If $r_4 = \frac{1}{2}$, then $r_3 = -\frac{1}{2} - \frac{3}{2} = -2$. $\left[\text{And if } r_4 = -2, \text{ then } r_3 = \frac{1}{2}\right]$
Therefore the other three zeros are $1 - i, \frac{1}{2}$ and -2.

Exercise 3.3

In questions 1–5, two polynomials P and D are given. Use either synthetic division or long division to divide $P(x)$ by $D(x)$, and express $P(x)$ in the form $P(x) = D(x)\cdot Q(x) + R(x)$.

1 $P(x) = 3x^2 + 5x - 5,\ D(x) = x + 3$

2 $P(x) = 3x^4 - 8x^3 + 9x + 5,\ D(x) = x - 2$

3 $P(x) = x^3 - 5x^2 + 3x - 7,\ D(x) = x - 4$

4 $P(x) = 9x^3 + 12x^2 - 5x + 1,\ D(x) = 3x - 1$

5 $P(x) = x^5 + x^4 - 8x^3 + x + 2,\ D(x) = x^2 + x - 7$

6 Given that $x - 1$ is a factor of the function $f(x) = 2x^3 - 17x^2 + 22x - 7$ factorize f completely.

7 Given that $2x + 1$ is a factor of the function $f(x) = 6x^3 - 5x^2 - 12x - 4$ factorize f completely.

8 Given that $x + \frac{2}{3}$ is a factor of the function $f(x) = 3x^4 + 2x^3 - 36x^2 + 24x + 32$ factorize f completely.

In questions 9–12, find the quotient and the remainder.

9 $\dfrac{x^2 - 5x + 4}{x - 3}$

10 $\dfrac{x^3 + 2x^2 + 2x + 1}{x + 2}$

11 $\dfrac{9x^2 - x + 5}{3x^2 - 7x}$

12 $\dfrac{x^5 + 3x^3 - 6}{x - 1}$

In questions 13–16, use synthetic division and the remainder theorem to evaluate $P(c)$.

13 $P(x) = 2x^3 - 3x^2 + 4x - 7, c = 2$

14 $P(x) = x^5 - 2x^4 + 3x^2 + 20x + 3, c = -1$

15 $P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14, c = -7$

16 $P(x) = x^3 - x + 1, c = \frac{1}{4}$

17 Given that $x = -6$ is a zero of the polynomial $x^3 + 2x^2 - 19x + 30$ find all remaining zeros of the polynomial.

18 Given that $x = 2$ is a double root of the polynomial $x^4 - 5x^3 + 7x^2 - 4$ find all remaining zeros of the polynomial.

19 Find the values of k such that -3 is a zero of $f(x) = x^3 - x^2 - k^2x$.

20 Find the values of a and b such that 1 and 4 are zeros of $f(x) = 2x^4 - 5x^3 - 14x^2 + ax + b$.

In questions 21–23, find a polynomial with real coefficients satisfying the given conditions.

21 Degree of 3; and zeros of -2, 1 and 4

22 Degree of 4; and zeros of -1, 3 (multiplicity of 2) and -2

23 Degree of 3; and 2 is the only zero (multiplicity of 3)

In questions 24–26, find a polynomial of lowest degree with real coefficients and the given zeros.

24 $x = -1$ and $x = 1 - i$

25 $x = 2, x = -4$ and $x = -3i$

26 $x = 3 + i$ and $x = 1 - 2i$

27 Given that $x = 2 - 3i$ is a zero of $f(x) = x^3 - 7x^2 + 25x - 39$ find the other remaining zeros.

28 The polynomial $6x^3 + 7x^2 + ax + b$ has a remainder of 72 when divided by $x - 2$ and is exactly divisible (i.e. remainder is zero) by $x + 1$.

a) Calculate a and b.
b) Show that $2x - 1$ is also a factor of the polynomial and, hence, find the third factor.

29 The polynomial $p(x) = (ax + b)^3$ leaves a remainder of -1 when divided by $x + 1$, and a remainder of 27 when divided by $x - 2$. Find the values of the real numbers a and b.

30 The quadratic polynomial $x^2 - 2x - 3$ is a factor of the quartic polynomial function $f(x) = 4x^4 - 6x^3 - 15x^2 - 8x - 3$. Find all of the zeros of the function f. Express the zeros exactly and completely simplified.

31 $x - 2$ and $x + 2$ are factors of $x^3 + ax^2 + bx + c$, and it leaves a remainder of 10 when divided by $x - 3$. Find the values of a, b and c.

32 Let $P(x) = x^3 + px^2 + qx + r$. Two of the zeros of $P(x) = 0$ are 3 and $1 + 4i$. Find the value of p, q and r.

33 When divided by $(x + 2)$ the expression $5x^3 - 3x^2 + ax + 7$ leaves a remainder of R. When the expression $4x^3 + ax^2 + 7x - 4$ is divided by $(x + 2)$ there is a remainder of $2R$. Find the value of the constant a.

34 The polynomial $x^3 + mx^2 + nx - 8$ is divisible by $(x + 1 + i)$. Find the value of m and n.

35 Given that the roots of the equation $x^3 - 9x^2 + bx - 216 = 0$ are consecutive terms in a geometric sequence, find the value of b and solve the equation.

36 a) Prove that when a polynomial $P(x)$ is divided by $ax - b$ the remainder is $P\left(\frac{b}{a}\right)$.
b) Hence, find the remainder when $9x^3 - x + 5$ is divided by $3x + 2$.

37 Find the sum and product of the roots of the following equations.

a) $x^4 - \frac{2}{3}x^3 + 3x^2 - 2x + 5 = 0$

b) $(x - 2)^3 = x^4 - 1$

c) $\frac{3}{x^2 + 2} = \frac{2x^2 - x}{2x^5 + 1}$

38 If α, β and γ are the three roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, show that $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$.

39 One of the zeros of the equation $x^3 - 63x + 162 = 0$ is double another zero. Find all three zeros.

40 Find the three zeros of the equation $x^3 - 6x^2 - 24x + 64 = 0$ given that they are consecutive terms in a geometric sequence. [Hint: let the zeros be represented by $\frac{\alpha}{r}$, α, αr where r is the common ratio.]

41 Consider the equation $x^5 - 12x^4 + 62x^3 - 166x^2 + 229x - 130 = 0$.
Given that two of the zeros of the equation are $x = 3 - 2i$ and $x = 2$, find the remaining three zeros.

42 Find the value of k such that the zeros of the equation $x^3 - 6x^2 + kx + 10 = 0$ are in arithmetic progression, that is, they can be represented by α, $\alpha + d$ and $\alpha + 2d$ for some constant d. [Hint: use the result from question 38.]

43 Find the value of k if the roots of the equation $x^3 + 3x^2 - 6x + k = 0$ are in geometric progression.

40 a) sum $= -3$, product $= -\frac{5}{2}$

b) sum $= -3$, product $= -1$

c) sum $= 0$, product $= -\frac{3}{2}$

d) sum $= a$, product $= -2a$

e) sum $= 6$, product $= -4$

f) sum $= \frac{1}{3}$, product $= -\frac{2}{3}$

41 $4x^2 + 5x + 4 = 0$

42 a) $\frac{1}{9}$ b) $\frac{1}{12}$ c) $\frac{55}{27}$

43 a) -2 and -6 b) $k = 12$

44 a) $-\frac{1}{4}$ b) $4x^2 + x + 1 = 0$

45 a) $x^2 - 19x + 25 = 0$ b) $25x^2 + 72x - 5 = 0$

Exercise 3.3

1 $3x^2 + 5x - 5 = (x + 3)(3x - 4) + 7$

2 $3x^4 - 8x^3 + 9x + 5 = (x - 2)(3x^3 - 2x^2 - 4x + 1) + 7$

3 $x^3 - 5x^2 + 3x - 7 = (x - 4)(x^2 - x - 1) - 11$

4 $9x^3 + 12x^2 - 5x + 1 = (3x - 1)(3x^2 + 5x) + 1$

5 $x^5 + x^4 - 8x^3 + x + 2 = (x^2 + x - 7)(x^3 - x + 1) + (-7x + 9)$

6 $(x - 7)(x - 1)(2x - 1)$ **7** $(x - 2)(2x + 1)(3x + 2)$

8 $(x - 2)^2(x + 4)(3x + 2)$ **9** $Q(x) = x - 2$, $R = -2$

10 $Q(x) = x^2 + 2$, $R = -3$ **11** $Q(x) = 3$, $R(x) = 20x + 5$

12 $Q(x) = x^4 + x^3 + 4x^2 + 4x + 4$, $R = -2$

13 $P(2) = 5$ **14** $P(-1) = -17$

15 $P(-7) = -483$ **16** $P\left(\frac{1}{4}\right) = \frac{49}{64}$

17 $x = 2 + i$ or $x = 2 - i$ **18** $x = \frac{1 + \sqrt{5}}{2}$ or $x = \frac{1 - \sqrt{5}}{2}$

19 $k = \sqrt{1 - x}\sqrt{3}$ or $k = -\sqrt{1 - x}\sqrt{3}$

20 $a = 5$, $b = 12$

21 $x^3 - 3x^2 - 6x + 8$ **22** $x^4 - 3x^3 - 7x^2 + 15x + 18$

23 $x^3 - 6x^2 + 12x - 8$ **24** $x^3 - x^2 + 2$

25 $x^4 + 2x^3 + x^2 + 18x - 72$ **26** $x^4 - 8x^3 + 27x^2 - 50x + 50$

27 $x = 2 + 3i$, $x = 3$

28 a) $a = -1$, $b = -2$ b) $3x + 2$

29 $a = \frac{4}{3}$, $b = \frac{1}{3}$

30 $x = 3$, $x = -1$, $x = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$, $x = -\frac{1}{4} - \frac{\sqrt{3}}{4}i$

31 $a = -1$, $b = -4$, $c = 4$ **32** $p = -5$, $q = 23$, $r = -51$

33 $a = -5$ **34** $m = -2$, $n = -6$

35 $b = 18$ **36** b) $R = 3$

37 a) sum $= \frac{2}{3}$, product $= 5$ b) sum $= 1$, product $= 7$

c) sum $= \frac{1}{3}$, product $= -\frac{1}{2}$

39 $-9, 3, 6$

40 $2, -4, 8$

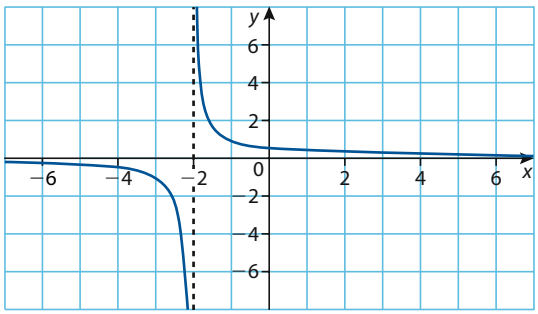
41 $3 + 2i, 2 + i, 2 - i$

42 $k = 3$

43 $k = -8$

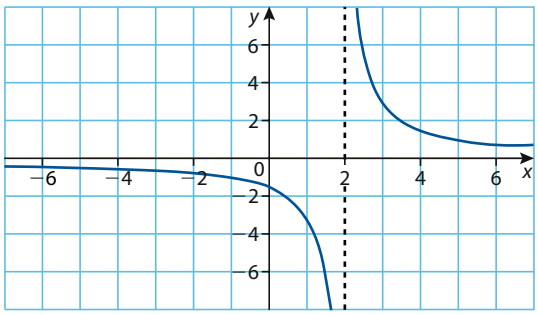
Exercise 3.4

1



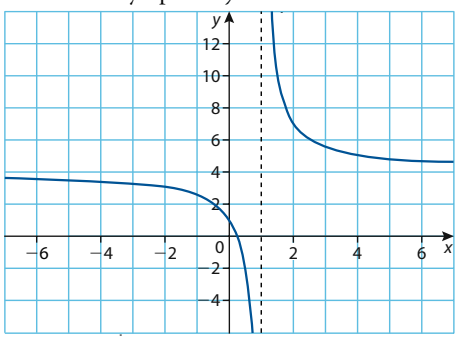
vertical asymptote: $x = -2$
horizontal asymptote: $y = 0$

2



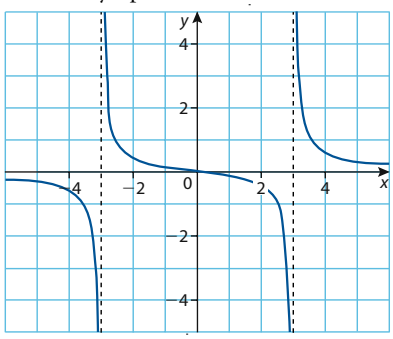
vertical asymptote: $x = 2$
horizontal asymptote: $y = 0$

3



x-intercept: $\left(\frac{1}{4}, 0\right)$, y-intercept: $(0, 1)$
vertical asymptote: $x = 1$ horizontal asymptote: $y = 4$

4



x- and y-intercept: $(0, 0)$
vertical asymptotes: $x = -3$, $x = 3$
horizontal asymptote: $y = 0$