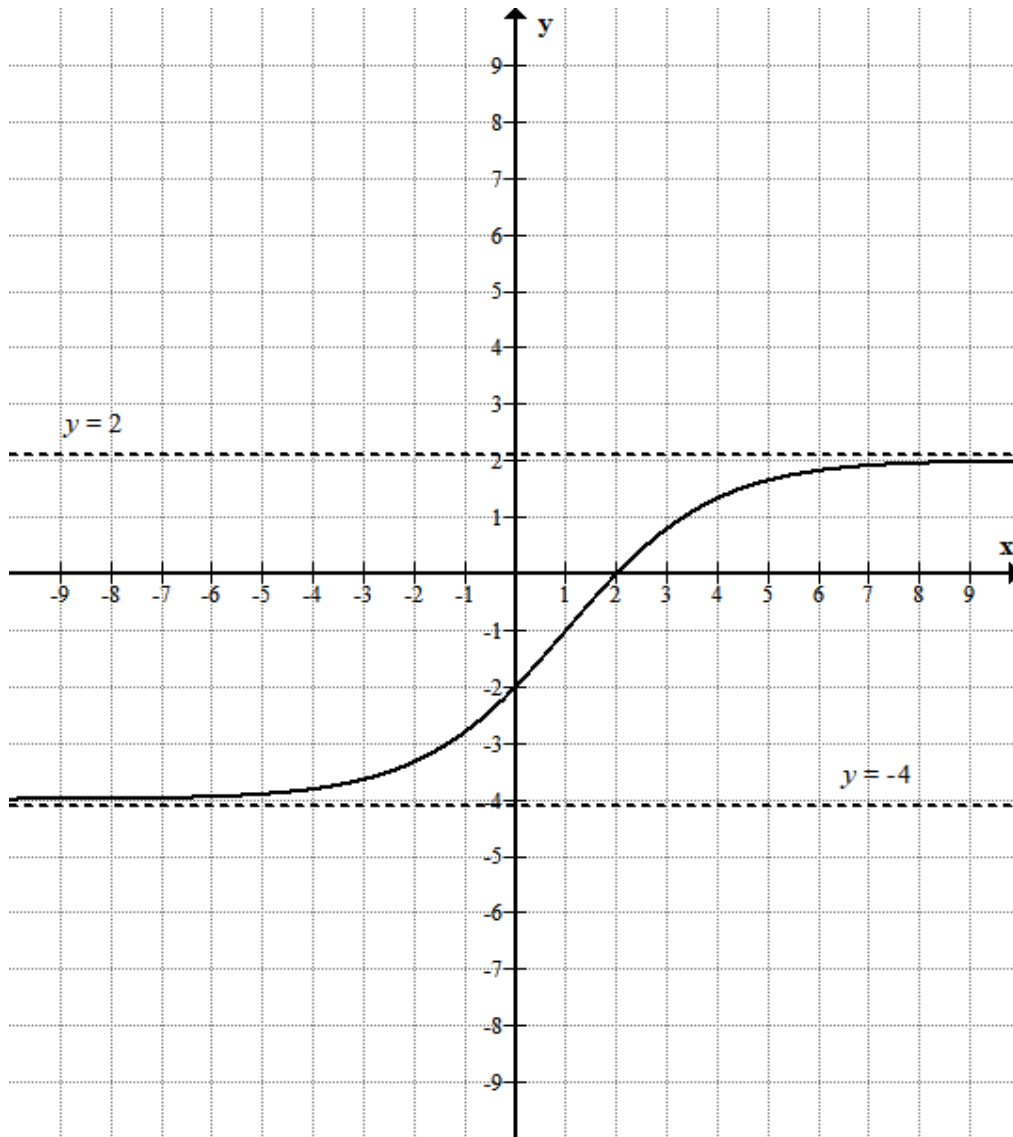
(b)   Write down the domain and the range

(i)   of $y = f(x)$ (ii)   of $y = \dfrac{8}{f(x)}$ [4]

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13. [Maximum mark: 7]   *[without GDC]*

The diagram shows the graph of $f(x)$. It has a maximum at A(1.5, 2), a vertical

asymptote at $x = 4$ and the y-intercept is at $(0, \frac{3}{2})$. Let $g(x) = \dfrac{1}{f(x)}$

(a) Write down the coordinates of

  (i) The y-intercept of $y = g(x)$ (ii) The local minimum of $y = g(x)$ [2]

(b) On the same diagram, sketch the graph of $y = g(x)$.   [5]

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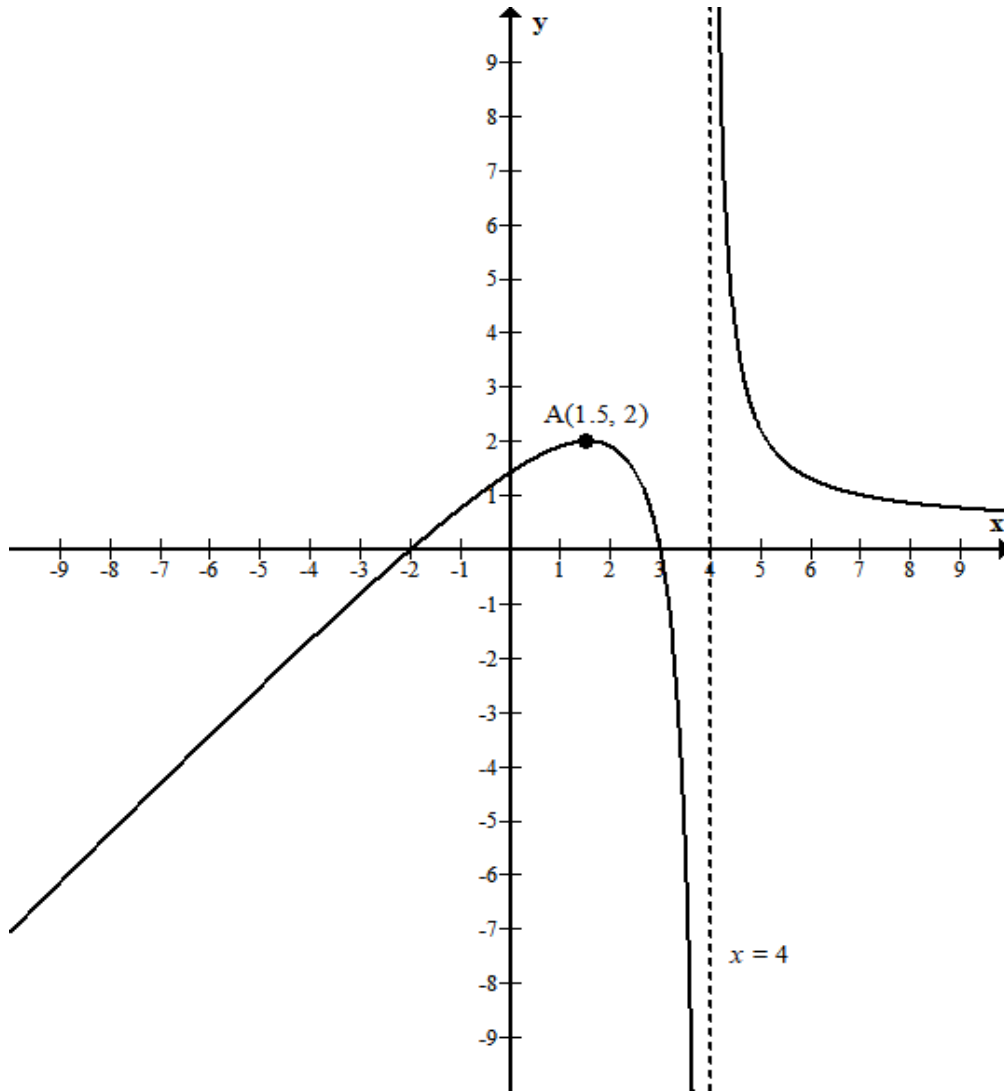
14*. [Maximum mark: 7]   *[without GDC]*

The graph of $y = f(x)$, where $-2 \le x \le 2$ is shown below.
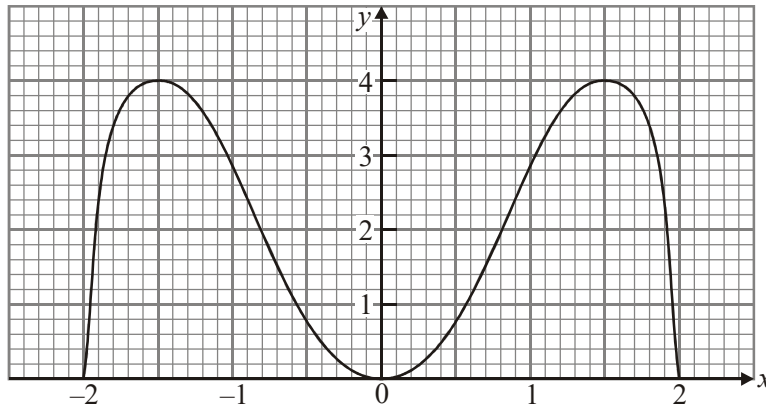
(a)   Write down the range of the following functions

Function	Range
$y = f(x)$	
$y = f(x)^2$	
$y = \sqrt{f(x)}$	

[3]

(b)   Sketch, on the axes provided below, the graph of $y^2 = f(x)$ for $-2 \le x \le 2$.
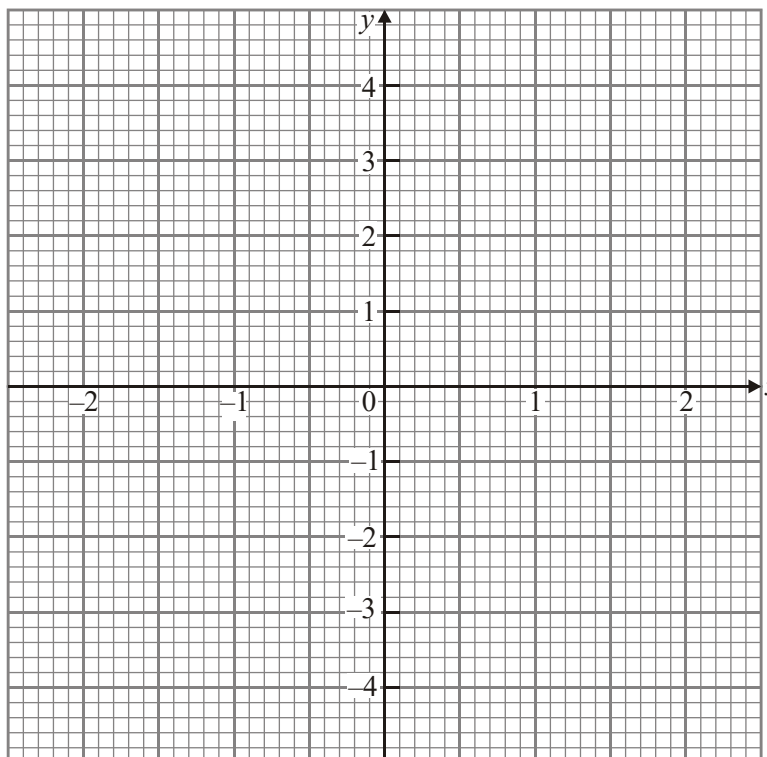
[4]

15*. [Maximum mark: 6]   *[without GDC]*

The diagram shows the graph of the functions y_1 and y_2.
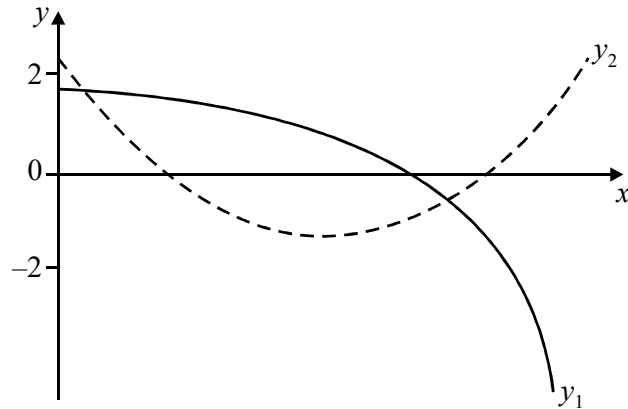
(a)   On the following diagram sketch the graph of $\dfrac{y_1}{y_2}$. Indicate clearly where the x-

intercepts and asymptotes occur.
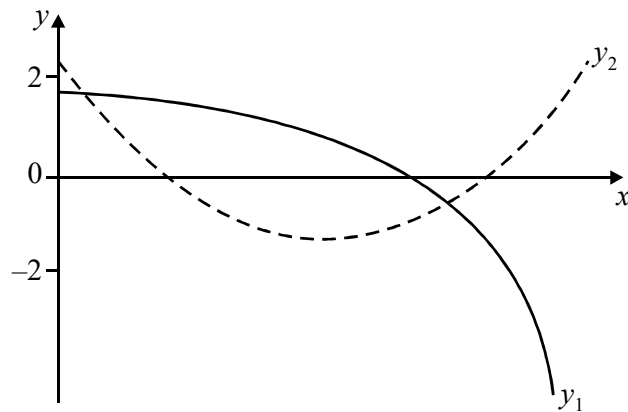
[4]

(b)   On the following diagram sketch the graph of $y_1 - y_2$. Indicate clearly where the
x-intercepts occur.

[2]

16. [Maximum mark: 7]   **[with GDC]**

Consider the graph of the function, f , defined by

$$f(x) = 3x^4 - 4x^3 - 30x^2 - 36x + 112, \quad -2 \le x \le 4.5$$

(a)   Given that $f(x) = 0$ has one solution at $x = 4$, find the other solution.   [1]

(b)   Find   (i) the coordinates of the minimum point   (ii) the range of f .   [2]

A sketch of the graph of $\dfrac{1}{f}$ is given below.

(c)   Write down the **equations** of the two vertical asymptotes.   [2]

(d)   The graph of $\dfrac{1}{f}$ has a maximum at P. Write down the *x*-coordinate of P.   [2]

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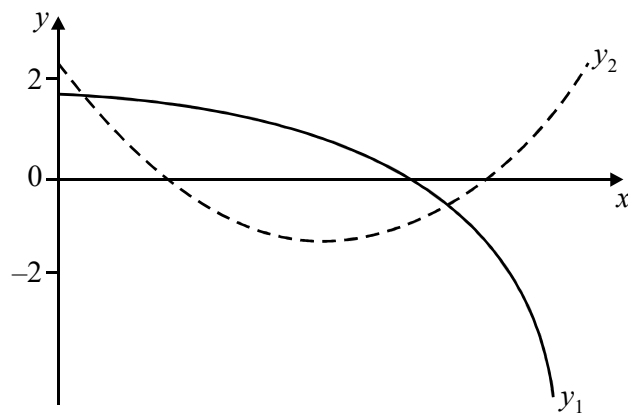..

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