Kinematics 1 [88 marks]

1. [Maximum mark: 8] 23N.1.AHL.TZ0.13

The velocity v of a particle at time t, as it moves along a straight line, can be modelled by the piecewise function

$$v(t) = \begin{cases} u_1(t), & 0 \le t \le T \\ u_2(t), & t \ge T \end{cases}$$

where $u_1(t) = 2t^2 - t^3$ and $u_2(t) = 8 - 4t$. It is required that $u_1(T) = u_2(T)$.

(a) Find the value of T. [2]

Markscheme

attempt to solve $u_1(t) = u_2(t)$ **OR** sketch of two graphs ***(M1)***

$(T =)\ 2$ ***A1***

Note: Award ***(M1)A0*** if additional values of T are seen **OR** if $T = -2$ is their final answer.

[2 marks]

(b) Show that $u_1\prime(T) = u_2\prime(T)$. [2]

Markscheme

$u_1\prime(t) = 4t - 3t^2$ ***A1***

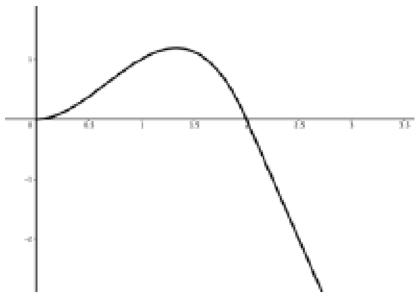
$u_1\prime(2) = -4 = u_2\prime(2)$ ***R1***

[2 marks]

The displacement of the particle at time $t = 0$ is zero.

(c) Find the time when the particle returns to its initial position. [4]

Markscheme



recognition of integrating **AND** equating to zero ***(M1)***

$\int_0^2 2t^2 - t^3 \, \mathrm{d}t + \int_2^k 8 - 4t \, \mathrm{d}t = 0$ ***(A1)***

$\frac{4}{3} + (8k - 2k^2) - 8 = 0$ **OR** $1.18350\ldots \left(\frac{6-\sqrt{6}}{3}\right)$ seen ***(A1)***

Note: Award ***(M1)(A1)A0*** if integration done correctly but limits are not substituted.

$(k =)\ 2.82\ \left(= 2.81649\ldots, \frac{6+\sqrt{6}}{3}\right)$ ***A1***

[4 marks]

2. [Maximum mark: 9] 22N.1.AHL.TZ0.14

A particle moves such that its velocity, v metres per second, at time t seconds, is given by $v = t\sin(t^2)$.

(a) Find an expression for the acceleration of the particle. [2]

Markscheme

attempt to use product rule ***(M1)***

$a = 2t^2 \cos(t^2) + \sin(t^2)$ ***A1***

[2 marks]

(b) Hence, or otherwise, find its greatest acceleration for $0 \le t \le 8$. [2]

Markscheme

graph of a ***(M1)***

$126\ (\text{ms}^{-2})\ \ (125.699\ldots)$ ***A1***

[2 marks]

The particle starts at the origin.

(c) Find an expression for the displacement of the particle. [3]

Markscheme

attempt at integration by substitution or inspection ***(M1)***

$s = -\frac{1}{2}\cos(t^2)\ (+c)$ ***A1***

($s = 0$ when $t = 0$) $\Rightarrow c = \frac{1}{2}$ ***A1***

$\left(s = -\frac{1}{2}\cos(t^2) + \frac{1}{2}\right)$

[3 marks]

(d) Hence show that the particle never has a negative displacement. [2]

Markscheme

$\cos\left(t^2\right) \leq 1$ ***A1***

$-\frac{1}{2}\cos\left(t^2\right) \geq -\frac{1}{2}$

so $\frac{1}{2} - \frac{1}{2}\cos\left(t^2\right) \geq 0$ ***R1***

hence the particle never has a negative displacement. ***AG***

Note: Do not accept reasoning based on a sketch of the graph.

[2 marks]

3. [Maximum mark: 15] 21M.2.AHL.TZ2.6

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v = -2t^2 + 16t - 24$ for $t \geq 0$.

(a) Find the times when P is at instantaneous rest. [2]

Markscheme

solving $v = 0$ ***M1***

$t = 2,\ t = 6$ ***A1***

[2 marks]

(b) Find the magnitude of the particle's acceleration at 6 seconds. [4]

Markscheme

use of power rule *(M1)*

$\frac{\mathrm{d}v}{\mathrm{d}t} = -4t + 16$ *(A1)*

$(t = 6)$

$\Rightarrow a = -8$ *(A1)*

magnitude $= 8\,\mathrm{m\,s^{-2}}$ *A1*

[4 marks]

(c) Find the greatest speed of P in the interval $0 \le t \le 6$. [2]

Markscheme

using a sketch graph of v *(M1)*

$24\,\mathrm{m\,s^{-1}}$ *A1*

[2 marks]

(d) The particle starts from the origin O. Find an expression for the displacement of P from O at time t seconds. [4]

Markscheme

METHOD ONE

$x = \int v \ \mathrm{d}\,t$

attempt at integration of v *(M1)*

$-\frac{2t^3}{3} + 8t^2 - 24t\ (+c)$ *A1* attempt to find c (use of $t = 0,\ x = 0$) *(M1)*

$c = 0$ *A1*

$\left(x = -\frac{2t^3}{3} + 8t^2 - 24t\right)$

METHOD TWO

$x = \int_0^t v\ \mathrm{d}\ t$

attempt at integration of v *(M1)*

$\left[-\frac{2t^3}{3} + 8t^2 - 24t\right]_0^t$ *A1*

attempt to substituted limits into their integral *(M1)*

$x = -\frac{2t^3}{3} + 8t^2 - 24t$ *A1*

[4 marks]

(e) Find the total distance travelled by P in the interval $0 \le t \le 4$. [3]

Markscheme

$\int_0^4 |v|\ \mathrm{d}\ t$ *(M1)(A1)*

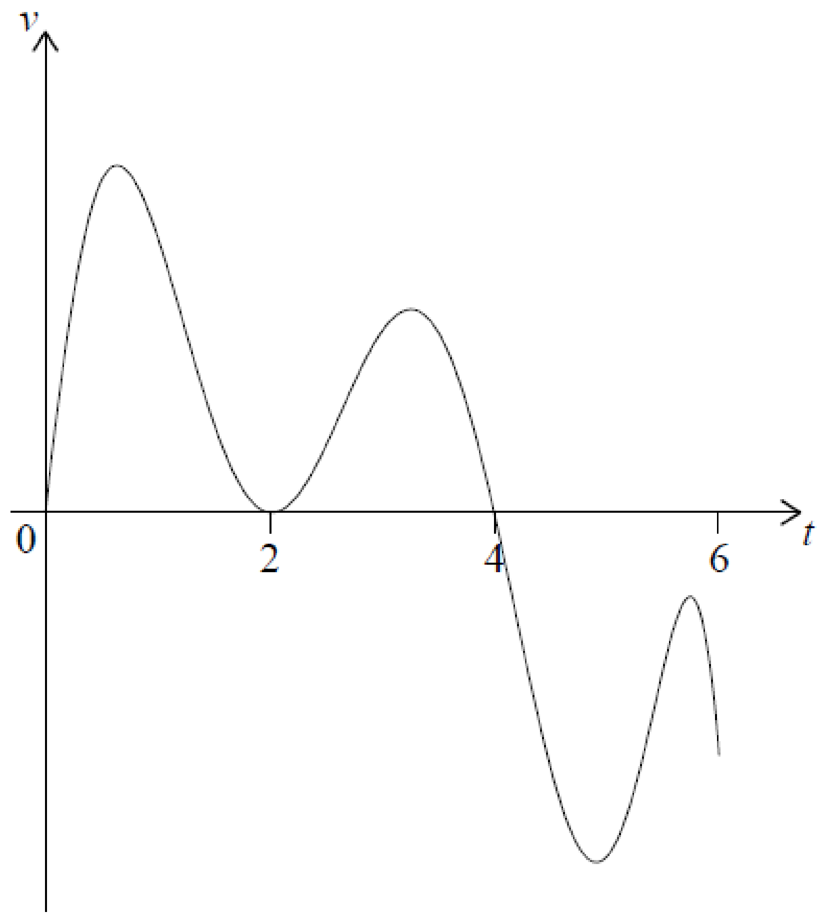
Note: Award ***M1*** for using the absolute value of v, or separating into two integrals, ***A1*** for the correct expression.

$= 32\,\mathrm{m}$ *A1*

[3 marks]

4. [Maximum mark: 7] 19M.1.SL.TZ1.S_7

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when $t = 0$, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v\,\mathrm{d}t = 9$.

(a) Find the value of $s(4) - s(2)$. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing relationship between v and s ***(M1)***

eg $\int v = s$, $s' = v$

$s(4) - s(2) = 9$ ***A1 N2***

[2 marks]

(b) Find the total distance travelled in the first 5 seconds. [5]

Markscheme

correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15$, $s(2) = 15$

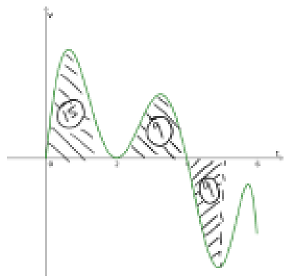
valid approach to find total distance travelled ***(M1)***

eg sum of 3 areas, $\int_0^4 v + \int_4^5 v$, shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 |v|$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v$, $\int_2^4 v = \int_4^5 |v|$, $\int_4^5 v\,\mathrm{d}t = -9$,
$s(4) - s(2) - [s(5) - s(4)]$,

equal areas 

correct working using $s(5) = s(2)$ ***(A1)***

eg $15 + 9 - (-9)$, $15 + 2[s(4) - s(2)]$, $15 + 2(9)$, $2 \times s(4) - s(2)$, $48 - 15$

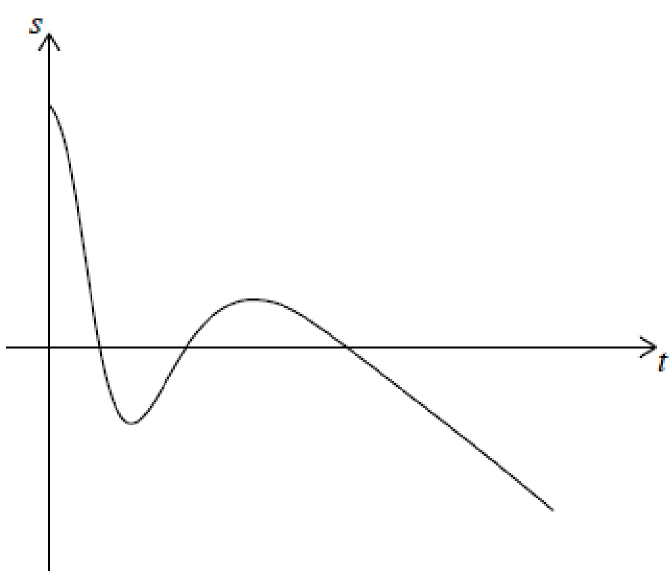
total distance travelled = 33 (m) ***A1 N2***

[5 marks]

5. [Maximum mark: 16] 19M.2.SL.TZ2.S_8

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_{\mathrm{A}} = 15 - t - 6t^3\mathrm{e}^{-0.8t}$, $0 \le t \le 25$. This is shown in the following diagram.



(a) Find the initial displacement of particle A from point P. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $s_{\mathrm{A}}(0),\ s(0),\ t=0$

15 (cm) ***A1 N2***

[2 marks]

(b) Find the value of t when particle A first reaches point P. [2]

Markscheme

valid approach ***(M1)***

eg $s_{\mathrm{A}}=0,\ s=0,\ 6.79321,\ 14.8651$

2.46941

t = 2.47 (seconds) ***A1 N2***

[2 marks]

(c) Find the value of t when particle A first changes direction. [2]

Markscheme

recognizing when change in direction occurs ***(M1)***

eg slope of s changes sign, $s'=0$, minimum point, 10.0144, (4.08, −4.66)

4.07702

t = 4.08 (seconds) ***A1 N2***

[2 marks]

(d) Find the total distance travelled by particle A in the first 3 seconds. [3]

Markscheme

METHOD 1 (using displacement)

correct displacement or distance from P at $t = 3$ (seen anywhere) ***(A1)***

eg −2.69630, 2.69630

valid approach ***(M1)***

eg 15 + 2.69630, $s(3) - s(0)$, −17.6963

17.6963

17.7 (cm) ***A1 N2***

METHOD 2 (using velocity)

attempt to substitute either limits or the velocity function into distance formula involving $|v|$ ***(M1)***

eg $\int_0^3 |v| \mathrm{d}t$, $\int \left|-1 - 18t^2\mathrm{e}^{-0.8t} + 4.8t^3\mathrm{e}^{-0.8t}\right|$

17.6963

17.7 (cm) ***A1 N2***

[3 marks]

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_{\mathrm{B}} = 8 - 2t$, $0 \le t \le 25$.

(e.i) Given that particles A and B start at the same point, find the displacement function s_{B} for particle B. [5]

Markscheme

recognize the need to integrate velocity *(M1)*

eg $\int v(t)$

$8t - \frac{2t^2}{2} + c$ (accept x instead of t and missing c) *(A2)*

substituting initial condition into their integrated expression (must have c) *(M1)*

eg $15 = 8(0) - \frac{2(0)^2}{2} + c,\ c = 15$

$s_{\mathrm{B}}(t) = 8t - t^2 + 15$ *A1 N3*

[5 marks]

(e.ii) Find the other value of t when particles A and B meet. [2]

Markscheme

valid approach *(M1)*

eg $s_{\mathrm{A}} = s_{\mathrm{B}}$, sketch, (9.30404, 2.86710)

9.30404

$t = 9.30$ (seconds) *A1 N2*

Note: If candidates obtain $s_{\mathrm{B}}(t) = 8t - t^2$ in part (e)(i), there are 2 solutions for part (e)(ii), 1.32463 and 7.79009. Award the last ***A1*** in part (e)(ii) only if both solutions are given.

[2 marks]

6. [Maximum mark: 6] 19M.2.AHL.TZ2.H_6

A particle moves along a horizontal line such that at time t seconds, $t \geq 0$, its acceleration a is given by $a = 2t - 1$. When $t = 6$, its displacement s from a fixed origin O is 18.25 m. When $t = 15$, its displacement from O is 922.75 m. Find an expression for s in terms of t. [6]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to integrate a to find v ***M1***

$v = \int a\,\mathrm{d}t = \int (2t - 1)\,\mathrm{d}t$

$= t^2 - t + c$ ***A1***

$s = \int v\,\mathrm{d}t = \int \left(t^2 - t + c\right)\mathrm{d}t$

$= \frac{t^3}{3} - \frac{t^2}{2} + ct + d$ ***A1***

attempt at substitution of given values ***(M1)***

at $t = 6,\ 18.25 = 72 - 18 + 6c + d$

at $t = 15,\ 922.75 = 1125 - 112.5 + 15c + d$

solve simultaneously: ***(M1)***

$c = -6,\ d = 0.25$ ***A1***

$\Rightarrow s = \frac{t^3}{3} - \frac{t^2}{2} + -6t + \frac{1}{4}$

[6 marks]

7. [Maximum mark: 7] 18N.2.SL.TZ0.S_4

A particle moves along a straight line so that its velocity, v m s^{-1}, after t seconds is given by $v\left(t\right) = 1.4^{t} - 2.7$, for $0 \le t \le 5$.

(a) Find when the particle is at rest. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $v\left(t\right) = 0$, sketch of graph

2.95195

$t = \log_{1.4}2.7$ (exact), $t = 2.95$ (s) ***A1 N2***

[2 marks]

(b) Find the acceleration of the particle when $t = 2$. [2]

Markscheme

valid approach ***(M1)***

eg $a\left(t\right) = v'\left(t\right),\ v'\left(2\right)$

0.659485

$a\left(2\right) = 1.96 \ln 1.4$ (exact), $a\left(2\right) = 0.659$ (m s^{-2}) ***A1 N2***

[2 marks]

(c) Find the total distance travelled by the particle. [3]

Markscheme

correct approach *(A1)*

eg $\int_0^5 |v(t)|\,\mathrm{d}t$, $\int_0^{2.95} (-v(t))\,\mathrm{d}t + \int_{295}^5 v(t)\,\mathrm{d}t$

5.3479

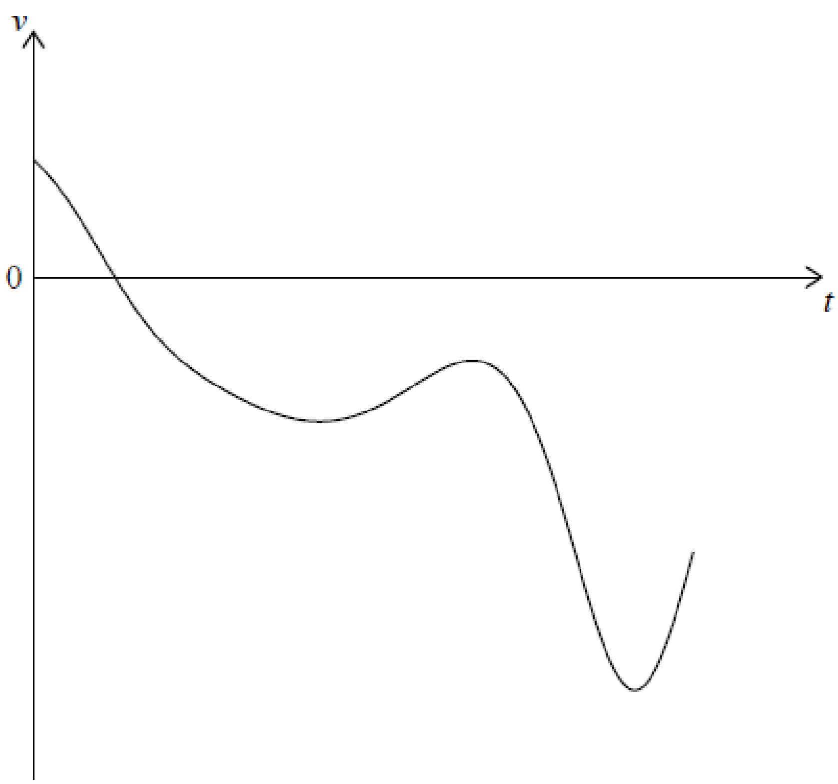
distance = 5.35 (m) ***A2 N3***

[3 marks]

8. [Maximum mark: 15] 18M.2.SL.TZ2.S_9

A particle P moves along a straight line. The velocity v m s^{-1} of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

initial velocity when $t = 0$ ***(M1)***

eg $v(0)$

$v = 17$ (m s^{-1}) ***A1 N2***

[2 marks]

(b) Find the maximum speed of P. [3]

Markscheme

recognizing maximum speed when $|v|$ is greatest ***(M1)***

eg minimum, maximum, $v' = 0$

one correct coordinate for minimum ***(A1)***

eg 6.37896, −24.6571

24.7 (ms^{-1}) ***A1 N2***

[3 marks]

(c) Write down the number of times that the acceleration of P is 0 m s^{-2}. [3]

Markscheme

recognizing a = v' ***(M1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, correct derivative of first term

identifying when a = 0 ***(M1)***

eg turning points of *v*, *t*-intercepts of *v*′

3 ***A1 N3***

[3 marks]

(d) Find the acceleration of P when it changes direction. [4]

Markscheme

recognizing P changes direction when $v = 0$ ***(M1)***

$t = 0.863851$ ***(A1)***

−9.24689

$a = -9.25$ (ms^{-2}) ***A2 N3***

[4 marks]

(e) Find the total distance travelled by P. [3]

Markscheme

correct substitution of limits or function into formula ***(A1)***

eg

$$\int_0^7 |\, v \,|,\ \int_0^{0.8638} v\mathrm{d}t - \int_{0.8638}^7 v\mathrm{d}t,\ \ \int |\, 7\cos x - 5x^{\cos x} \,|\, dx,\ \ 3.32 = 60.6$$

63.8874

63.9 (metres) ***A2 N3***

[3 marks]

9. [Maximum mark: 5] 18M.2.AHL.TZ2.H_7

A point P moves in a straight line with velocity v ms^{-1} given by $v\left(t\right) = \mathrm{e}^{-t} - 8t^2\mathrm{e}^{-2t}$ at time t seconds, where $t \geq 0$.

(a) Determine the first time t_1 at which P has zero velocity. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to solve $v\left(t\right) = 0$ for t or equivalent ***(M1)***

$t_1 = 0.441$(s) ***A1***

[2 marks]

(b.i) Find an expression for the acceleration of P at time t. [2]

Markscheme

$a\left(t\right) = \frac{\mathrm{d}v}{\mathrm{d}t} = -\mathrm{e}^{-t} - 16t\mathrm{e}^{-2t} + 16t^2\mathrm{e}^{-2t}$ ***M1A1***

Note: Award ***M1*** for attempting to differentiate using the product rule.

[2 marks]

(b.ii) Find the value of the acceleration of P at time t_1. [1]

Markscheme

$a\left(t_1\right) = -2.28$ (ms^{-2}) ***A1***

[1 mark]