Multiples

Things you need to learn

- Write down a general form of a number given the information about its division with remainder.
- Use these form to solve simple problems and write simple proofs.

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
   $x = 2k \qquad k \in \mathbb{Z}$

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

c) x is a product of three consecutive even numbers.

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

c) x is a product of three consecutive even numbers.
$x = (2k - 2) \times 2k \times (2k + 2) \qquad k \in \mathbb{Z}$

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

c) x is a product of three consecutive even numbers.
$x = (2k - 2) \times 2k \times (2k + 2) \qquad k \in \mathbb{Z}$

d) x is a product of three consecutive odd numbers.

Example 1 - even and odd numbers

Write down an integer x if

a) x is an even number.
$x = 2k \qquad k \in \mathbb{Z}$

b) x is an odd number.
$x = 2k + 1 \qquad k \in \mathbb{Z}$

c) x is a product of three consecutive even numbers.
$x = (2k - 2) \times 2k \times (2k + 2) \qquad k \in \mathbb{Z}$

d) x is a product of three consecutive odd numbers.
$x = (2k - 1) \times (2k + 1) \times (2k + 3) \qquad k \in \mathbb{Z}$

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
$x = 123k \qquad k \in \mathbb{Z}$

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
$x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
$x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.
$x = 10k \qquad k \in \mathbb{Z}$

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
$x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.
$x = 10k \qquad k \in \mathbb{Z}$

d) x is divisible by 2 and 6

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
$x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
$x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.
$x = 10k \qquad k \in \mathbb{Z}$

d) x is divisible by 2 and 6
$x = 6k \qquad k \in \mathbb{Z}$

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
   $x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
   $x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.
   $x = 10k \qquad k \in \mathbb{Z}$

d) x is divisible by 2 and 6
   $x = 6k \qquad k \in \mathbb{Z}$

e) x is divisible by 4 and 6

Example 2 - divisibility

Write down an integer x if

a) x is divisible by 7.
   $x = 7k \qquad k \in \mathbb{Z}$

b) x is divisible by 123.
   $x = 123k \qquad k \in \mathbb{Z}$

c) x is divisible by 2 and 5.
   $x = 10k \qquad k \in \mathbb{Z}$

d) x is divisible by 2 and 6
   $x = 6k \qquad k \in \mathbb{Z}$

e) x is divisible by 4 and 6
   $x = 12k \qquad k \in \mathbb{Z}$

Reminder

Remember, if a number is divisible by m and n, then it is divisible by $lcm(m, n)$, but not necessarily by mn.

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.
$x = 5k + 3 \qquad k \in \mathbb{N}$

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.
$x = 5k + 3 \qquad k \in \mathbb{N}$

b) the remainder when x is divided 11 is equal to 2.

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.
$x = 5k + 3 \qquad k \in \mathbb{N}$

b) the remainder when x is divided 11 is equal to 2.
$x = 11k + 2 \qquad k \in \mathbb{N}$

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.
$x = 5k + 3 \qquad k \in \mathbb{N}$

b) the remainder when x is divided 11 is equal to 2.
$x = 11k + 2 \qquad k \in \mathbb{N}$

c) the remainder when x is divided 7 is equal to 6.

Example 3 - remainders

Write a natural number x if

a) the remainder when x is divided 5 is equal to 3.
$x = 5k + 3 \qquad k \in \mathbb{N}$

b) the remainder when x is divided 11 is equal to 2.
$x = 11k + 2 \qquad k \in \mathbb{N}$

c) the remainder when x is divided 7 is equal to 6.
$x = 7k + 6 \qquad k \in \mathbb{N}$

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1
$6k+1, 6k+7, 6k+13 \qquad k \in \mathbb{Z}$

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1

$6k+1, 6k+7, 6k+13 \qquad k \in \mathbb{Z}$

Note: you could have also written for example:

$6k-5, 6k+1, 6k+7 \qquad k \in \mathbb{Z}$

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1
$6k + 1, 6k + 7, 6k + 13 \qquad k \in \mathbb{Z}$

Note: you could have also written for example:
$6k - 5, 6k + 1, 6k + 7 \qquad k \in \mathbb{Z}$

b) the remainder when they are divided by 13 is 5 .

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1
$6k+1, 6k+7, 6k+13 \qquad k \in \mathbb{Z}$

Note: you could have also written for example:
$6k-5, 6k+1, 6k+7 \qquad k \in \mathbb{Z}$

b) the remainder when they are divided by 13 is 5 .
$13k+5, 13k+18, 13k+31 \qquad k \in \mathbb{Z}$

Example 4 - remainders

Write down three consecutive integers such that

a) the remainder when they are divided by 6 is 1
$6k+1, 6k+7, 6k+13 \qquad k \in \mathbb{Z}$

Note: you could have also written for example:
$6k-5, 6k+1, 6k+7 \qquad k \in \mathbb{Z}$

b) the remainder when they are divided by 13 is 5 .
$13k+5, 13k+18, 13k+31 \qquad k \in \mathbb{Z}$

Note: again, another possible way would be:
$13k-8, 13k+5, 13k+18 \qquad k \in \mathbb{Z}$

Example 5

Find three consecutive odd numbers whose sum is 159

Example 5

Find three consecutive odd numbers whose sum is 159

$$(2k-1)+(2k+1)+(2k+3)=159$$

$$6k=156$$

$$k=26$$

Example 5

Find three consecutive odd numbers whose sum is 159

$$(2k-1)+(2k+1)+(2k+3)=159$$

$$6k=156$$

$$k=26$$

$$2k-1=2\times 26-1=51$$

The numbers are 51, 53 and 55.

Example 6

Find four consecutive numbers, which when divide by 4 give remainder equal to 3, and their sum is 116.

Example 6

Find four consecutive numbers, which when divide by 4 give remainder equal to 3, and their sum is 116.

$$(4k-1)+(4k+3)+(4k+7)+(4k+11)=116$$

$$16k=96$$

$$k=6$$

Example 6

Find four consecutive numbers, which when divide by 4 give remainder equal to 3, and their sum is 116.

$$(4k-1)+(4k+3)+(4k+7)+(4k+11)=116$$

$$16k=96$$

$$k=6$$

$$4k-1=4\times 6-1=23$$

The numbers are 23, 27, 31 and 35.

Example 7

Show that a square of an odd number is an odd number.

Example 7

Show that a square of an odd number is an odd number. We start with an odd number $2k + 1$,

Example 7

Show that a square of an odd number is an odd number. We start with an

odd number $2k+1$, we want to show that after we square it, we will still be able to write it in this form.

Example 7

Show that a square of an odd number is an odd number. We start with an

odd number $2k+1$, we want to show that after we square it, we will still be able to write it in this form.

$$(2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1$$

where $m = 2k^2 + 2k$, and so m is a integer.

Example 7

Show that a square of an odd number is an odd number. We start with an

odd number $2k+1$, we want to show that after we square it, we will still be able to write it in this form.

$$(2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 = 2m + 1$$

where $m = 2k^2 + 2k$, and so m is a integer. So $(2k+1)^2$ is in the form $2m+1$, with m integer, so it is odd.

□

Example 7

A number n has a remainder of 2, when divided by 3. Find the remainder of n^2, when divided by 3.

Example 7

A number n has a remainder of 2, when divided by 3. Find the remainder of n^2, when divided by 3.

We start with $n = 3k + 2$, we will square it and see what we get.

Example 7

A number n has a remainder of 2, when divided by 3. Find the remainder of n^2, when divided by 3.

We start with $n = 3k + 2$, we will square it and see what we get.

$$n^2 = (3k+2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1 = 3m + 1$$

where $m = 3k^2 + 4k + 1$, so it's an integer.

Example 7

A number n has a remainder of 2, when divided by 3. Find the remainder of n^2, when divided by 3.

We start with $n = 3k + 2$, we will square it and see what we get.

$$n^2 = (3k + 2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1 = 3m + 1$$

where $m = 3k^2 + 4k + 1$, so it's an integer. So n^2 is of the form $3m + 1$, with m integer, so it has a remainder of 1, when divided by 3.

Example 8

Show that a product of two odd numbers is an odd number.

Example 8

Show that a product of two odd numbers is an odd number.

We start with two odd numbers $2k+1$ and $2m+1$, where k and m are some integers. We want to multiply them:

Example 8

Show that a product of two odd numbers is an odd number.

We start with two odd numbers $2k+1$ and $2m+1$, where k and m are some integers. We want to multiply them:

$$(2k+1)(2m+1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1 = 2n + 1$$

where $n = 2km + k + m$, so it's an integer.

Example 8

Show that a product of two odd numbers is an odd number.

We start with two odd numbers $2k+1$ and $2m+1$, where k and m are some integers. We want to multiply them:

$$(2k+1)(2m+1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1 = 2n + 1$$

where $n = 2km + k + m$, so it's an integer. So we got the form $2n+1$, with n being an integer, which means that we got an odd number.

□

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Any integer has to be in of the following forms: $4k$, $4k+1$, $4k+2$ or $4k+3$.

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Any integer has to be in of the following forms: $4k$, $4k+1$, $4k+2$ or $4k+3$. In other words, given any integer it is divisible by 4 or gives remainder 1, 2 or 3, when divided by 4.

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Any integer has to be in of the following forms: $4k$, $4k + 1$, $4k + 2$ or $4k + 3$. In other words, given any integer it is divisible by 4 or gives remainder 1, 2 or 3, when divided by 4. Let's square all these numbers and see what happens:

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Any integer has to be in of the following forms: $4k$, $4k+1$, $4k+2$ or $4k+3$. In other words, given any integer it is divisible by 4 or gives remainder 1, 2 or 3, when divided by 4. Let's square all these numbers and see what happens:

$$(4k)^2 = 16k^2 = 4 \times 4k^2 = 4m$$
$$(4k+1)^2 = 16k^2 + 8k + 1 = 4(4k^2 + 2k) + 1 = 4n + 1$$
$$(4k+2)^2 = 16k^2 + 16k + 4 = 4(4k^2 + 4k + 1) = 4s$$
$$(4k+3)^2 = 16k^2 + 24k + 9 = 4(4k^2 + 6k + 2) + 1 = 4t + 1$$

Example 9

Show that a square number is divisible by 4 or has remainder 1 when divided by 4.

Any integer has to be in of the following forms: $4k$, $4k+1$, $4k+2$ or $4k+3$. In other words, given any integer it is divisible by 4 or gives remainder 1, 2 or 3, when divided by 4. Let's square all these numbers and see what happens:

$$\begin{aligned}
(4k)^2 &= 16k^2 = 4\times 4k^2 = 4m \\
(4k+1)^2 &= 16k^2+8k+1 = 4(4k^2+2k)+1 = 4n+1 \\
(4k+2)^2 &= 16k^2+16k+4 = 4(4k^2+4k+1) = 4s \\
(4k+3)^2 &= 16k^2+24k+9 = 4(4k^2+6k+2)+1 = 4t+1
\end{aligned}$$

in each case we have a number of the form $4m$ or $4m+1$, so we either have a number divisible by 4 or that gives remainder 1 when divided by 1.

If there are any questions or doubts, you can contact me via Librus or MS Teams.