1.

METHOD 1

$z = (2 - \mathrm{i})(z + 2)$ M1

$\quad = 2z + 4 - \mathrm{i}z - 2\mathrm{i}$

$z(1 - \mathrm{i}) = -4 + 2\mathrm{i}$

$z = \dfrac{-4 + 2\mathrm{i}}{1 - \mathrm{i}}$ A1

$z = \dfrac{-4 + 2\mathrm{i}}{1 - \mathrm{i}} \times \dfrac{1 + \mathrm{i}}{1 + \mathrm{i}}$ M1

$= -3 - \mathrm{i}$ A1

METHOD 2

let $z = a + \mathrm{i}b$

$\dfrac{a + \mathrm{i}b}{a + \mathrm{i}b + 2} = 2 - \mathrm{i}$ M1

$a + \mathrm{i}b = (2 - \mathrm{i})((a + 2) + \mathrm{i}b)$

$a + \mathrm{i}b = 2(a + 2) + 2b\mathrm{i} - \mathrm{i}(a + 2) + b$

$a + \mathrm{i}b = 2a + b + 4 + (2b - a - 2)\mathrm{i}$

attempt to equate real and imaginary parts M1

$a = 2a + b + 4 (\Rightarrow a + b + 4 = 0)$

and $b = 2b - a - 2 (\Rightarrow -a + b - 2 = 0)$ A1

Note: Award A1 for two correct equations.

$b = -1; a = -3$

$z = -3 - \mathrm{i}$ A1

[4]

2.

$a^2 + 2\mathrm{i}ab - b^2 = 3 + 4\mathrm{i}$

Equate real and imaginary parts (M1)

$a^2 - b^2 = 3,\ 2ab = 4$ A1

Since $b = \dfrac{2}{a}$

$\Rightarrow a^2 - \dfrac{4}{a^2} = 3$ (M1)

$\Rightarrow a^4 - 3a^2 - 4 = 0$ A1

Using factorisation or the quadratic formula (M1)

$\Rightarrow a = \pm 2$

$\Rightarrow b = \pm 1$

$\Rightarrow \sqrt{3 + 4\mathrm{i}} = 2 + \mathrm{i}, -2 - \mathrm{i}$ A1A1

[7]

3.

$iz_1 + 2z_2 = 3 \Rightarrow z_2 = -\frac{1}{2}iz_1 + \frac{3}{2}$

$z_1 + (1-i)z_2 = 4$

$\Rightarrow z_1 + (1-i)\left(-\frac{1}{2}iz_1 + \frac{3}{2}\right) = 4$ M1A1

$\Rightarrow z_1 - \frac{1}{2}iz_1 + \frac{3}{2} + \frac{1}{2}i^2 z_1 - \frac{3}{2}i = 4$

$\Rightarrow \frac{1}{2}z_1 - \frac{1}{2}iz_1 = \frac{5}{2} + \frac{3}{2}i$

$\Rightarrow z_1 - iz_1 = 5 + 3i$ A1

EITHER

Let $z_1 = x + iy$ (M1)

$\Rightarrow x + iy - ix - i^2 y = 5 + 3i$

Equate real and imaginary parts M1

$\Rightarrow x + y = 5$

$\underline{-x + y = 3}$

$2y = 8$

$y = 4 \Rightarrow x = 1$ i.e. $z_1 = 1 + 4i$ A1A1

$z_2 = -\frac{1}{2}i(1+4i) + \frac{3}{2}$ M1

$z_2 = -\frac{1}{2}i - 2i^2 + \frac{3}{2}$

$z_2 = \frac{7}{2} - \frac{1}{2}i$ A1

OR

$z_1 = \frac{5+3i}{1-i}$ M1

$z_1 = \frac{(5+3i)(1+i)}{(1-i)(1+i)} \left(= \frac{5+8i-3}{2}\right)$ M1A1

$z_1 = 1 + 4i$ A1

$z_2 = -\frac{1}{2}i(1+4i) + \frac{3}{2}$ M1

$z_2 = -\frac{1}{2}i - 2i^2 + \frac{3}{2}$

$z_2 = \frac{7}{2} - \frac{1}{2}i$ A1

[9]

4.

METHOD 1

$20 + 10b\text{i} = (1 - b\text{i})(-7 + 9\text{i})$	(M1)
$20 + 10b\text{i} = (-7 + 9b) + (9 + 7b)\text{i}$	A1A1
Equate real and imaginary parts	(M1)

EITHER

$-7 + 9b = 20$

$b = 3$ (M1)A1

OR

$10b = 9 + 7b$

$3b = 9$

$b = 3$ (M1)A1

METHOD 2

$= \dfrac{(2+b\text{i})(1+b\text{i})}{(1-b\text{i})(1+b\text{i})} = \dfrac{-7+9\text{i}}{10}$ (M1)

$\dfrac{2-b^2+3b\text{i}}{1+b^2} = \dfrac{-7+9\text{i}}{10}$ A1

Equate real and imaginary parts (M1)

$\dfrac{2-b^2}{1+b^2} = -\dfrac{7}{10}$ Equation A

$\dfrac{3b}{1+b^2} = \dfrac{9}{10}$ Equation B

From equation A

$20 - 10b^2 = -7 - 7b^2$

$3b^2 = 27$

$b = \pm 3$ A1

From equation B

$30b = 9 + 9b^2$

$3b^2 - 10b + 3 = 0$

By factorisation or using the quadratic formula

$b = \dfrac{1}{3}$ or 3 A1

Since 3 is the common solution to both equations $b = 3$ R1

[6]