

Divisibility

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Rational numbers $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\}$. In other words rational numbers are numbers that **can be** written as a fraction where both the numerator and the denominator are integers and the denominator is not 0.

Divisibility (Podzielność)

We say that an integer n is divisible (**podzielny**) by an integer m , if there exists an integer k , such that $n = k \cdot m$.

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We have $6|42$, because $42 = 7 \cdot 6$ (and 7 is an integer).

We will write $m \nmid n$ to indicate that n is not divisible by m . So for instance we have $8 \nmid 42$, because there is no integer k such that $42 = k \cdot 8$.

Definitions

Prime numbers

A prime number is a natural number greater than 1, which cannot be written as a product of two smaller natural numbers.

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Equivalent definition:

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A prime number is a natural number, which has exactly two positive divisors (1 and itself).

Try and prove (or at least convince yourself) that these two definitions are indeed equivalent.

Prime numbers

There are 25 prime numbers less than 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

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Theorem

There are infinitely many prime numbers.

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Theorem

There are infinitely many prime numbers.

We will prove the above theorem in class - it's very important, make sure that you understand the proof.

Prime numbers - bonus material

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The bonus question is how large can the gap between two consecutive prime numbers be? If you have some thoughts on this, come and talk to me after class.

Composite numbers

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Note that 1 is neither prime nor composite.

Fundamental Theorem of Arithmetic

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Example: $20 = 2 \times 2 \times 5$.

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Example: $20 = 2 \times 2 \times 5$.

In some statements of this theorem the option when the number is prime is not treated separately, but it is considered a product of a single number.

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For example we have $20 = 4 \times 5$, but also $20 = 2 \times 10$, so 20 can be written as a product of two natural numbers in more than one way.

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For example we have $20 = 4 \times 5$, but also $20 = 2 \times 10$, so 20 can be written as a product of two natural numbers in more than one way.

However if we wanted to express 20 as a product of prime numbers then it can be done and it can be done in only one way: $20 = 2 \times 2 \times 5$

Prime decomposition

In order to decompose a number into its prime factors we do the following. We divide the given number by the least possible prime number that divides it. We then repeat this for the quotient (result of the division) until our quotient is 1.

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Lets decompose 20 into primes. The least prime number that divides 20 is **2**. $20 \div 2 = 10$. Now we work with 10. The least prime number that divides 10 is again **2**. $10 \div 2 = 5$. Now 5, the least prime number that divides 5 is **5** itself. $5 \div 5 = 1$. We got to 1, we're done.

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Finally we have: $20 = 2 \times 2 \times 5$.

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The whole thing can be written as:

$$\begin{array}{r|l} 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

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So (for the fourth time): $20 = 2 \times 2 \times 5$.

Prime decomposition - examples

Write 378 as a product of prime numbers.

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378		2
189		3
63		3
21		3
7		7
1		

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So: $378 = 2 \times 3 \times 3 \times 3 \times 7$.

Prime decomposition - examples

Write 14300 as a product of prime numbers.

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14300		2
7150		2
3575		5
715		5
143		11
13		13
1		

Prime decomposition - examples

Write 14300 as a product of prime numbers.

14300		2
7150		2
3575		5
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So: $14300 = 2 \times 2 \times 5 \times 5 \times 11 \times 13$.

Now we move on to divisibility rules. In other words - how do we know that some number is divisible by, say, 3?

Divisibility rules

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- $2|n$ iff the units digit (**cyfra jendości**) of n is 0,2,4,6 or 8.

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- $5|n$ iff the units digit of n is 0, or 5.
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- $8|n$ iff the last three digits of n represent a number divisible by 8 or are all 0.

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- $9|n$ iff the sum of the digits of n is divisible by 9.

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- $9|n$ iff the sum of the digits of n is divisible by 9.
- $11|n$ iff the difference between the sums of every second digit and the sum of the remaining digits is divisible by 11.

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Note that iff stands for *if and only if*.

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- not divisible by 2, the last digit is not 0,2,4,6 or 8;

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- not divisible by 2, the last digit is not 0,2,4,6 or 8;
- not divisible by 3, the sum of all its digits is
 $1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$, which is not divisible by 3;

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- not divisible by 3, the sum of all its digits is $1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$, which is not divisible by 3;
- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4.

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- not divisible by 5, last digit not 0 or 5;

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- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4. Of course it cannot be divisible by 4 also because it's not divisible by 2;
- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);

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- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4. Of course it cannot be divisible by 4 also because it's not divisible by 2;
- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);

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- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4. Of course it cannot be divisible by 4 also because it's not divisible by 2;
- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);
- not divisible by 9, the sum of its digits is 16, which is not divisible by 9. And also the number is not divisible by 3;

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- not divisible by 4, the last two digits make up a number 21, which is not divisible by 4. Of course it cannot be divisible by 4 also because it's not divisible by 2;
- not divisible by 5, last digit not 0 or 5;
- not divisible by 6, because it's not divisible by 2 (and 3);
- not divisible by 8, 321 is not divisible by 8 and also the number is not divisible by 2 (and 4);
- not divisible by 9, the sum of its digits is 16, which is not divisible by 9. And also the number is not divisible by 3;
- divisible by 11, we have $(1 + 3 + 3 + 1) - (2 + 4 + 2) = 0$, which is divisible by 11.

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Divisibility rules - example

The number 659442 is:

- divisible by 2, the last digits 2;
- divisible by 3, the sum of all its digits is $6 + 5 + 9 + 4 + 4 + 2 = 30$, which is divisible by 3;

Divisibility rules - example

The number 659442 is:

- divisible by 2, the last digits 2;
- divisible by 3, the sum of all its digits is $6 + 5 + 9 + 4 + 4 + 2 = 30$, which is divisible by 3;
- not divisible by 4, the last two digits make up a number 42, which is not divisible by 4.

Divisibility rules - example

The number 659442 is:

- divisible by 2, the last digits 2;
- divisible by 3, the sum of all its digits is $6 + 5 + 9 + 4 + 4 + 2 = 30$, which is divisible by 3;
- not divisible by 4, the last two digits make up a number 42, which is not divisible by 4.
- not divisible by 5, last digit not 0 or 5;

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- not divisible by 5, last digit not 0 or 5;
- divisible by 6, because it's divisible by both 2 and 3;
- not divisible by 8, 442 is not divisible by 8 and also the number is not divisible by 4;

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- not divisible by 5, last digit not 0 or 5;
- divisible by 6, because it's divisible by both 2 and 3;
- not divisible by 8, 442 is not divisible by 8 and also the number is not divisible by 4;
- not divisible by 9, the sum of its digits is 30, which is not divisible by 9.

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- not divisible by 4, the last two digits make up a number 42, which is not divisible by 4.
- not divisible by 5, last digit not 0 or 5;
- divisible by 6, because it's divisible by both 2 and 3;
- not divisible by 8, 442 is not divisible by 8 and also the number is not divisible by 4;
- not divisible by 9, the sum of its digits is 30, which is not divisible by 9.
- not divisible by 11, we have $(6 + 9 + 4) - (5 + 4 + 2) = 8$, which is not divisible by 11.

If there are any questions or doubts, you can contact me via Librus or MS Teams.