# Kinematics 2 [84 marks]

**1.** [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time $t$ seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

Markscheme

use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is 16.5 cm (to the left of O) ***A1***

***[3 marks]***

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

Markscheme

attempt to find time when particle changes direction *eg* considering the first maximum on the graph of $s$ or the first $t$ – intercept on the graph of $s'$. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of $s'$ for **their** value of $t$, $s''(1.51986...)$ ***(M1)***

$= -8.92$ (cm/s$^2$) ***A1***

***[4 marks]***

**2.** [Maximum mark: 5]

A particle moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, at time $t$ seconds is given by

$$v = 4t^2 - 6t + 9 - 2\sin(4t),\ 0 \le t \le 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of $T$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting

compared to formal exam papers.

attempts either graphical or symbolic means to find the value of $t$ when $\frac{dv}{dt} = 0$ **(M1)**

$T = 0.465\,(\mathrm{s})$ **A1**

**[2 marks]**

(b) Let $s_1$ be the distance travelled by the particle from $t = 0$ to $t = T$ and let $s_2$ be the distance travelled by the particle from $t = T$ to $t = 1$.

Show that $s_2 > s_1$. [3]

Markscheme

attempts to find the value of either $s_1 = \int_0^{0.46494\ldots} v\,\mathrm{d}t$ or $s_2 = \int_{0.46494\ldots}^{1} v\,\mathrm{d}t$ **(M1)**

$s_1 = 3.02758\ldots$ and $s_2 = 3.47892\ldots$ **A1A1**

**Note:** Award as above for obtaining, for example, $s_2 - s_1 = 0.45133\ldots$ or $\frac{s_2}{s_1} = 1.14907\ldots$

**Note:** Award a maximum of **M1A1A0FT** for use of an incorrect value of $T$ from part (a).

so $s_2 > s_1$ **AG**

**[3 marks]**

**3.** [Maximum mark: 5]

A particle moves along a straight line. Its displacement, $s$ metres, from a fixed point O after time $t$ seconds is given by $s(t) = 4.3\sin\left(\sqrt{3t+5}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after $q$ seconds.

(a) Find the value of $q$. [2]

Markscheme

recognizing at rest when $\frac{ds}{dt} = 0$ OR $s$ is a minimum *(M1)*

$q = 5.73553\ldots$

$= 5.74$ ***A1***

**Note:** If no working shown, award ***(M1)A0*** for $q = 5.7$ (2sf).

***[2 marks]***

(b) Find the total distance that the particle travels in the first $q$ seconds. [3]

Markscheme

**METHOD 1**

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{5.73\ldots} |v(t)|\,\mathrm{d}t$ OR $\int_0^{5.73\ldots} \left|\frac{\mathrm{d}}{\mathrm{d}t}s(t)\right|\mathrm{d}t$ OR $\left|\int_0^{5.73\ldots} v(t)\,\mathrm{d}t\right|$ OR $-\int_0^{5.73\ldots} v(t)\,\mathrm{d}t$ ***(A1)***

**Note**: Condone absence of $\mathrm{d}t$.

Only accept $\left|\int_0^{q} v(t)\,\mathrm{d}t\right|$ if their value of $q$ does not result in the particle changing direction in the first $q$ seconds.

$= 7.68302\ldots$

$= 7.68\ (\mathrm{m})$ ***A1***

**Note:** Special Cases:
Award a maximum of ***(M1)(A1FT)A0FT*** if the candidate obtains $q = 1.62320\ldots$ in part (a), and uses that value to find the total distance to be $3.38302\ldots$ ($3.37644\ldots$ from 3sf).

Award ***(M1)(A0)A1*** if the candidate writes $\int_0^{5.73\ldots} v(t)\,\mathrm{d}t$ followed by the correct answer.

**METHOD 2**

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.38302\ldots$ AND at minimum is $-4.3$ ***(A1)***
total distance travelled $= 3.38302\ldots - (-4.3)$

$= 7.68302\ldots$

$= 7.68\ (\mathrm{m})$ ***A1***

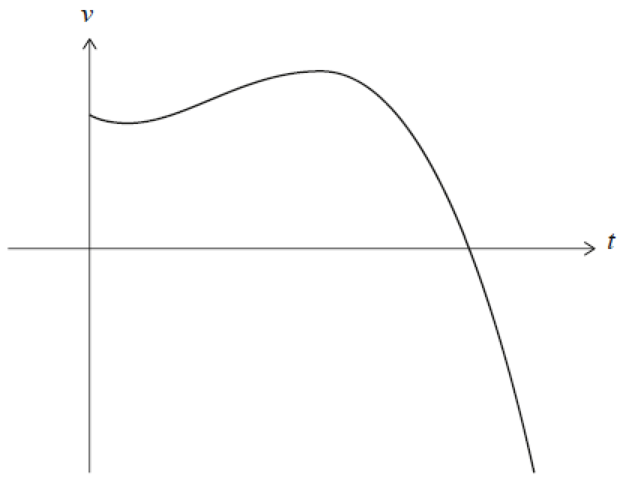
**Note:** If no working shown, award ***(M1)(A0)A0*** for $7.7$ (2sf).

***[3 marks]***

**4.** [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\mathrm{m\,s}^{-1}$, at time $t$ seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of $v$ is shown in the following diagram.




At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

Markscheme

attempt to integrate $v$ (integration of at least one term) ***(M1)***

$(s(t) =) - \frac{1}{4}t^4 + \frac{7}{6}t^3 - t^2 + 6t\ (+C)$ ***A2***

**Note:** Award ***A1*** for at least two correct terms.

substitution of $t = 1$ into their integrated expression ***(M1)***

displacement $= 5\frac{11}{12} \left(= \frac{71}{12}\right)$ (m) ***A1***

***[5 marks]***

(b) Find an expression for the acceleration of the object. [2]

Markscheme

attempt to differentiate $v$ (differentiation of at least one term) ***(M1)***

$a(t) = -3t^2 + 7t - 2$ ***A1***

***[2 marks]***

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

Markscheme

setting their $v\prime(t) = 0$ ***(M1)***

$-3t^2 + 7t - 2 = 0$

valid attempt to solve quadratic ***(M1)***

$(3t - 1)(t - 2) = 0$ OR $\frac{-7 \pm \sqrt{49 - 4(-3)(-2)}}{-6}$

$t = \frac{1}{3},\ 2$ ($t = \frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive $t$-value into $v(t)$ ***(M1)***

greatest speed is $8\ (\mathrm{ms}^{-1})$ ***A1***

***[5 marks]***

(d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]

Markscheme

attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6\ (= -10)$

greatest speed is $10\ \mathrm{ms}^{-1}$ ***A1***

***[2 marks]***

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

Markscheme

identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

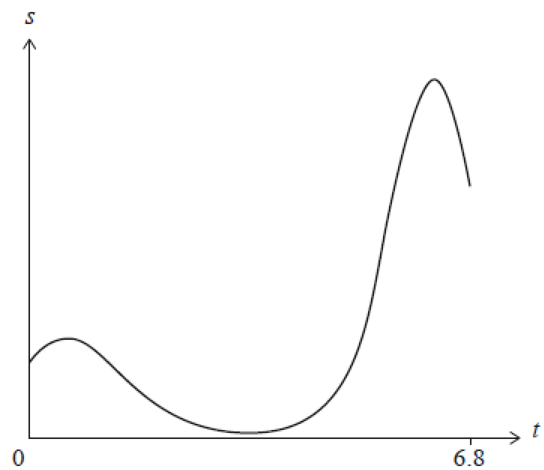
$\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t + \int_{k}^{4}|v(t)|\mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\mathrm{d}t + \left|\int_{\mathrm{k}}^{4} \mathrm{vd}\,t\right|$ OR $\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t - \int_{k}^{4} v\left(t\right)\mathrm{d}t$ ***A1***

**Note:** Condone missing $\mathrm{d}t$.

***[3 marks]***

**5.** [Maximum mark: 16]

A particle moves in a straight line. Its displacement, $s$ metres, from a fixed point $\mathrm{P}$ at time $t$ seconds is given by $s\left(t\right) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.




(a) Find the particle's initial displacement from the point $\mathrm{P}$. [2]

Markscheme

initial displacement is $s(0)$ ***(M1)***

$6\,(\mathrm{m})$ ***A1***

***[2 marks]***

(b) Find the particle's velocity when $t = 2$. [2]

Markscheme

velocity is $s\prime$ ***(M1)***

$-2.29920$

$-2.30$ (m/s) *A1*

***[2 marks]***

(c) Determine the intervals of time when the particle is moving away from the point $\mathrm{P}$. [5]

Markscheme

attempting to find $t$ when the particle changes direction *(M1)*

$t = 0.433007\ldots$ AND $3.25575\ldots$ AND $6.33965\ldots$ (may be seen on a graph) *(A1)*

particle travels away from $\mathrm{P}$ when $v > 0$ OR when $s' > 0$ *(M1)*

$0 \leq t < 0.433007\ldots,\ 3.25575\ldots < t < 6.33965$

$0 \leq t < 0.433,\ 3.26 < t < 6.34$ *A1A1*

***[5 marks]***

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of $b$ and the value of $c$. [4]

Markscheme

recognizing that acceleration is $a(t) = v'(t)$ OR $a(t) = s''(t)$ *(M1)*

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration *(M1)*

$b = 1.23140\ldots,\ c = 5.68959\ldots$

$b = 1.23,\ c = 5.69$ *A1A1*

***[4 marks]***

(e) Find the total distance travelled by the particle for $b \leq t \leq c$. [3]

Markscheme

**METHOD 1** (using integral of velocity)

correct integral (accept absence of $\mathrm{d}\ t$) *(A1)*

$\int_{1.23140...}^{5.68959...} |v(t)| \, \mathrm{d}\, t$ OR $\int_b^c |s\prime(t)| \, \mathrm{d}\, t$ OR $-\int_{1.23140...}^{3.25575...} v(t) \, \mathrm{d}\, t + \int_{3.25575...}^{5.68959...} v(t) \, \mathrm{d}\, t$ OR $3.8560 + 15.696$

$19.5525...$

total distance $= 19.6 \text{ (m)}$ ***A2***

**METHOD 2** (using differences in displacement)

finding displacement at $b$, $c$ **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734),\ (3.25575,\ 0.577001)$ OR $4.43306,\ 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 - 0.577001) + (16.2734 - 0.577001)$ OR towards $\mathrm{P}\ 3.85606 +$ away from $\mathrm{P}\ 15.696$

$19.5525...$

total distance $= 19.6 \text{ (m)}$ ***A1***

***[3 marks]***

**6.** [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v \,\mathrm{m\,s^{-1}}$, after $t$ seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4 \sin t$ for $0 \le t \le 6$.

(a) Find the value of $t$ when the particle is at rest. [2]

Markscheme

recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692...$

$t = 3.35$ (seconds) ***A1***

**Note:** Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

***[2 marks]***

(b) Find the acceleration of the particle when it changes direction. [3]

Markscheme

recognizing particle changes direction when $v = 0$ OR when $t = 3.34692\ldots$ ***(M1)***

$a = -4.71439\ldots$

$a = -4.71\ (\mathrm{ms}^{-2})$ ***A2***

***[3 marks]***

(c) Find the total distance travelled by the particle. [2]

Markscheme

distance travelled $= \int_0^6 |v|\ \mathrm{d}\ t$ OR

$\int_0^{3.34\ldots} \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\ \mathrm{d}\ t - \int_{3.34\ldots}^{6} \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\ \mathrm{d}\ t\ \ (= 14.3104\ldots + 6.44300\ldots)$ ***(A1)***

$= 20.7534\ldots$

$= 20.8$ (metres) ***A1***

***[2 marks]***

**7.** [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s}^{-1}$, at time $t$ seconds is given by $v = \frac{(t^2+1)\cos t}{4},\ 0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

Markscheme

recognises the need to find the value of $t$ when $v = 0$ ***(M1)***

$t = 1.57079\ldots\ \left(= \frac{\pi}{2}\right)$

$t = 1.57\ \left(= \frac{\pi}{2}\right)$ (s) ***A1***

*[2 marks]*

(b) Find the times when the particle's acceleration is $-1.9\,\mathrm{m\,s^{-2}}$. [3]

Markscheme

recognises that $a(t) = v\prime(t)$ ***(M1)***

$t_1 = 2.26277\ldots,\ t_2 = 2.95736\ldots$

$t_1 = 2.26,\ t_2 = 2.96$ (s) ***A1A1***

**Note:** Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

*[3 marks]*

(c) Find the particle's acceleration when its speed is at its greatest. [2]

Markscheme

speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778\ldots$

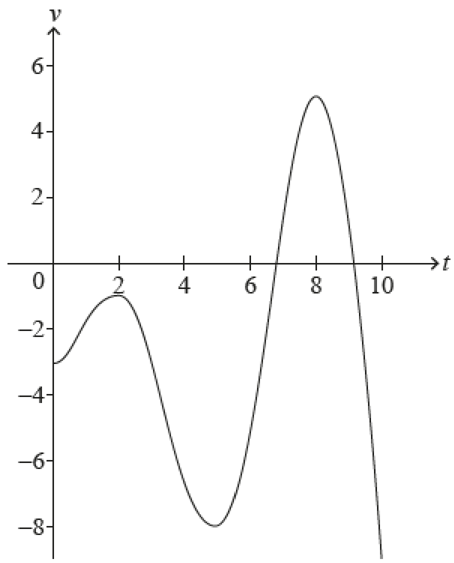
$a = -1.84\ (\mathrm{m\,s^{-2}})$ ***A1***

*[2 marks]*

**8.** [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\mathrm{m\,s^{-1}}$, of the particle at time $t$ seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of $v$.




(a) Find the smallest value of $t$ for which the particle is at rest. [2]

Markscheme

recognising $v=0$ ***(M1)***

$t=6.74416\ldots$

$=6.74$ (sec) ***A1***

**Note:** Do not award ***A1*** if additional values are given.

***[2 marks]***

(b) Find the total distance travelled by the particle. [2]

Markscheme

$\int_0^{10}|v(t)|\ \mathrm{d}\ t$ OR $-\int_0^{6.74416\ldots} v(t)\ \mathrm{d}\ t+\int_{6.74416\ldots}^{9.08837\ldots} v(t)\ \mathrm{d}\ t-\int_{9.08837\ldots}^{10} v(t)\ \mathrm{d}\ t$ ***(A1)***

$=37.0968\ldots$

$=37.1\ (\mathrm{m})$ ***A1***

***[2 marks]***

(c) Find the acceleration of the particle when $t=7$. [2]

Markscheme

recognizing acceleration at $t = 7$ is given by $v\prime(7)$ *(M1)*

acceleration $= 5.93430\ldots$

$= 5.93\ \left(\mathrm{ms}^{-2}\right)$ ***A1***

***[2 marks]***

**9.** [Maximum mark: 14]

A rocket is travelling in a straight line, with an initial velocity of $140$ m s$^{-1}$. It accelerates to a new velocity of $500$ m s$^{-1}$ in two stages.

During the first stage its acceleration, $a$ m s$^{-2}$, after $t$ seconds is given by $a\left(t\right) = 240\sin\left(2t\right)$, where $0 \leqslant t \leqslant k$.

(a) Find an expression for the velocity, $v$ m s$^{-1}$, of the rocket during the first stage. [4]

Markscheme

recognizing that $v = \int a$ *(M1)*

correct integration ***A1***

*eg* $-120\cos\left(2t\right) + c$

attempt to find $c$ using their $v\left(t\right)$ *(M1)*

*eg* $-120\cos\left(0\right) + c = 140$

$v\left(t\right) = -120\cos\left(2t\right) + 260$ ***A1 N3***

***[4 marks]***

The first stage continues for $k$ seconds until the velocity of the rocket reaches $375$ m s$^{-1}$.

(b) Find the distance that the rocket travels during the first stage. [4]

Markscheme

evidence of valid approach to find time taken in first stage *(M1)*

*eg* graph, $-120\cos\left(2t\right) + 260 = 375$

$k = 1.42595$ ***A1***

attempt to substitute **their** $v$ and/or **their** limits into distance formula ***(M1)***

*eg* $\int_0^{1.42595} |v|$, $\int 260 - 120\cos(2t)$, $\int_0^k (260 - 120\cos(2t))\,\mathrm{d}t$

$353.608$

distance is $354$ (m) ***A1 N3***

***[4 marks]***

(c) During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage.

Find the total time taken for the two stages. [6]

Markscheme

recognizing velocity of second stage is linear (seen anywhere) ***R1***

*eg* graph, $s = \frac{1}{2}h(a+b)$, $v = mt + c$

valid approach ***(M1)***

*eg* $\int v = 353.608$

correct equation ***(A1)***

*eg* $\frac{1}{2}h(375 + 500) = 353.608$

time for stage two $= 0.808248$ ($0.809142$ from 3 sf) ***A2***

$2.23420$ ($2.23914$ from 3 sf)

$2.23$ seconds ($2.24$ from 3 sf) ***A1 N3***

***[6 marks]***