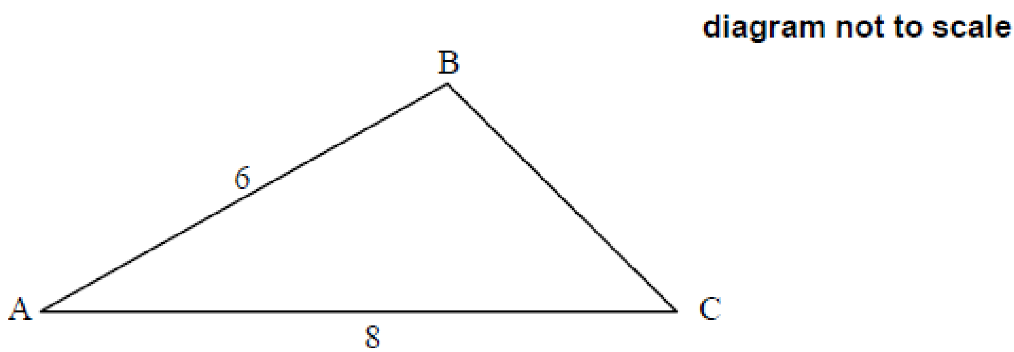
Trigonometry review (research days) [120 marks]

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



(a) Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3]

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) Find the area of triangle ABC. [2]

Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

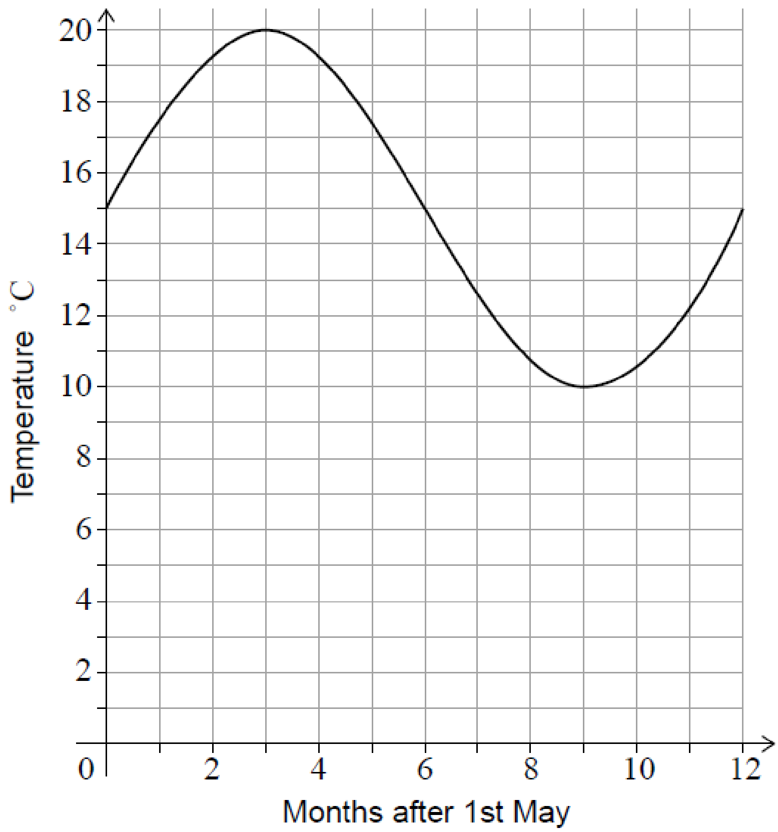
[2 marks]

2. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least $15\,^{\circ}\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\sin bx + c$ for $0 \le x \le 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

Markscheme

$12b = 2\pi$ OR $(b =)\frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$ ***A1***

$b = \frac{\pi}{6}$ ***AG***

[1 mark]

(b) Write down the value of

(b.i) a; [1]

Markscheme

$a = 5$ ***A1***

[1 mark]

(b.ii) c. [1]

Markscheme

$c = 15$ ***A1***

[1 mark]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

Markscheme

attempt to substitute $x = 5$ into $g(x)$ ***(M1)***

$g(5) = 3.5 \sin \frac{5\pi}{6} + 11$

$\sin \frac{5\pi}{6} = \frac{1}{2}$ ***(A1)***

$g(5) = 3.5 \times \frac{1}{2} + 11$

$g(5) = 12.75 (= \frac{51}{4})$ ***A1***

[3 marks]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

Markscheme

METHOD 1 (finding maximum temperature)

considering the maximum value of $\sin \frac{\pi}{6} x (= 1)$ OR $g\prime(x) = 0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) ***(M1)***

$g_{\text{max}} = 3.5 + 11$ OR $\frac{\pi}{6} \cdot 3.5 \cos \left(\frac{\pi}{6} x\right) = 0$ OR $x = 3$

$g_{\text{max}} = 14.5$ ***A1***

$14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

Worded conclusions are acceptable for the ***R1***, as long as the reasoning is clear that the water does not reach 15°, so not warm enough for Alex.

METHOD 2 (working with inequality)

$3.5 \sin \left(\frac{\pi}{6} x\right) + 11 \geq 15$ ***(M1)***

$\sin\left(\frac{\pi}{6}x\right) \geq \frac{8}{7}$ ***A1***

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

If candidate works with an equation throughout, the ***M1*** and ***A1*** may be awarded, if appropriate. A correct inequality is required for the ***R1*** to be awarded.

[3 marks]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

Markscheme

EITHER

attempt to find $0.7f(x)$ OR $0.7f(x) + q$ ***(M1)***

$0.7f(x) = 3.5 \sin \frac{\pi}{6}x + 10.5$ OR
$0.7f(x) + q = 3.5 \sin \frac{\pi}{6}x + 10.5 + q$ OR $10.5 + q = 11$ ***(A1)***

OR

attempt to find $0.7f(x)$ for a particular value of x ***(M1)***

eg maximum $20 \times 0.7 = 14$ ***(A1)***

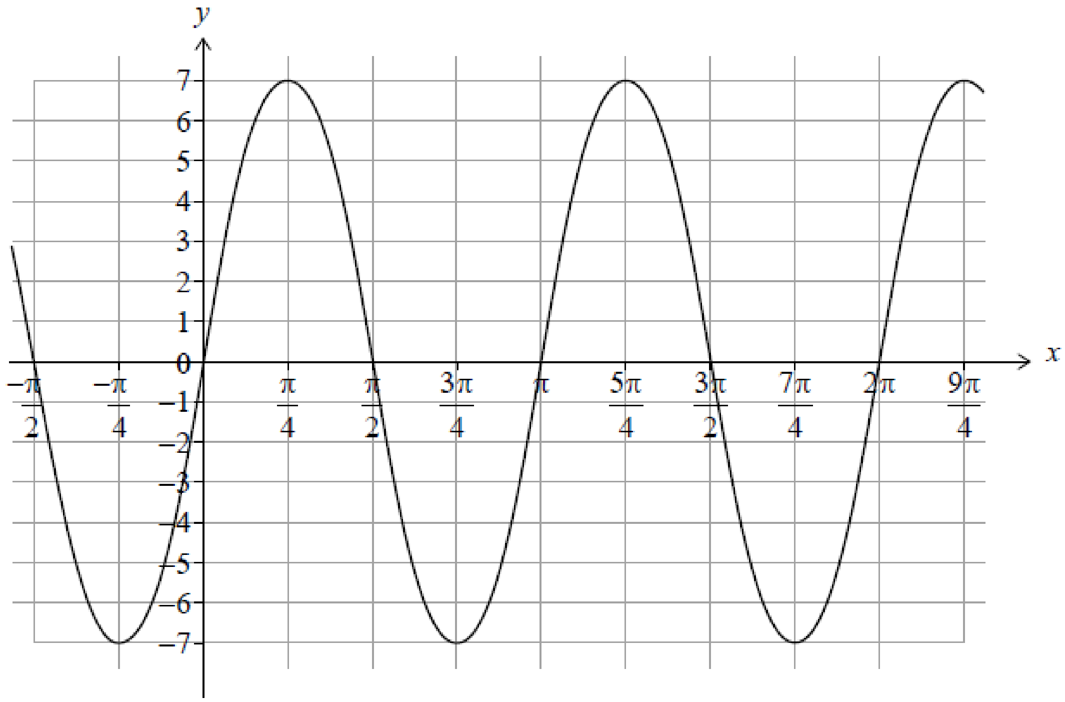
THEN

$q = 0.5$ ***A1***

[3 marks]

3. [Maximum mark: 7]

Consider the function $f(x) = a\ \sin\ (bx)$ with $a,\ b \in \mathbb{Z}^+$.The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

Markscheme

$a = 7$ ***A1***

[1 mark]

(b.i) Write down the period of f. [1]

Markscheme

period = π ***A1***

[1 marks]

(b.ii) Hence, find the value of b. [2]

Markscheme

$b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{b}$ ***(A1)***

$= 2$ ***A1***

[2 marks]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

Markscheme

substituting $\frac{\pi}{12}$ into their $f(x)$ ***(M1)***

$f\left(\frac{\pi}{12}\right) = 7\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ ***(A1)***

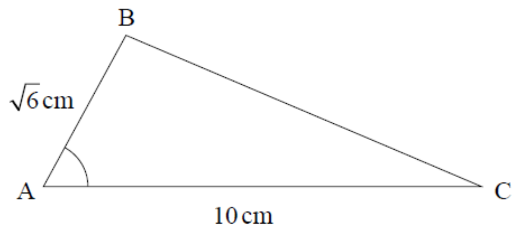
$= \frac{7}{2}$ **A1**

[3 marks]

4. [Maximum mark: 6]

In the following triangle ABC, $\mathrm{AB} = \sqrt{6\ \mathrm{cm}}$, $\mathrm{AC} = 10\ \mathrm{cm}$ and $\cos \mathrm{B\hat{A}C} = \frac{1}{5}$.

diagram not to scale



[6]

Find the area of triangle ABC.

Markscheme

METHOD 1

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{5^2-1^2}=\right)\sqrt{24}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2\alpha+\sin^2\alpha=1$ ***(M1)***

$\sin^2 \mathrm{B\hat{A}C}=1-\left(\frac{1}{5}\right)^2$ ***(A1)***

THEN

$\sin\ \mathrm{B\hat{A}C}=\frac{\sqrt{24}}{5}$ (may be seen in area formula) ***A1***

attempt to use 'Area$=\frac{1}{2}\ ab\sin\ C$' (must include their calculated value of $\sin\ \mathrm{B\hat{A}C}$) ***(M1)***

$=\frac{1}{2}\times 10\times\sqrt{6}\ \times\frac{\sqrt{24}}{5}$ ***(A1)***

$=12$ (cm^2) ***A1***

[6 marks]

METHOD 2

attempt to find perpendicular height of triangle BAC *(M1)*

EITHER

$\text{height} = \sqrt{6} \times \sin \mathrm{B}\hat{\mathrm{A}}\mathrm{C}$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ *(M1)*

$\text{height} = \sqrt{6} \times \sqrt{1 - \left(\frac{1}{5}\right)^2}$ **(A1)**

$= \sqrt{6} \times \frac{\sqrt{24}}{5} \left(= \frac{12}{5}\right)$ *A1*

OR

$\text{adjacent} = \frac{\sqrt{6}}{5}$ *(A1)*

attempt to use Pythagoras' theorem in a right-angled triangle. *(M1)*

$\text{height} = \sqrt{6 - \frac{6}{25}} \left(= \frac{12}{5}\right)$ (may be seen in area formula) *(A1)*

THEN

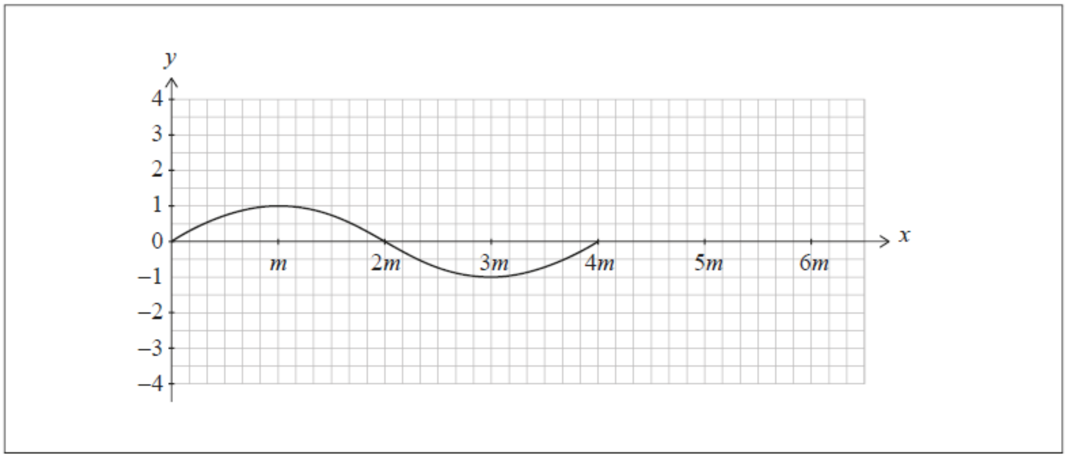
attempt to use 'Area$= \frac{1}{2}\text{base} \times \text{height}$' (must include their calculated value for height) *(M1)*

$= \frac{1}{2} \times 10 \times \frac{12}{5}$

$= 12$ (cm^2) *A1*

[6 marks]

5. [Maximum mark: 6]
The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0, 2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

Markscheme

recognition that period is $4m$ OR substitution of a point on f (except the origin) ***(M1)***

$4m = \frac{2\pi}{q}$ OR $1 = \sin qm$

$m = \frac{\pi}{2q}$ ***A1***

[2 marks]

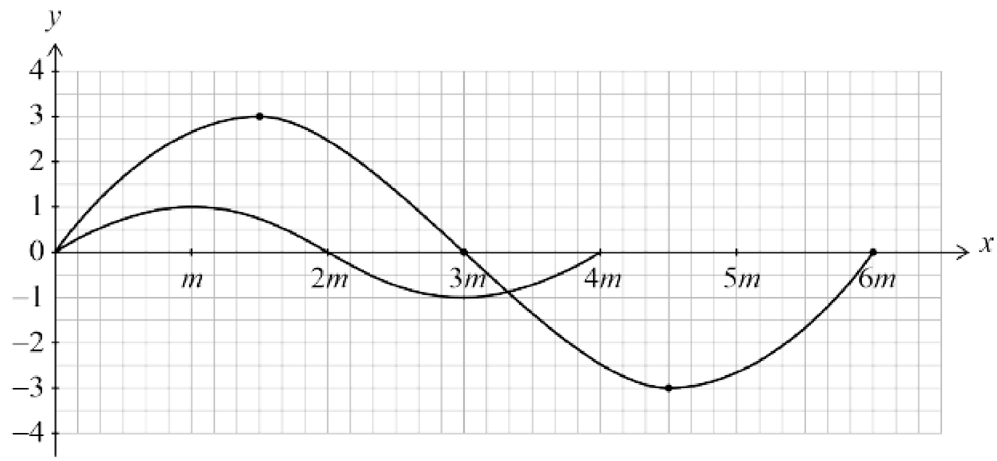
The function g is defined by $g\left(x\right) = 3\sin\frac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

Markscheme

horizontal scale factor is $\frac{3}{2}$ (seen anywhere) ***(A1)***

Note: This ***(A1)*** may be earned by seeing a period of $6m$, half period of $3m$ or the correct x-coordinate of the maximum/minimum point.



A1A1A1

Note:
Curve must be an approximate sinusoidal shape (sine or cosine).
Only in this case, award the following:
A1 for correct amplitude.
A1 for correct domain.
A1 for correct max and min points **and** correct x-intercepts.

[4 marks]

6. [Maximum mark: 14]
Consider an acute angle θ such that $\cos\,\theta = \frac{2}{3}$.

(a) Find the value of

(a.i) $\sin\,\theta$; [2]

Markscheme

attempt to use Pythagoras ***(M1)***

$\sin^2 \theta + \left(\frac{2}{3}\right)^2 = 1$ OR $x^2 + 2^2 = 3^2$ OR right triangle with side 2 and hypotenuse 3

$\sin \theta = \frac{\sqrt{5}}{3}$ ***A1***

[2 marks]

(a.ii) $\sin 2\theta$. [2]

Markscheme

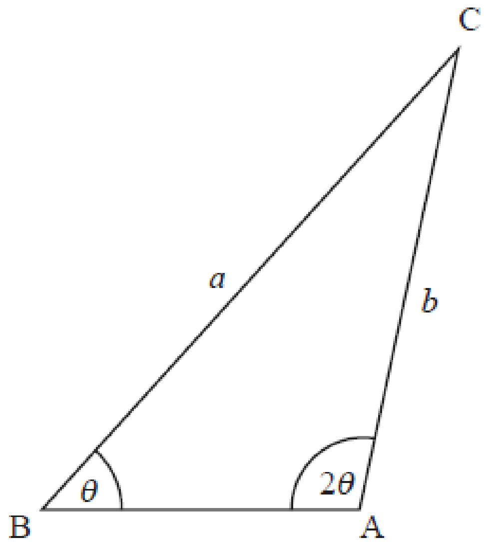
attempt to substitute into double-angle identity using their value of $\sin \theta$ ***(M1)***

$\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$ ***A1***

[2 marks]

The following diagram shows triangle ABC, with $\widehat{\mathrm{B}} = \theta, \widehat{\mathrm{A}} = 2\theta, \mathrm{BC} = a$ and $\mathrm{AC} = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

Markscheme

METHOD 1 (using values from part (a))

$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$

attempt to use sine rule with their values from part (a) ***(M1)***

$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)}$ OR $\frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$

correct working that leads to ***AG*** ***A1***

$\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3} = \frac{4b}{9}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

METHOD 2 (double-angle identity)

$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$

using double-angle identity ***(A1)***

$\frac{b}{\sin\theta} = \frac{a}{2\sin\theta\cos\theta}$ OR $b = \frac{a\sin\theta}{2\sin\theta\cos\theta}$ OR $b = \frac{a}{2\cos\theta}$

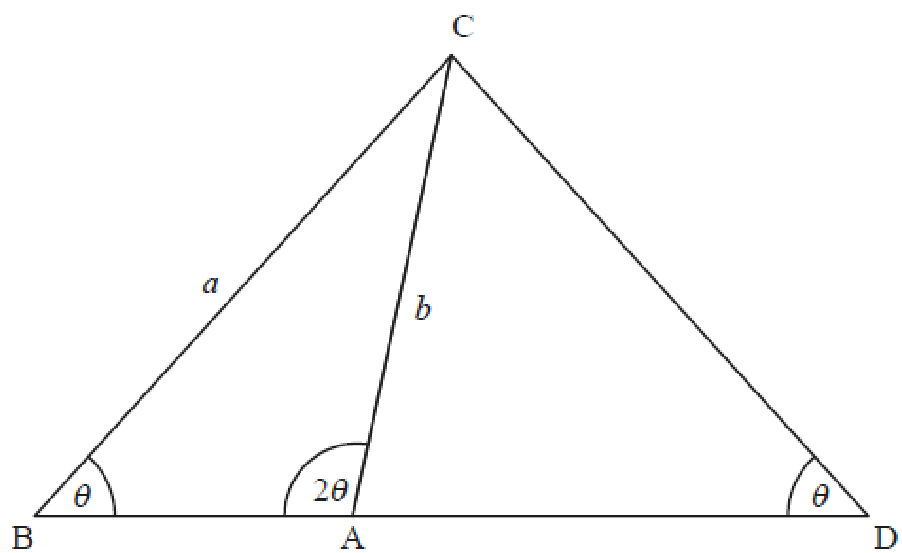
correct working (involving substituting $\cos\theta = \frac{2}{3}$) that leads to ***AG*** ***A1***

$b = \frac{a\sin\theta}{2\sin\theta\left(\frac{2}{3}\right)}$ OR $b = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)}$ OR $b = \frac{a}{2\left(\frac{2}{3}\right)}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

[2 marks]

$[\mathrm{BA}]$ is extended to form an isosceles triangle DAC, with $\hat{\mathrm{D}} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin\,\mathrm{C\hat{A}D}$. [3]

Markscheme

METHOD 1 (using supplementary angles)

recognizing $\mathrm{C\hat{A}D}$ and $\mathrm{B\hat{A}C}$ are supplementary ***(M1)*** recognizing supplementary angles have the same sine value ***(A1)***

$$\sin \mathrm{C\hat{A}D} = \sin 2\theta$$

$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$ ***A1***

METHOD 2 (using sine rule)

recognizing $\mathrm{CD} = a$ ***(M1)***

$$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{b}{\sin \theta}$$

correct substitution of $\sin \theta \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)***

$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)}$ **OR** $\sin \mathrm{C\hat{A}D} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)}$ (or equivalent)

$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$ ***A1***

[3 marks]

(d) Find the area of triangle DAC, in terms of a. [5]

Markscheme

METHOD 1 (using $\mathrm{C\hat{A}D}$ in area formula)

recognizing $\mathrm{D\hat{C}A} = \theta$ ***(A1)***

recognizing $AD = b \left(= \frac{3a}{4}\right)$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \mathrm{C\hat{A}D}$) ***(M1)***

$\text{area} = \frac{1}{2}\left(b\right)\left(b\right)\left(\frac{4\sqrt{5}}{9}\right)$ **OR** $\text{area} = \frac{1}{2}(b)(b)\sin 2\theta$ **OR**
$\text{area} = \frac{1}{2}\left(b\right)\left(b\right)\sin \text{C}\widehat{\text{A}}\text{D}$

correct substitution in terms of a *(A1)*

$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$

$\text{area} = \frac{\sqrt{5}a^2}{8}$ ***A1***

METHOD 2 (using $\text{A}\widehat{\text{C}}\text{D}$ or $\text{A}\widehat{\text{D}}\text{C}$ in area formula)

recognizing $\text{CD} = a$ *(A1)*

recognizing $AD = b\ \left(= \frac{3a}{4}\right)$ and/or $\text{D}\widehat{\text{C}}\text{A} = \theta$ *(A1)*

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin\ \text{A}\widehat{\text{D}}\text{C}$ or $\sin\ \text{A}\widehat{\text{C}}\text{D}$) *(M1)*

$\text{area} = \frac{1}{2}\left(a\right)\left(b\right)\left(\frac{\sqrt{5}}{3}\right)$ OR $\text{area} = \frac{1}{2}(a)(b)\sin\theta$ OR
$\text{area} = \frac{1}{2}\left(a\right)\left(b\right)\sin \text{A}\widehat{\text{D}}\text{C}$

OR $\text{area} = \frac{1}{2}\left(a\right)\left(b\right)\sin \text{A}\widehat{\text{C}}\text{D}$

correct substitution in terms of a *(A1)*

$\text{area} = \frac{1}{2}\left(a\right)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\text{area} = \frac{\sqrt{5}a^2}{8}$ ***A1***

[5 marks]

7. [Maximum mark: 5]

Let $f(x) = \cos(x - k)$, where $0 \le x \le a$ and $a,\ k \in \mathbb{R}^+$.

(a) Consider the case where $k = \frac{\pi}{2}$.

By sketching a suitable graph, or otherwise, find the largest value of a for which the inverse function f^{-1} exists. [2]

Markscheme

$a = \frac{\pi}{2}$ ***A2***

Note: For sinusoidal graph through the origin seen with incorrect a, or use of horizontal line test with incorrect a, award ***A1A0***

[2 marks]

(b) Find the largest value of a for which the inverse function f^{-1} exists in the case where $k = \pi$. [1]

Markscheme

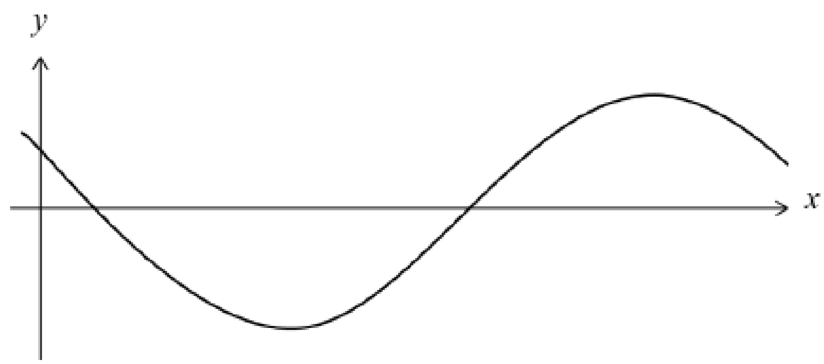
$a = \pi$ ***A1***

[1 mark]

(c) Find the largest value of a for which the inverse function f^{-1} exists in the case where $\pi < k < 2\pi$. Give your answer in terms of k.

[2]

Markscheme



sketch showing sinusoidal shape decreasing as it crosses the y-axis

(below or above the origin) ***(A1)***

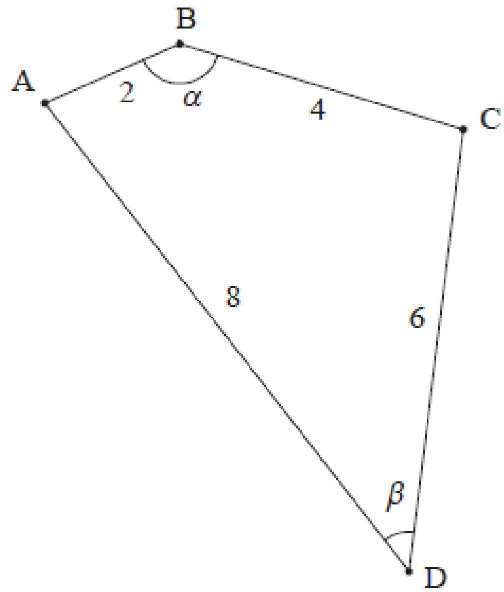
$a = k - \pi$ ***A1***

[2 marks]

8. [Maximum mark: 8]

Consider a quadrilateral ABCD such that $\mathrm{AB} = 2,\ \mathrm{BC} = 4,\ \mathrm{CD} = 6$ and $\mathrm{DA} = 8$, as shown in the following diagram. Let $\alpha = \mathrm{A\hat{B}C}$ and $\beta = \mathrm{A\hat{D}C}$.

diagram not to scale



(a.i) Find AC in terms of α. [2]

Markscheme

attempt to use the cosine rule *(M1)*

$$\mathrm{AC} = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha}\left(= \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha}\right)$$

A1

[2 marks]

(a.ii) Find AC in terms of β. [1]

Markscheme

$$\mathrm{AC} = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta}\left(= \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta}\right)$$

A1

[1 mark]

(a.iii) Hence or otherwise, find an expression for α in terms of β. [1]

Markscheme

$5 - 4\cos\alpha = 25 - 24\cos\beta$

$\alpha = \arccos(6\cos\beta - 5)$ ***A1***

[1 mark]

(b) Find the maximum area of the quadrilateral ABCD. [4]

Markscheme

attempt to find the sum of two triangle areas using $A = \frac{1}{2}ab\sin C$ ***(M1)***

Note: Do not award this ***M1*** if the triangle is assumed to be right angled.

$\text{Area} = \frac{1}{2}(8)\sin\alpha + \frac{1}{2}(48)\sin\beta$ ***(A1)***

attempt to express the area in terms of one variable only ***(M1)***

$= 4\sqrt{1 - (6\cos\beta - 5)^2} + 24\sin\beta$ or
$4\sin(\arccos(6\cos\beta - 5)) + 24\sin\beta$ OR

$4\sin\alpha + 24\sqrt{1 - \left(\frac{5+\cos\alpha}{6}\right)^2}$ or
$4\sin\alpha + 24\sin\left(\arccos\left(\frac{5+\cos\alpha}{6}\right)\right)$

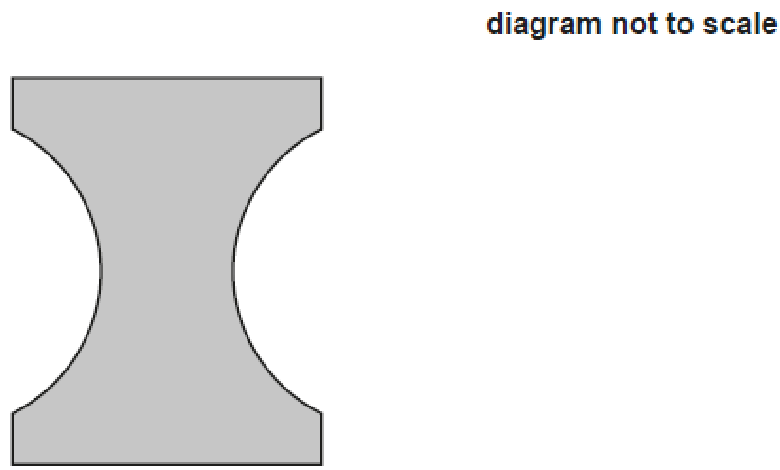
Max area $= 19.5959\ldots$

$= 19.6$ ***A1***

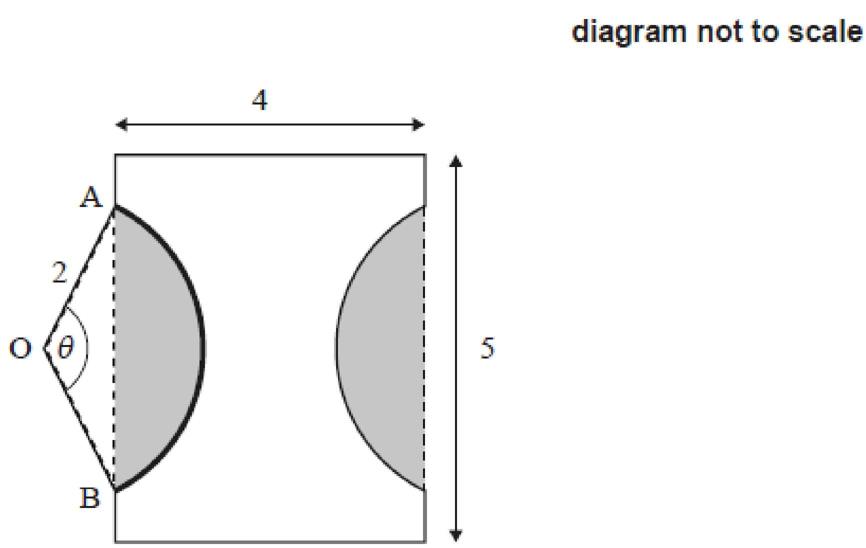
[4 marks]

9. [Maximum mark: 6]

A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.



(a) Find the area of one of the shaded segments in terms of θ. [3]

Markscheme

valid approach to find area of segment by finding area of sector – area of triangle ***(M1)***

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta$

$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2 \sin \theta$ ***(A1)***

area $= 2\theta - 2 \sin \theta$ ***A1***

[3 marks]

(b) Given that the area of the logo is 13.4cm^2, find the value of θ. [3]

Markscheme

EITHER

area of logo = area of rectangle – area of segments ***(M1)***

$5 \times 4 - 2 \times (2\theta - 2 \sin \theta) = 13.4$ ***(A1)***

OR

area of one segment $= \frac{20-13.4}{2} (= 3.3)$ ***(M1)***

$2\theta - 2 \sin \theta = 3.3$ ***(A1)***

THEN

$\theta = 2.35672\ldots$

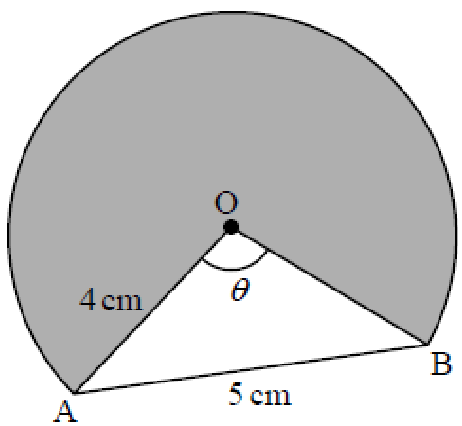
$\theta = 2.36$ (do not accept an answer in degrees) ***A1***

Note: Award ***(M1)(A1)A0*** if there is more than one solution.
Award ***(M1)(A1FT)A0*** if the candidate works in degrees and obtains a final answer of $135.030\ldots$

[3 marks]

10. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and AÔB = θ.

(a) Find the value of θ, giving your answer in radians. [3]

Markscheme

METHOD 1

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2+4^2-5^2}{2\times4\times4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) Find the area of the shaded region. [3]

Markscheme

attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35\ldots)$ ***A1***

$= 39.5$ (cm^2) ***A1***

[3 marks]

11. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

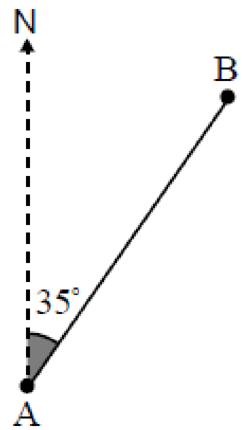
(a) Find the distance from point A to point B. [2]

Markscheme

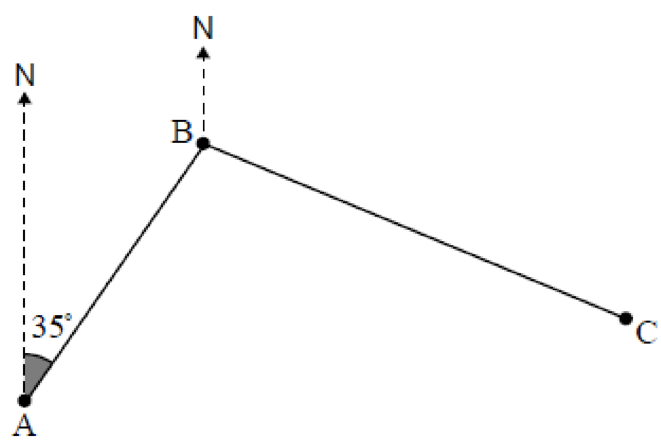
$\frac{4.2}{60} \times 45$ ***A1***

AB = 3.15 (km) ***A1***

[2 marks]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



(b.i) Show that $\hat{ABC}$ is 101°. [2]

Markscheme

66° or (180 – 114) ***A1***

35 + 66 ***A1***

$\hat{ABC}$ = 101° ***AG***

[2 marks]

(b.ii) Find the distance from the camp to point C. [3]

Markscheme

attempt to use cosine rule ***(M1)***

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101°$ (or equivalent) ***A1***

$AC = 6.05$ (km) ***A1***

[3 marks]

(c) Find $\hat{BCA}$. [3]

Markscheme

valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(\hat{BCA}\right)}{3.15} = \frac{\sin 101}{6.0507\ldots}$

$\hat{BCA} = 30.7°$ ***A1***

[3 marks]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C .

(d) Find the bearing that Jacob must take to point C. [3]

Markscheme

$\mathrm{B\hat{A}C}$ = 48.267 (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg 48.267 + 35

bearing = 83.3° (accept 083°) ***A1***

[3 marks]

(e) Jacob hikes at an average speed of 3.9 km/h.

Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Markscheme

attempt to use $\mathrm{time} = \frac{\mathrm{distance}}{\mathrm{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

t = 93 (minutes) ***A1***

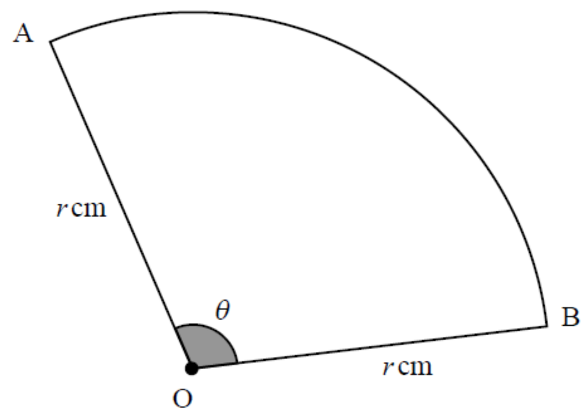
[3 marks]

12. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius $r\ \mathrm{cm}$ with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\ \text{cm}$ and the area of the sector is $6.25\ \text{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

Markscheme

$2r + r\theta = 10$ ***A1***

$\frac{1}{2}r^2\theta = 6.25$ ***A1***

attempt to eliminate θ to obtain an equation in r ***M1***

correct intermediate equation in r ***A1***

$10 - 2r = \frac{25}{2r}$ OR $\frac{10}{r} - 2 = \frac{25}{2r^2}$ OR $\frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25$ OR $1.25 + 2r^2 = 10r$

$4r^2 - 20r + 25 = 0$ ***AG***

[4 marks]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

Markscheme

attempt to solve quadratic by factorizing or use of formula or completing the square ***(M1)***

$(2r-5)^2 = 0$ OR $r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \left(= \frac{20 \pm \sqrt{400 - 400}}{8} \right)$

$r = \frac{5}{2}$ ***A1***

attempt to substitute their value of r into their perimeter or area equation ***(M1)***

$\theta = \frac{10 - 2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$ or $\theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$

$\theta = 2$ ***A1***

[4 marks]

13. [Maximum mark: 16]

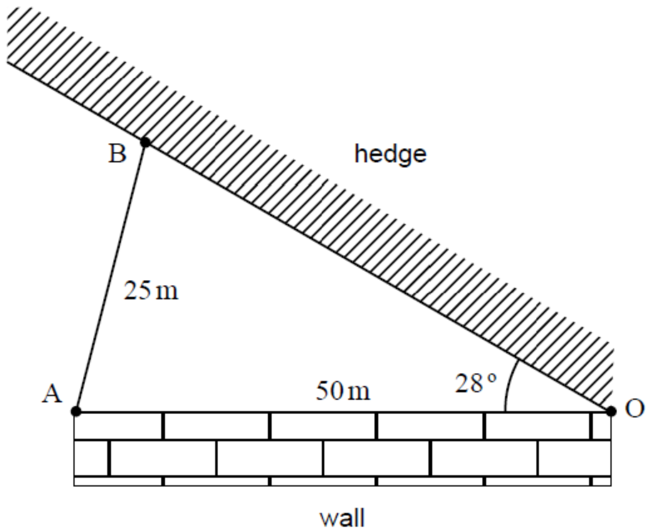
All angles in this question are given in degrees.

A farmer owns land which lies between a wall and a hedge. The wall has a length of $50\,\mathrm{m}$ and lies between points O and A. The hedge meets the wall at O and the angle between the wall and the hedge is 28°.

The farmer plans to form a triangular field for her prizewinning goats by placing a fence with a fixed length of 25 metres from point A to the hedge. The fence meets the hedge at a point B.

The information is shown on the following diagram.

diagram not to scale



(a.i) Find the two possible sizes of $\mathrm{O\hat{A}B}$, giving your answers in degrees. [5]

Markscheme

METHOD 1

attempt to use the sine rule to find $\mathrm{A\hat{B}O}$ ***(M1)***

$$\frac{25}{\sin 28^\circ} = \frac{50}{\sin \mathrm{A\hat{B}O}}$$

$\left(\mathrm{A\hat{B}O} =\right) 69.8748\ldots^\circ$ or $\left(\mathrm{A\hat{B}O} =\right) 110.125\ldots^\circ$ ***(A1)(A1)***

Note: Award ***A1*** for each value.

attempt to find at least one possible angle for $\mathrm{O\hat{A}B}$ ***(M1)***

$\left(\mathrm{O\hat{A}B} =\right) 180^\circ - 28^\circ - 69.8748\ldots^\circ$ OR
$\left(\mathrm{O\hat{A}B} =\right) 180^\circ - 28^\circ - 110.125\ldots^\circ$

$\mathrm{O\hat{A}B} = 82.1251\ldots^\circ,\ 41.8748\ldots^\circ$

$\text{OAB} = 82.1^\circ,\ 41.9^\circ$ ***A1***

METHOD 2

attempt to use the cosine rule to find OB ***(M1)***

$25^2 = 50^2 + \text{OB}^2 - (50)(\text{OB})\ \cos\ 28^\circ$

$\text{OB} = 52.7491\ldots$ or $35.5455\ldots$ ***(A1)(A1)***

attempt to use the sine rule to find $\text{O}\widehat{\text{A}}\text{B}$ ***(M1)***

$\frac{25}{\sin\ 28^\circ} = \frac{\text{OB}}{\sin\ \text{O}\widehat{\text{A}}\text{B}}$

$\text{O}\widehat{\text{A}}\text{B} = 82.1251\ldots^\circ,\ 41.8748\ldots^\circ$

$\text{O}\widehat{\text{A}}\text{B} = 82.1^\circ,\ 41.9^\circ$ ***A1***

[5 marks]

(a.ii) Hence, find the two possible areas of the triangular field. [3]

Markscheme

attempt to substitute two sides of triangle OAB and one of their angles into the area formula $\frac{1}{2}ab\ \sin\ \text{C}$ ***(M1)***

$\frac{1}{2}(\text{OA})(\text{AB})\ \sin\ 82.1251\ldots^\circ$ OR $\frac{1}{2}(50)(25)\ \sin\ 41.8748\ldots^\circ$
OR

$\frac{1}{2}(50)(52.7491\ldots)\ \sin\ 28^\circ$ OR $\frac{1}{2}(50)(35.5455)\ \sin\ 28^\circ$

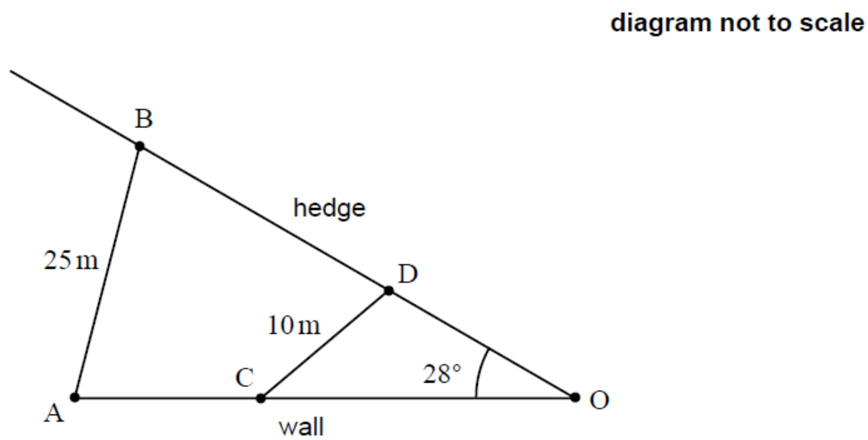
Area $= 619.106\ldots$ OR $= 417.190\ldots$

$= 619\,(\text{m})^2$ OR $= 417\left(\text{m}^2\right)$ ***A1A1***

[3 marks]

One of the goats, Brenda, fights with the other goats. The farmer plans to place a second fence with a fixed length of 10 metres between the wall and the hedge to form a small triangular field inside OAB for Brenda.

The information is shown on the following diagram.



The small triangular field OCD has an area of $60\,\mathrm{m}^2$.

Let x be the distance OC and let y be the distance OD.

(b) Show that $x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$. [5]

Markscheme

attempt to use the cosine rule in triangle OCD ***(M1)***

$10^2 = x^2 + y^2 - 2xy\cos 28^\circ$ ***A1***

attempt to use the area formula in triangle OCD ***(M1)***

$\frac{1}{2}xy\sin 28^\circ = 60$ ***A1***

Note: Award ***(M1)A1*** for use of the area formula independently of the ***(M1)A1*** for use of the cosine rule.

$xy = \frac{120}{\sin 28^\circ}$

$100 = x^2 + y^2 - \frac{240 \cos 28^\circ}{\sin 28^\circ}$ ***A1***

$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$ ***AG***

[5 marks]

(c) Hence, determine the two possible lengths of OC. [3]

Markscheme

EITHER

attempt to eliminate y or x ***(M1)***

$100 = x^2 + \left(\frac{120}{x \sin 28^\circ}\right)^2 - \frac{240}{\tan 28^\circ}$

OR

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula ***(M1)***

Note: Award ***(M1)*** only if their graphs are of functions with the same subject e.g. both "$y = \ldots$" or both "$y^2 = \ldots$".

THEN

$x = 13.1300\ldots$ or $x = 19.4673\ldots$

$x = 13.1$ or $x = 19.5$ ***A1A1***

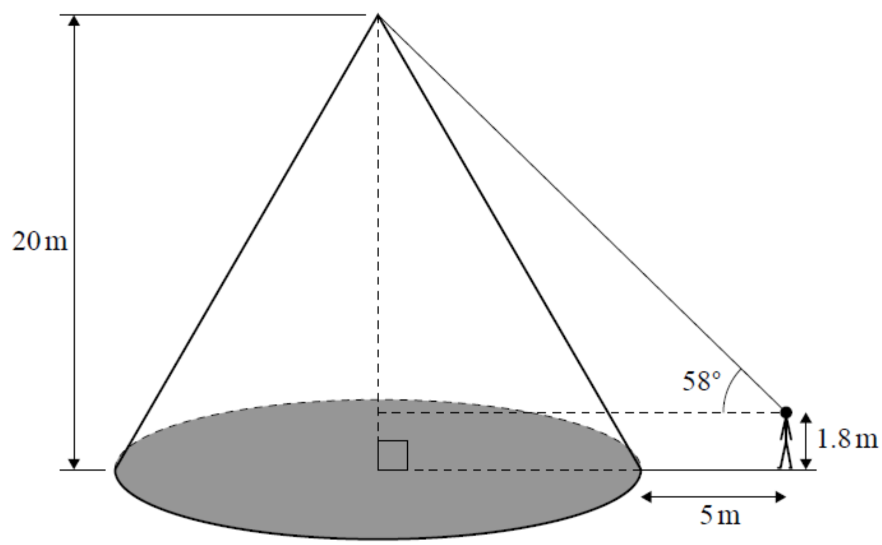
$\mathrm{OC} = 13.1\,(\mathrm{m})$ or $19.5\,(\mathrm{m})$

[3 marks]

14. [Maximum mark: 5]

A monument is in the shape of a right cone with a vertical height of 20 metres. Oliver stands 5 metres from the base of the monument. His eye level is 1.8 metres above the ground and the angle of elevation from Oliver's eye level to the vertex of the cone is 58°, as shown on the following diagram.

diagram not to scale



(a) Find the radius of the base of the cone. [3]

Markscheme

attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r + 5)$ ***(M1)***

$\tan 58^\circ = \frac{18.2}{r+5}$ OR $\frac{r+5}{\sin 32^\circ} = \frac{18.2}{\sin 58^\circ}$ OR $(r + 5 =) 11.3726\ldots$ ***(A1)***

$r = 6.37262\ldots$ (m)

$(r =)\ 6.37$ (m) ***A1***

[3 marks]

(b) Find the volume of the monument. [2]

Markscheme

attempt to substitute $h = 20$ and their radius into the correct volume of cone formula ***(M1)***

$V = \frac{\pi(6.37262...)^2(20)}{3}$

$= 850.540\ldots$

$= 851\,(\mathrm{m})^3$ ***(A1)***

Note: Accept $849.840\ldots\ (850)$ obtained from using $r = 6.37$.

[2 marks]