Revision questions [170 marks]

1. [Maximum mark: 6] 23M.1.AHL.TZ1.7

Consider $P(z) = 4m - mz + \frac{36}{m}z^2 - z^3$, where $z \in \mathbb{C}$ and $m \in \mathbb{R}^+$.

Given that $z - 3\text{i}$ is a factor of $P(z)$, find the roots of $P(z) = 0$. [6]

2. [Maximum mark: 7] 22N.1.AHL.TZ0.5

Consider the equation $z^4 + pz^3 + 54z^2 - 108z + 80 = 0$ where $z \in \mathbb{C}$ and $p \in \mathbb{R}$.

Three of the roots of the equation are $3 + \text{i}$, α and α^2, where $\alpha \in \mathbb{R}$.

(a) By considering the product of all the roots of the equation, find the value of α. [4]

(b) Find the value of p. [3]

3. [Maximum mark: 16] 22N.1.AHL.TZ0.11

Consider a three-digit code abc, where each of a, b and c is assigned one of the values $1, 2, 3, 4$ or 5.

Find the total number of possible codes

(a.i) assuming that each value can be repeated (for example, 121 or 444). [2]

(a.ii) assuming that no value is repeated. [2]

Let $P(x) = x^3 + ax^2 + bx + c$, where each of a, b and c is assigned one of the values $1, 2, 3, 4$ or 5. Assume that no value is repeated.

Consider the case where $P(x)$ has a factor of $(x^2 + 3x + 2)$.

(b.i) Find an expression for b in terms of a. [6]

(b.ii) Hence show that the only way to assign the values is $a = 4,\ b = 5$ and $c = 2$. [2]

(b.iii) Express $P(x)$ as a product of linear factors. [1]

(b.iv) Hence or otherwise, sketch the graph of $y = P(x)$, clearly showing the coordinates of any intercepts with the axes. [3]

4. [Maximum mark: 5] 21M.1.AHL.TZ2.7

The cubic equation $x^3 - kx^2 + 3k = 0$ where $k > 0$ has roots $\alpha,\ \beta$ and $\alpha + \beta$.

Given that $\alpha\beta = -\frac{k^2}{4}$, find the value of k. [5]

5. [Maximum mark: 17] 19M.2.AHL.TZ1.H_11

Consider the equation $x^5 - 3x^4 + mx^3 + nx^2 + px + q = 0$, where m, n, p, $q \in \mathbb{R}$.

The equation has three distinct real roots which can be written as $\log_2 a$, $\log_2 b$ and $\log_2 c$.

The equation also has two imaginary roots, one of which is $d\mathrm{i}$ where $d \in \mathbb{R}$.

(a) Show that $abc = 8$. [5]

The values a, b, and c are consecutive terms in a geometric sequence.

(b) Show that one of the real roots is equal to 1. [3]

(c) Given that $q = 8d^2$, find the other two real roots. [9]

6. [Maximum mark: 7] 18N.1.AHL.TZ0.H_8

Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$.

Two of the roots of the equation are $\log_2 6$ and $i\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.

Show that $6a + d + 12 = 0$. [7]

7. [Maximum mark: 5] 18M.1.AHL.TZ1.H_1

Let $f(x) = x^4 + px^3 + qx + 5$ where p, q are constants.

The remainder when $f(x)$ is divided by $(x + 1)$ is 7, and the remainder when $f(x)$ is divided by $(x - 2)$ is 1. Find the value of p and the value of q. [5]

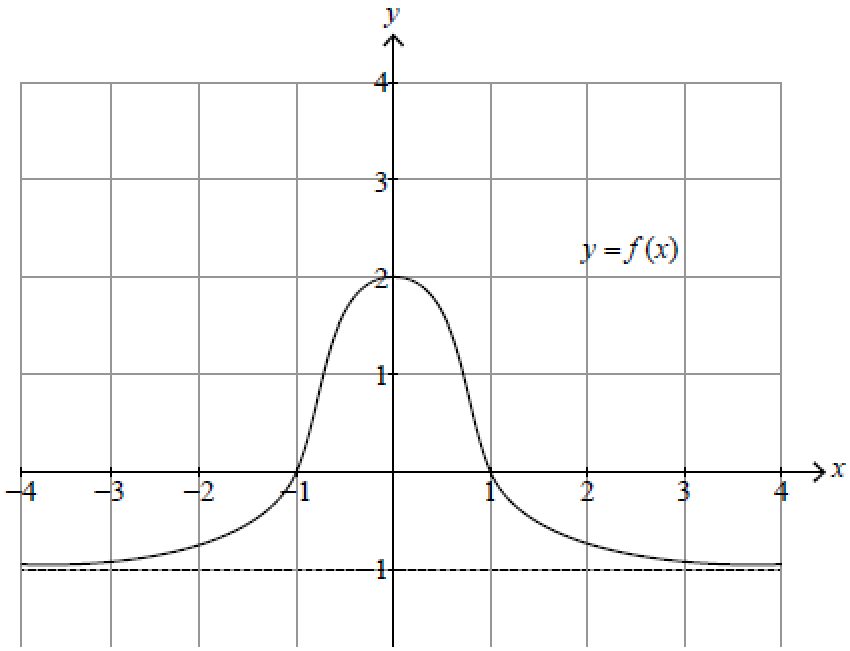
8. [Maximum mark: 5] 18M.2.AHL.TZ2.H_2

The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x - 1)$, $(x - 2)$ and $(x - 3)$.

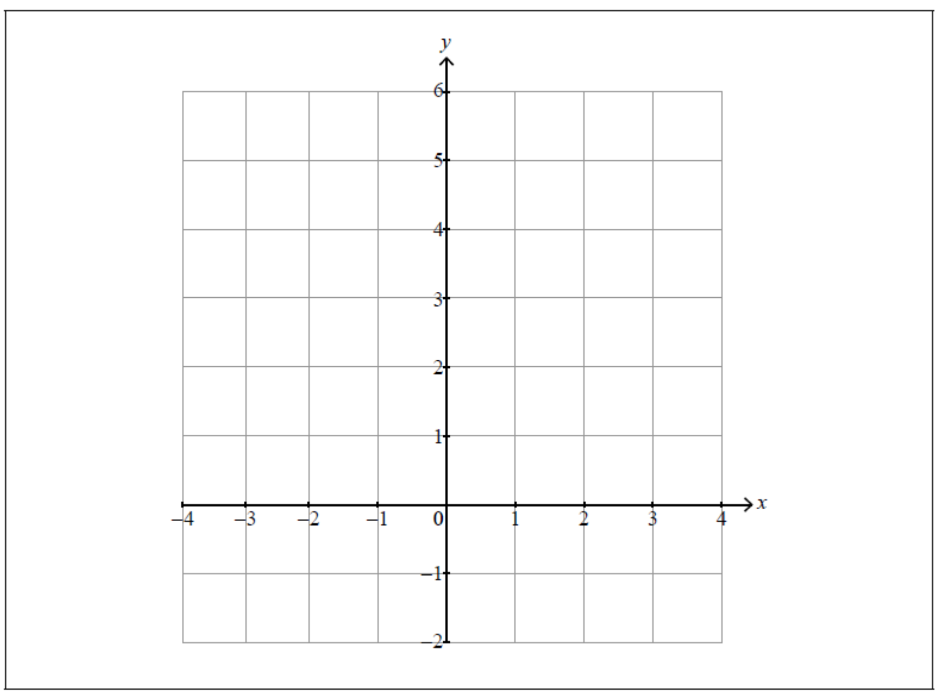
Find the values of p, q and r. [5]

9. [Maximum mark: 5] SPM.1.AHL.TZ0.4

The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$.



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



[5]

10. [Maximum mark: 5] SPM.2.AHL.TZ0.9

Consider the graphs of $y = \frac{x^2}{x-3}$ and $y = m\left(x+3\right), m \in \mathbb{R}$.

Find the set of values for m such that the two graphs have no intersection points. [5]

11. [Maximum mark: 9] EXM.1.AHL.TZ0.5

Let $f\left(x\right) = \frac{2x^2-5x-12}{x+2},\ x \in \mathbb{R},\ x \neq -2$.

(a) Find all the intercepts of the graph of $f\left(x\right)$ with both the x and y axes. [4]

(b) Write down the equation of the vertical asymptote. [1]

(c) As $x \rightarrow \pm\infty$ the graph of $f\left(x\right)$ approaches an oblique straight line asymptote.

Divide $2x^2 - 5x - 12$ by $x + 2$ to find the equation of this asymptote. [4]

12. [Maximum mark: 19] 23N.2.AHL.TZ1.11

Consider the function defined by $f(x) \frac{x^2-14x+24}{2x+6}$, where $x \in \mathbb{R}, x \neq -3$.

(a) State the equation of the vertical asymptote on the graph of f. [1]

(b) Find the coordinates of the points where the graph of f crosses the x-axis. [2]

The graph of f also has an oblique asymptote of the form $y = ax + b$, where $a,\ b \in \mathbb{Q}$.

(c) Find the value of a and the value of b. [4]

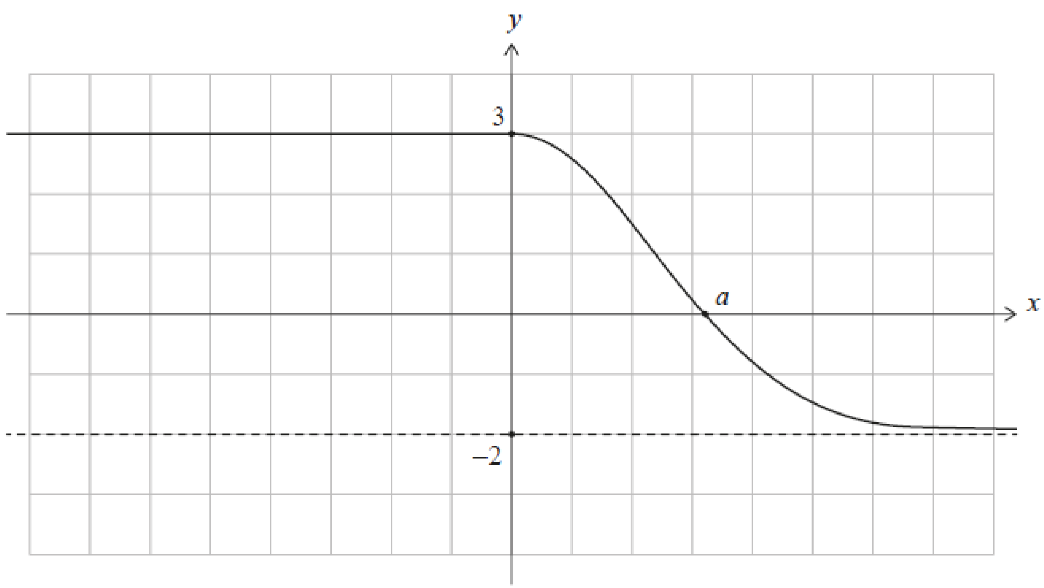
(d) Sketch the graph of f for $-50 \leq x \leq 50$, showing clearly the asymptotes and any intersections with the axes. [4]

(e) Find the range of f. [4]

(f) Solve the inequality $f(x) > x$. [4]

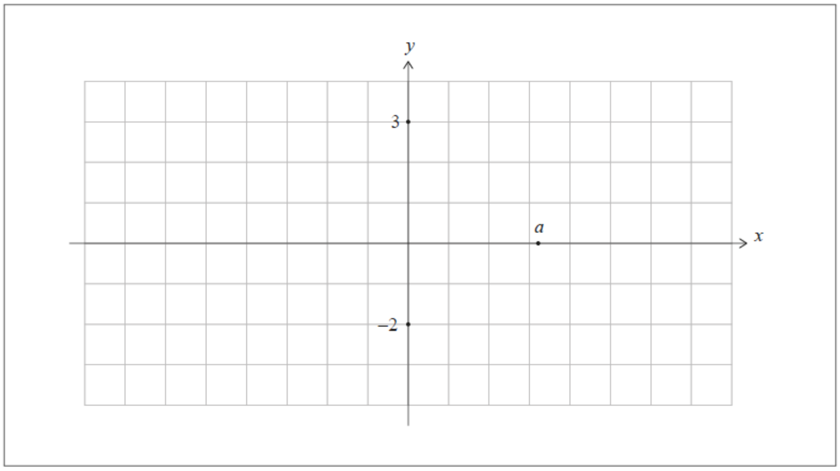
13. [Maximum mark: 7] 23M.1.AHL.TZ1.8

Part of the graph of a function, f, is shown in the following diagram. The graph of $y = f(x)$ has a y-intercept at $(0,\ 3)$, an x-intercept at $(a,\ 0)$ and a horizontal asymptote $y = -2$.



Consider the function $g(x) = |f(|x|)|$.

(a) On the following grid, sketch the graph of $y = g(x)$, labelling any axis intercepts and giving the equation of the asymptote.



[4]

(b) Find the possible values of k such that $\left(g(x)\right)^2 = k$ has exactly two solutions.

[3]

14. [Maximum mark: 8] 22M.1.AHL.TZ2.3

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

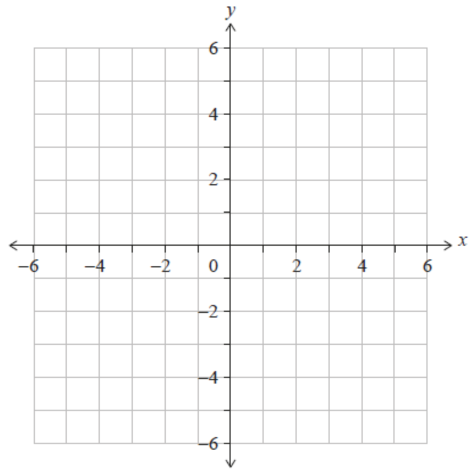
The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

(a.ii) Write down the equation of the horizontal asymptote. [1]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]

(d) Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. [2]

15. [Maximum mark: 11] 21N.2.AHL.TZ0.10

Consider the function $f(x) = \frac{x^2-x-12}{2x-15},\ x \in \mathbb{R},\ x \neq \frac{15}{2}$.

Find the coordinates where the graph of f crosses the

(a.i) x-axis. [2]

(a.ii) y-axis. [1]

(a.iii) Write down the equation of the vertical asymptote of the graph of f. [1]

(a.iiii) The oblique asymptote of the graph of f can be written as $y = ax + b$ where $a,\ b \in \mathbb{Q}$.

Find the value of a and the value of b. [4]

(a.iiiii) Sketch the graph of f for $-30 \leq x \leq 30$, clearly indicating the points of intersection with each axis and any asymptotes.

[3]

16. [Maximum mark: 7] SPM.2.AHL.TZ0.8

The complex numbers w and z satisfy the equations

$\frac{w}{z} = 2\text{i}$

$\text{z}^* - 3w = 5 + 5\text{i}$.

Find w and z in the form $a + b\text{i}$ where $a, \text{b} \in \mathbb{Z}$. [7]

17. [Maximum mark: 5] 23N.1.AHL.TZ1.7

It is given that $z = 5 + q\text{i}$ satisfies the equation
$z^2 + \text{i}z = -p + 25\text{i}$, where $p,\ q\ \in \mathbb{R}$.

Find the value of p and the value of q. [5]

18. [Maximum mark: 8] 21M.1.AHL.TZ1.7

Consider the quartic equation
$z^4 + 4z^3 + 8z^2 + 80z + 400 = 0,\ z \in \mathbb{C}$.

Two of the roots of this equation are $a + b\text{i}$ and $b + a\text{i}$, where
$a,\ b \in \mathbb{Z}$.

Find the possible values of a. [8]

19. [Maximum mark: 5] 20N.1.AHL.TZ0.H_4

Consider the equation $\frac{2z}{3-z^*} = \text{i}$, where $z = x + \text{i}y$ and
$x,\ y \in \mathbb{R}$.

Find the value of x and the value of y. [5]

20. [Maximum mark: 7] 19N.1.AHL.TZ0.H_5

Consider the equation $z^4 = -4$, where $z \in \mathbb{C}$.

(a) Solve the equation, giving the solutions in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$. [5]

(b) The solutions form the vertices of a polygon in the complex plane. Find the area of the polygon. [2]

21. [Maximum mark: 6] 19N.2.AHL.TZ0.H_6

Let $P(z) = az^3 - 37z^2 + 66z - 10$, where $z \in \mathbb{C}$ and $a \in \mathbb{Z}$.

One of the roots of $P(z) = 0$ is $3 + \mathrm{i}$. Find the value of a. [6]