

# Mathematics mind map

## Analysis and approaches SL

# Mathematics mind map

## Analysis and approaches AHL

## Analysis and approaches SL

## Common content

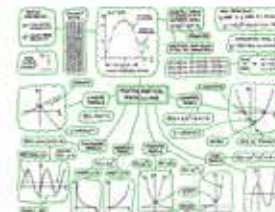
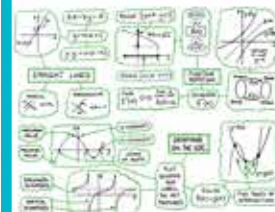
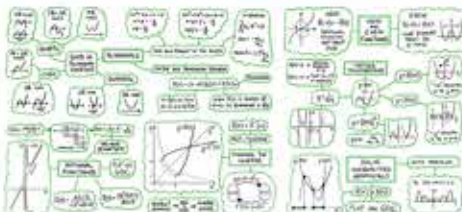
## Applications and interpretation SL

## Applications and interpretation AHL

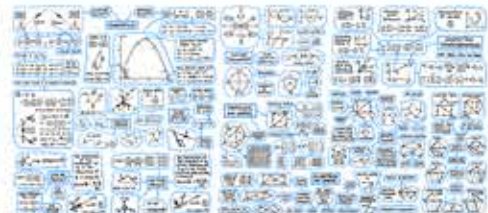
### Number and algebra



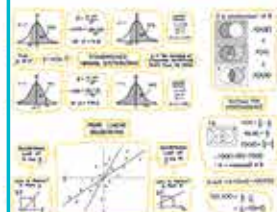
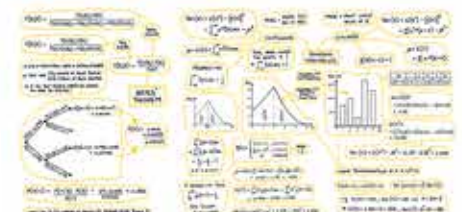
### Functions



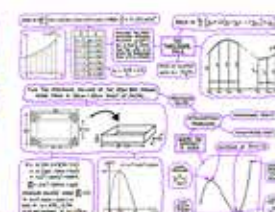
### Geometry and trigonometry



### Statistics and probability



### Calculus



$u_1 = 27$   
 $r = \frac{2}{3}$   
 $S_\infty = 27 + 18 + 12 + 8 + \dots = \frac{27}{1 - \frac{2}{3}} = 81$

**INFINITE GEOMETRIC SERIES**

ONLY CONVERGES FOR  $-1 < r < 1$

$S_\infty = u_1 + u_1 r + u_1 r^2 + \dots = \frac{u_1}{1-r}$

e.g.  $16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$

e.g.  $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} = \frac{3}{2}$

**RATIONAL EXPONENTS**  
 $a^{\frac{1}{m}} = \sqrt[m]{a} \Rightarrow a^{\frac{n}{m}} = (\sqrt[m]{a})^n = \sqrt[m]{a^n}$

**CHANGE OF BASE**  
 $\log_b a = \frac{\log_c a}{\log_c b}$

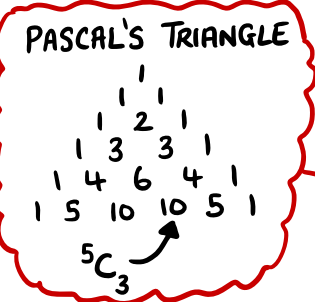
**EQUATIONS**  
e.g.  $(\frac{1}{3})^x = 9^{x+1}$   
 $(3^{-1})^x = (3^2)^{x+1}$   
 $3^{-x} = 3^{2x+2}$   
 $-x = 2x+2$   
 $-2 = 3x \Rightarrow x = -\frac{2}{3}$

**MORE EXPONENTIAL AND LOGARITHM**

e.g.  $\log 24 = \log 8 + \log 3$   
 $\log_3 \frac{10}{4} = \log_3 10 - \log_3 4$   
 $\log_4 3^5 = 5 \log_4 3$

**LAWS OF LOGS**  
 $\log_a x + \log_a y = \log_a xy$   
 $\log_a x - \log_a y = \log_a \frac{x}{y}$   
 $\log_a x^n = n \log_a x$

CONSTANT TERM =  ${}^9C_3 (x^2)^3 (\frac{-3}{x})^6 = 84 \cancel{x^6} \cdot \frac{3^6}{\cancel{x^6}} = 84 \times 3^6 = 61,236$



$n! = n(n-1)(n-2)\dots 3.2.1$

**BINOMIAL THEOREM**  
 $n \in \mathbb{Z}^+$

FIND THE CONSTANT TERM  
 $(x^2 - \frac{3}{x})^9$

${}^n C_r = \frac{n!}{r!(n-r)!}$

$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n$

e.g.  $(x-2)^5 = (x)^5 + 5(x)^4(-2)^1 + 10(x)^3(-2)^2 + 10(x)^2(-2)^3 + 5(x)^1(-2)^4 + (-2)^5$   
 $= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

SHOW THAT  $\frac{1}{m+1} + \frac{1}{m^2+m} = \frac{1}{m}$

LHS  $\equiv \frac{1}{m+1} + \frac{1}{m^2+m}$   
 $\equiv \frac{1}{m+1} + \frac{1}{m(m+1)}$   
 $\equiv \frac{m+1}{m(m+1)}$   
 $\equiv \frac{1}{m}$   
 $\equiv$  RHS

**IDENTITY**  
 $x(x+1) \equiv x^2+x$   
TRUE FOR ALL  $x$

**DEDUCTIVE PROOF**

**EQUALITY**  
 $x^2 + 3x - 10 = 0$   
TRUE FOR SOME  $x$

$\Delta > 0$   
TWO DISTINCT REAL ROOTS

$\Delta = 0$   
A REPEATED REAL ROOT

$\Delta < 0$   
NO REAL ROOTS

$3kx^2 + 2x + k = 0 \Rightarrow \Delta = 2^2 - 4(3k)(k) = 4 - 12k^2$

TWO DISTINCT REAL ROOTS  
 $4 - 12k^2 > 0$   
 $4 > 12k^2$   
 $\frac{1}{3} > k^2$

$-\frac{1}{\sqrt{3}} < k < \frac{1}{\sqrt{3}}$

DISCRIMINANT  
 $\Delta = b^2 - 4ac$

QUADRATIC INEQUALITIES

QUADRATICS

$y = ax^2 + bx + c$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

"QUADRATICS IN HIDING"  
eg  $e^{2x} - 5e^x + 4 = 0$   
 $(e^x - 4)(e^x - 1) = 0$   
 $e^x = 4, e^x = 1$   
 $x = \ln 4, x = 0$

$a > 0$

$a < 0$

x-INTERCEPTS  
(p, 0) AND (q, 0)

VERTEX (h, k)

$y = a(x-h)^2 + k$

y-INTERCEPT  
 $\Rightarrow x = 0$

AXIS OF SYMMETRY  
 $x = \frac{-b}{2a}$

HORIZONTAL ASYMPTOTE  
as  $x \rightarrow \infty, y = \frac{4}{2} = 2$

eg SKETCH  
 $y = \frac{4x+7}{2x+3}$

VERTICAL ASYMPTOTE  
 $2x+3=0$   
 $2x=-3$   
 $x = -\frac{3}{2}$

$x=0 \Rightarrow y = \frac{7}{3}$

$y=0$   
 $4x=-7$   
 $x = -\frac{7}{4}$

RATIONAL FUNCTIONS

RECIPROCAL FUNCTION  
 $y = \frac{1}{x}, x \neq 0$

g f

$f \circ g(x) = f(g(x))$

IDENTITY FUNCTION  
 $f^{-1} \circ f(x) = f \circ f^{-1}(x) = x$

COMPOSITE AND INVERSE

$f(x) = \frac{3x+2}{x-5} \Rightarrow$  LET  $x = \frac{3y+2}{y-5}$

$x(y-5) = 3y+2$   
 $xy-5x = 3y+2$   
 $xy-3y = 5x+2$   
 $y(x-3) = 5x+2$   
 $y = \frac{5x+2}{x-3}$

$f^{-1}(x) = \frac{5x+2}{x-3}$

ONE-TO-ONE  
 $\Rightarrow f^{-1}$  EXISTS

MANY-TO-ONE  
 $\Rightarrow$  NO  $f^{-1}$

$a f(x)$

$-f(x)$

REFLECT IN x AXIS

$f(-x)$

REFLECT IN y AXIS

$f(x) + a$

$f(ax)$

TRANSFORMATIONS OF GRAPHS

COMPOSITE TRANSFORMATIONS

$y = x^2$   $y = 3x^2$   $y = 3x^2 + 2$

VERTICAL STRETCH SCALE FACTOR 3

VERTICAL TRANSLATION UP 2

$f(x+a)$

GRAPHS OF EXPONENTIALS AND LOGARITHMS

$x \in \mathbb{R}$   $a^x$   $x > 0$   $\log_a x$

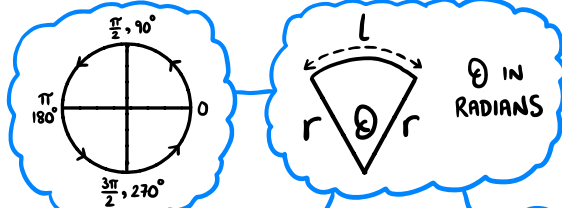
**2 sin x = 1, 0 ≤ x ≤ 2π**  
 $\sin x = \frac{1}{2}$   
 $x = \frac{\pi}{6}, \frac{5\pi}{6}$

**DOUBLE ANGLE**

$2\sin 2x = 3\cos x, 0 \leq x \leq 180^\circ$   
 $4\sin x \cos x = 3\cos x$   
 $4\sin x \cos x - 3\cos x = 0$   
 $\cos x (4\sin x - 3) = 0$   
 $\cos x = 0, \sin x = \frac{3}{4}$   
 $x = 90^\circ, x = 48.6^\circ$   
 $x = 131.4^\circ$

**DEGREES** ↔ **RADIANS**

$L = r\theta$      $A = \frac{1}{2}\theta r^2$

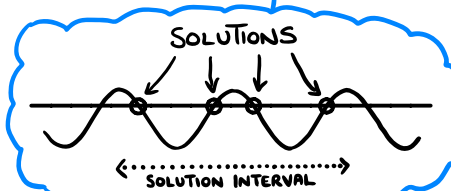


**EQUATIONS**

**TAN**

$\sqrt{3} \sin x = \cos x, 0 \leq x \leq 2\pi$   
 $\frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$   
 $\tan x = \frac{1}{\sqrt{3}}$   
 $x = \frac{\pi}{6}, \frac{7\pi}{6}$

**GRAPHICALLY**



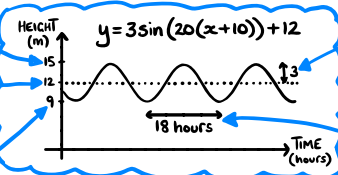
**APPLICATIONS e.g. TIDES**

$y = 3\sin(20(x+10)) + 12$

HEIGHT (m) vs TIME (hours)

AMPLITUDE = a  
 PRINCIPAL AXIS y = d  
 MINIMUM VALUE d - a  
 PERIOD =  $\frac{360^\circ}{b} = \frac{2\pi}{b}$

$y = a \sin(b(x+c)) + d$



**IDENTITIES**

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

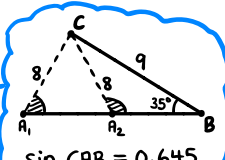
$\cos^2 \theta + \sin^2 \theta = 1$

$\cos^2 \theta = 1 - \sin^2 \theta$      $\sin^2 \theta = 1 - \cos^2 \theta$

$\sin 2\theta = 2\sin \theta \cos \theta$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta = 2\cos^2 \theta - 1$

**SINE RULE AMBIGUOUS CASE**

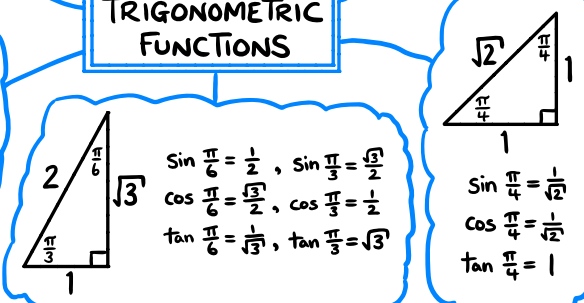


$\sin \theta = \frac{3}{4} \Rightarrow \theta \text{ IS OBTUSE} \Rightarrow \cos \theta = -\frac{\sqrt{7}}{4}$   
 FIND  $\cos \theta$      $\tan \theta = -\frac{3}{\sqrt{7}}$

**TRIGONOMETRIC FUNCTIONS**

$y = \sin x$      $y = \cos x$      $y = \tan x$

$\sin \frac{\pi}{6} = \frac{1}{2}, \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$   
 $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \cos \frac{\pi}{3} = \frac{1}{2}$   
 $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}, \tan \frac{\pi}{3} = \sqrt{3}$



$\sigma = ?$   
75%

$$Z = \frac{X - \mu}{\sigma}$$

$$-0.674 = \frac{130 - 140}{\sigma}$$

$$\Rightarrow \sigma = 14.8$$

$\sigma = 1$   
75%

INVERSE NORMAL  
 $p = 0.25$   
 $\mu = 0$   
 $\sigma = 1$   
 $\Rightarrow Z = -0.674$

FIND  $\mu$  OR  $\sigma$     
  $Z \sim N(0,1)$     
 STANDARDISED NORMAL DISTRIBUTION    
 Z IS THE NUMBER OF STANDARD DEVIATIONS AWAY FROM THE MEAN

$\sigma = 17$   
90%

$$Z = \frac{X - \mu}{\sigma}$$

$$1.282 = \frac{170 - \mu}{17}$$

$$\Rightarrow \mu = 148.2$$

$\sigma = 1$   
90%

INVERSE NORMAL  
 $p = 0.9$   
 $\mu = 0$   
 $\sigma = 1$   
 $\Rightarrow Z = 1.282$

MORE LINEAR REGRESSION

REGRESSION LINE OF  $x$  ON  $y$

REGRESSION LINE OF  $y$  ON  $x$

USED TO PREDICT  $x$  FROM  $y$

USED TO PREDICT  $y$  FROM  $x$

A IS INDEPENDENT OF B

$P(A/B')$

||

$P(A)$

||

$P(A/B)$

TESTING FOR INDEPENDENCE

e.g.

$P(A) = \frac{8}{12} = \frac{2}{3}$   
 $P(A/B) = \frac{2}{3}$   
 $P(A/B') = \frac{6}{9} = \frac{2}{3}$

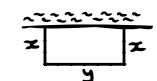
∴  $P(A/B') = P(A) = P(A/B)$   
 ∴ A IS INDEPENDENT OF B

A IND. B  $\Leftrightarrow P(A \cap B) = P(A) \cdot P(B)$

$$P(A) \cdot P(B) = \frac{2}{3} \cdot \frac{1}{4}$$


$$= \frac{2}{12} = P(A \cap B)$$

**A RECTANGULAR FIELD IS BUILT NEXT TO A RIVER USING 120M OF FENCING. FIND THE MAX. AREA.**



LENGTH =  $2x + y = 120$   
 $\Rightarrow y = 120 - 2x$   
 AREA =  $xy = x(120 - 2x)$   
 $A = 120x - 2x^2$   
 $\frac{dA}{dx} = 120 - 4x$   
 MAX. WHEN  $\frac{dA}{dx} = 0$   
 $4x = 120 \Rightarrow x = 30m$   
 MAX. AREA =  $30 \times 60 = 1800m^2$

**LOCAL MAXIMUM**  
 $f''(x) < 0 \Rightarrow \begin{matrix} + & 0 & - \\ & f'(x) & \end{matrix}$

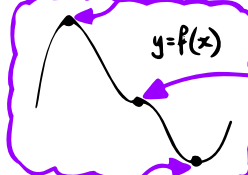


$f''(x) < 0$   
 $f''(x) > 0$

**eg.  $y = x^2 \sin x$**   
 $\frac{dy}{dx} = x^2 \cos x + 2x \sin x$

**eg.  $y = \sin(x^2) \Rightarrow \frac{dy}{dx} = 2x \cdot \cos(x^2)$**   
DIFF.

**POINTS OF INFLEXION**  
 $f''(x) = 0$   
 AND CHANGES SIGN



**PRODUCT RULE**  
 $y = uv$   
 $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

**CHAIN RULE**  
 $y = g(u), u = f(x)$   
 $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

**LOCAL MINIMUM**  
 $f''(x) > 0 \Rightarrow \begin{matrix} - & 0 & + \\ & f'(x) & \end{matrix}$

**THE SECOND DERIVATIVE**  
 $\frac{d^2y}{dx^2}$  OR  $f''(x)$

**QUOTIENT RULE**  
 $y = \frac{u}{v}$   
 $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

**OPTIMISATION**

$f'(x) = 0$

**FURTHER DIFFERENTIATION**

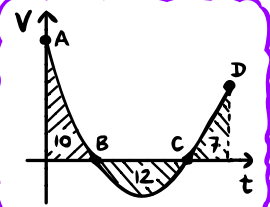
$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$   
 $f(x) = e^x \Rightarrow f'(x) = e^x$   
 $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$   
 $f(x) = \sin x \Rightarrow f'(x) = \cos x$   
 $f(x) = \cos x \Rightarrow f'(x) = -\sin x$

**RELATIONSHIP BETWEEN GRAPHS**

$f(x)$   
 $\updownarrow$   
 $f'(x)$   
 $\updownarrow$   
 $f''(x)$

**KINEMATICS**

$\frac{d}{dt} \left( \begin{matrix} S = \text{DISPLACEMENT} \\ v = \text{VELOCITY} \\ a = \text{ACCELERATION} \end{matrix} \right) \int dt$



DISPLACEMENT FROM A TO D =  $\int v dt = 10 - 12 + 7 = 5m$   
 DIST. TRAVELLED =  $\int |v| dt = 10 + 12 + 7 = 29m$

**REVERSE CHAIN RULE**

REPLACE  $x \rightarrow ax + b$

eg  $\int e^{3x+2} dx = \frac{1}{3} e^{3x+2} + c$   
 $\int \cos(2x-1) dx = \frac{1}{2} \sin(2x-1) + c$

**FURTHER INTEGRATION**

**FIND AREA**

WITHOUT GDC  
 $\int_a^b g'(x) dx = g(b) - g(a)$

**ENCLOSED BETWEEN TWO CURVES**

AREA =  $\int_a^b |f(x) - g(x)| dx$

FIND THE TOTAL AREA ENCLOSED BETWEEN THE CURVES  
 $f(x) = x^3 - 5x$  AND  $g(x) = x^2$   
 CURVES INTERSECT WHEN  $f(x) = g(x)$   
 GDC  $\Rightarrow x = -1.791, 0, 2.791$   
 TOTAL AREA =  $\int_{-1.791}^{2.791} |(x^3 - 5x) - (x^2)| dx$   
 GDC = 15.08 units<sup>2</sup>

**OF THE FORM  $\int g'(x)f(g(x))dx$**

$\int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + c$   
 $\int 2x(x^2+1)^4 dx = \frac{1}{5}(x^2+1)^5 + c$

**SHADED AREA**

$\int_2^5 x^2 + 3 dx$   
 $= \left[ \frac{1}{3}x^3 + 3x \right]_2^5$   
 $= \left( \frac{125}{3} + 15 \right) - \left( \frac{8}{3} + 6 \right)$   
 $= 48 \text{ units}^2$

