PROBLEM 2 Deadline: 13.11

Given that

$$x^4 + px^2 + qx + r = (x^2 - ax + b)(x^2 + ax + c),$$

express p, q and r in terms of a, b and c.

Show also that a^2 is a root of the cubic equation

$$u^3 + 2pu^2 + (p^2 - 4r)u - q^2 = 0.$$

Explain why this equation always has a non-negative root, and verify that $u = 9$ is a root in the case $p = -1$, $q = -6$, $r = 15$.

Hence, or otherwise, express

$$y^4 - 8y^3 + 23y^2 - 34y + 39$$

as a product of two quadratic factors.