

# Financial Mathematics

# Introduction

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In almost all cases you will want to use the GDC for these problems. We will review this in detail, but we will also discuss a more algebraic approach.

# Interest rates

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## Simple interest

Fixed amount is added each period.

## Compound interest

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# Simple interest

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If we invest 1000 PLN in a fund that pays 5% simple interest per year, it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) each year. So after 5 years we will have  $1000 + 5 \cdot 50 = 1250$  PLN.

## Simple interest - exercise

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After 7 years his investment will be worth:

$$20000 + 7 \cdot 120 = 28400$$

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If we invest 1000 PLN in a fund that pays 5% compound interest (compounded annually), it means that we will get 50 PLN (because 5% of 1000 PLN is 50 PLN) after the first year, but we will get 52.5 PLN (because 5% of 1050 PLN is 52.5 PLN) in the second year and the amount we receive increases as we have more money in the account.

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Because the amount we have increases by a fixed percentage, we can multiply it by 1.05 each year.

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Because the amount we have increases by a fixed percentage, we can multiply it by 1.05 each year. Note that multiplying by 1.05 denotes the increase by 5%.

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Because the amount we have increases by a fixed percentage, we can multiply it by 1.05 each year. Note that multiplying by 1.05 denotes the increase by 5%.

So after 5 years we will have:

$$1000 \cdot 1.05^5 = 1276.28$$

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We multiplied by 1.03 to denote the 3% increase. 1.03 is to the 8th power because we want to know the value of the investment after 8 years.

# Compound interest GDC

To solve compound interest problems on **Ti-84** you need to press APPS and find Finance.

# Compound interest GDC

When using GDC you need to input the following data:

**N** - number of periods/payments,

**I%** - annual interest rate,

**PV** - present value,

**PMT** - payments,

**FV** - future value,

**P/Y** - payments (or periods) per year,

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Note that we use sign (+ or -) to indicate the direction of the money. For example -1000 means that we lose (or invest) the money, +1000 means that we get (or borrow, or withdraw) the money.

# Compound interest GDC

In our example we have

$N = 8$  (our period is a year and we want 8 years)

$I\% = 3$

$PV = -50000$  (minus sign indicates that we invest this money)

$PMT = 0$  (there are no payments)

$FV = ?$  (this doesn't matter, we need find it)

$P/Y = 1$

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You need to highlight FV and press ALPHA-ENTER (Solve).

You should get  $FV = 63338.50$



Note that  $P/Y$  denotes payments per year. In the cases we will consider there will be no payments (no money is added or withdrawn during the investment).

$P/Y$  is directly linked to  $N$ . If  $P/Y$  is set to 12, then  $N$  is measured in months.

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Tomasz invests 20000 PLN. The bank pays 4% annual interest rate and the interest is compounded every 6 months. Calculate the value of the investment after 7 years.

The annual interest rate is 4%, but it is compounded twice a year, so every 6 months we receive 2%. In 7 years there are 14 6-month periods, so we can calculate the value of the investment as follows:

$$20000 \cdot 1.02^{14} = 26389.58$$

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$$PMT = 0 \text{ (there are no payments)}$$

$$FV = ? \text{ (this is what we're looking for)}$$

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**P/Y** = 2 (there are 2 6-month periods in a year)

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We get the same result  $FV = 26389.58$

## Compound interest - exercise 2

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We should get  $r = 2.50$ . Think about this equation. It helps if you understand what is happening here. The 12 in the denominator comes from the fact that we are looking for annual interest rate, but since it is compounded monthly than each month we get  $\frac{r}{12}\%$ .



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We should get  $r = 2.50$ . Think about this equation. It helps if you understand what is happening here. The 12 in the denominator comes from the fact that we are looking for annual interest rate, but since it is compounded monthly than each month we get  $\frac{r}{12}\%$ . It is, of course, much easier to simply use TVM on GDC.

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$$N = 5 \text{ (5 years)}$$

$$I\% = ? \text{ (unknown)}$$

$$PV = -40000 \text{ (remember about the minus sign)}$$

$$PMT = 0 \text{ (there are no payments)}$$

$$FV = 45320.04 \text{ (no minus here, we're getting this money)}$$

$$P/Y = 1 \text{ (12 months in a year)}$$

$$C/Y = 12 \text{ (interest is compounded 12 times a year)}$$

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$$FV = 45320.04 \text{ (no minus here, we're getting this money)}$$

$$P/Y = 1 \text{ (12 months in a year)}$$

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We get that the interest rate is  $r = 2.50$ .

## Compound interest - exercise 2

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$N = 60$  (we have 60 months in 5 years)

$I\% = ?$  (unknown)

$PV = -40000$  (remember about the minus sign)

$PMT = 0$  (there are no payments)

$FV = 45320.04$  (no minus here, we're getting this money)

$P/Y = 12$  (12 months in a year)

$C/Y = 12$  (interest is compounded 12 times a year)

## Compound interest - exercise 2

We can also do:

$N = 60$  (we have 60 months in 5 years)

$I\% = ?$  (unknown)

$PV = -40000$  (remember about the minus sign)

$PMT = 0$  (there are no payments)

$FV = 45320.04$  (no minus here, we're getting this money)

$P/Y = 12$  (12 months in a year)

$C/Y = 12$  (interest is compounded 12 times a year)

We also get  $r = 2.50$ .

## Compound interest - exercise 3

Tomasz would like to have 100 000 PLN on his account in 10 years time. How much does he need to invest into savings account that pays an interest of 2% compounded quarterly?



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Algebraically we would do it as follows:

$$100000 = x \cdot (1.005)^{40}$$

## Compound interest - exercise 3

Tomasz would like to have 100 000 PLN on his account in 10 years time. How much does he need to invest into savings account that pays an interest of 2% compounded quarterly?

Algebraically we would do it as follows:

$$100000 = x \cdot (1.005)^{40}$$

We multiply by 1.005 because we earn 0.5% per quarter. The power is 40, because we have 40 quarters in 10 years. The answer we get is  $x = 81913.89$ .

# Compound interest - exercise 3

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$$N = 10 \text{ (10 years)}$$

$$I\% = 2 \text{ (annual interest rate)}$$

$$PV = ? \text{ (this will be negative)}$$

$$PMT = 0 \text{ (there are no payments)}$$

$$FV = 100\,000 \text{ (we want to have this much)}$$

$$P/Y = 1$$

$$C/Y = 4 \text{ (compounded 4 times a year)}$$

## Compound interest - exercise 3

Now let's use TVM:

$$N = 10 \text{ (10 years)}$$

$$I\% = 2 \text{ (annual interest rate)}$$

$$PV = ? \text{ (this will be negative)}$$

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$$FV = 100\,000 \text{ (we want to have this much)}$$

$$P/Y = 1$$

$$C/Y = 4 \text{ (compounded 4 times a year)}$$

We get that  $PV = -81913.89$ . So he should invest 81913.89 PLN. Do not use the negative sign in your final answer.

# Compound interest - exercise 3

Another approach:

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$N = 40$  (we have 40 quarters in 10 years)

$I\% = 2$  (annual interest rate)

$PV = ?$  (this will be negative)

$PMT = 0$  (there are no payments)

$FV = 100\ 000$  (we want to have this much)

$P/Y = 4$  (4 quarters in a year)

$C/Y = 4$  (compounded 4 times a year)

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Another approach:

$N = 40$  (we have 40 quarters in 10 years)

$I\% = 2$  (annual interest rate)

$PV = ?$  (this will be negative)

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$P/Y = 4$  (4 quarters in a year)

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We also get that  $PV = -81913.89$ .



In case of any questions you can message me via Librus or MS Teams.