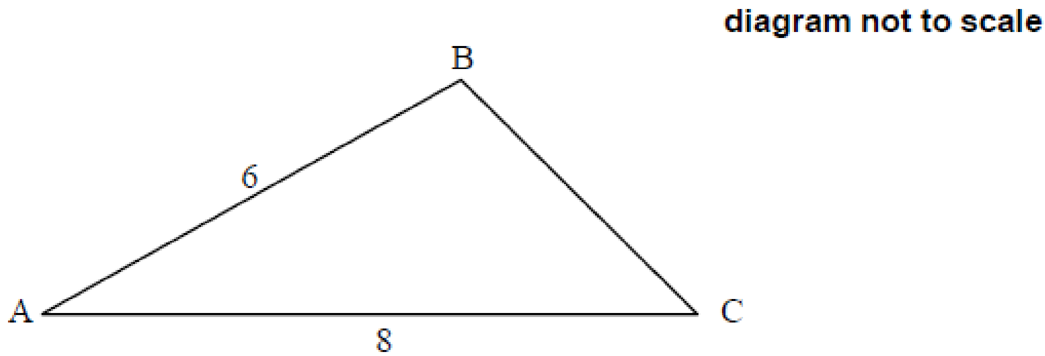
Trigonometry review (research days) [120 marks]

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



(a) Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3]

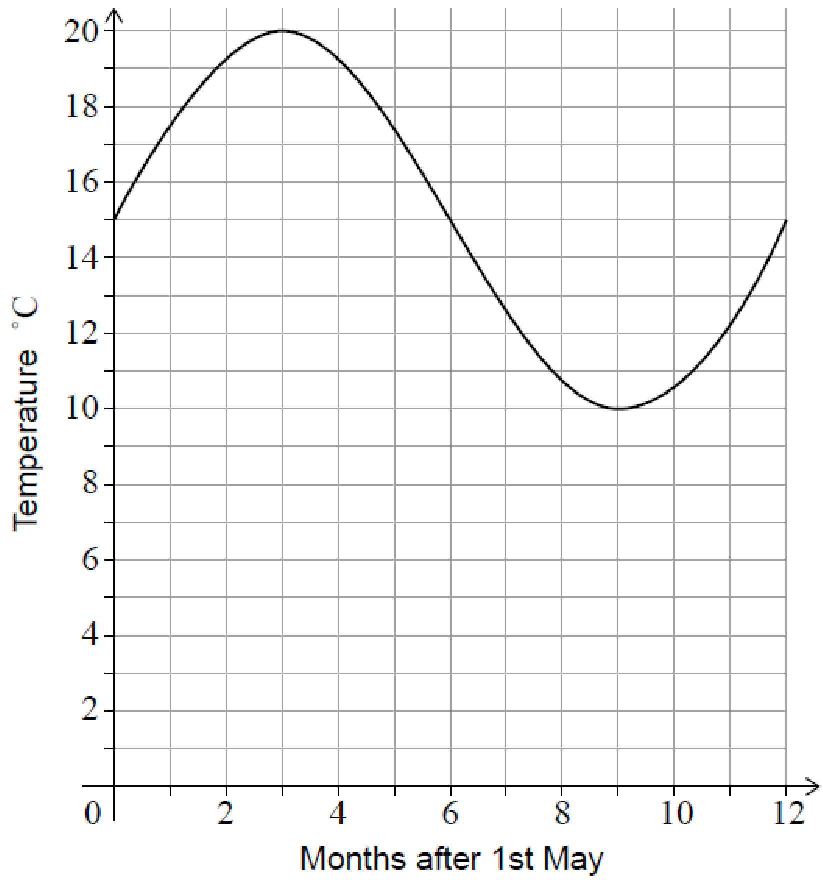
(b) Find the area of triangle ABC. [2]

2. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least 15°C. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a \sin bx + c$ for $0 \leq x \leq 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(b.i) a; [1]

(b.ii) c. [1]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6}x + 11$, where $0 \leq x \leq 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

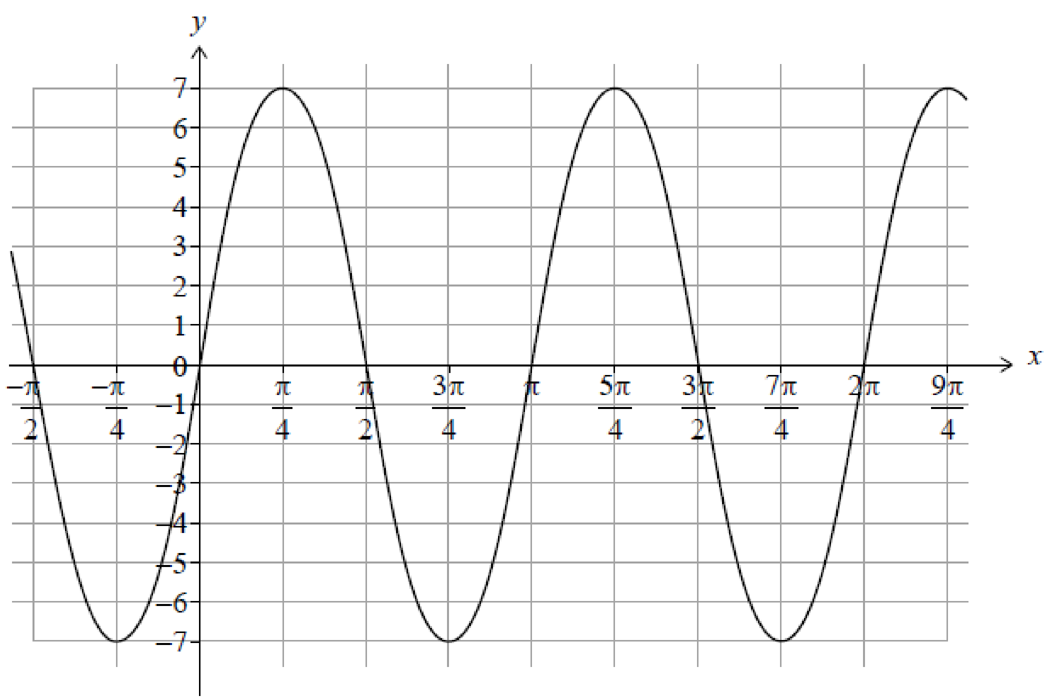
(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

3. [Maximum mark: 7]

Consider the function $f(x) = a \sin (bx)$ with $a,\ b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

(b.i) Write down the period of f. [1]

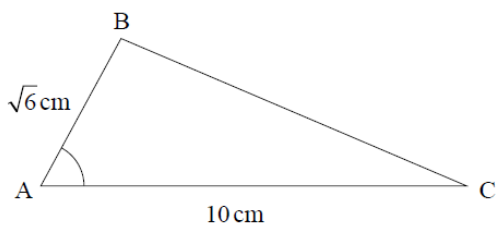
(b.ii) Hence, find the value of b. [2]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

4. [Maximum mark: 6]

In the following triangle ABC, $\mathrm{AB} = \sqrt{6\ \mathrm{cm}}$, $\mathrm{AC} = 10\ \mathrm{cm}$ and $\cos\ \mathrm{B\hat{A}C} = \frac{1}{5}$.

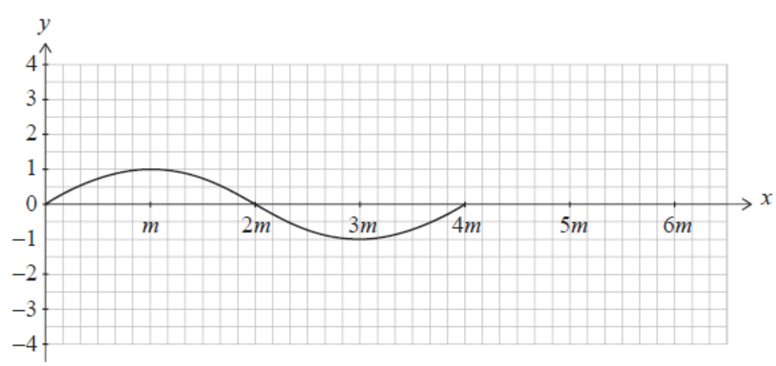
diagram not to scale



Find the area of triangle ABC. [6]

5. [Maximum mark: 6]

The function f is defined by $f(x) = \sin\ qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0, 2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

The function g is defined by $g\left(x\right) = 3\ \sin\ \frac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

6. [Maximum mark: 14]

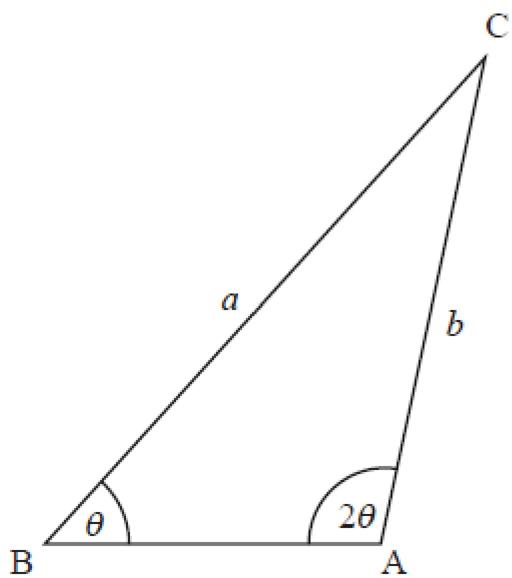
Consider an acute angle θ such that $\cos\,\theta = \frac{2}{3}$.

(a) Find the value of

(a.i) $\sin\,\theta$; [2]

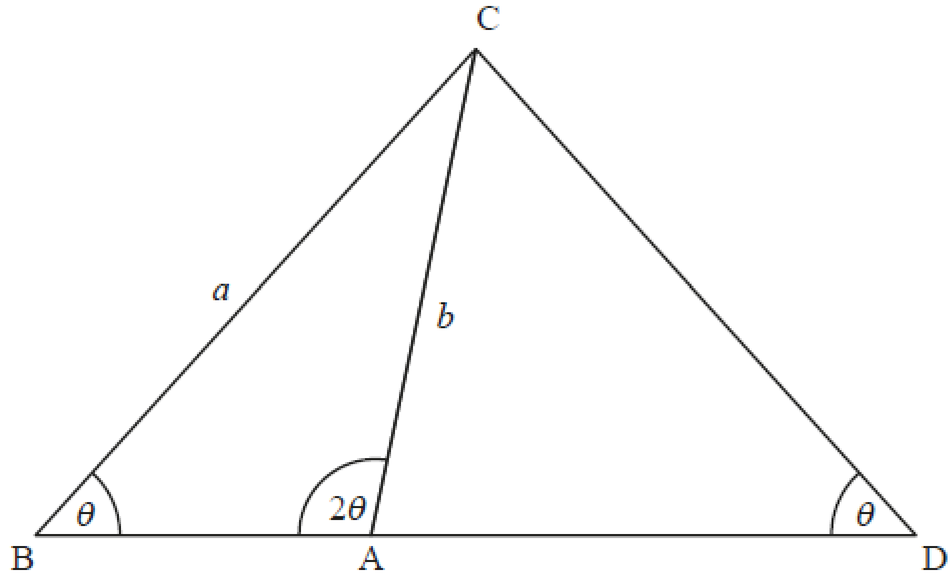
(a.ii) $\sin\,2\theta$. [2]

The following diagram shows triangle ABC, with $\widehat{\mathrm{B}} = \theta$, $\widehat{\mathrm{A}} = 2\theta$, $\mathrm{BC} = a$ and $\mathrm{AC} = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

$[\mathrm{BA}]$ is extended to form an isosceles triangle DAC, with $\widehat{\mathrm{D}} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin \mathrm{C\hat{A}D}$. [3]

(d) Find the area of triangle DAC, in terms of a. [5]

7. [Maximum mark: 5]

Let $f(x) = \cos(x - k)$, where $0 \le x \le a$ and $a,\ k \in \mathbb{R}^+$.

(a) Consider the case where $k = \frac{\pi}{2}$.

By sketching a suitable graph, or otherwise, find the largest value of a for which the inverse function f^{-1} exists. [2]

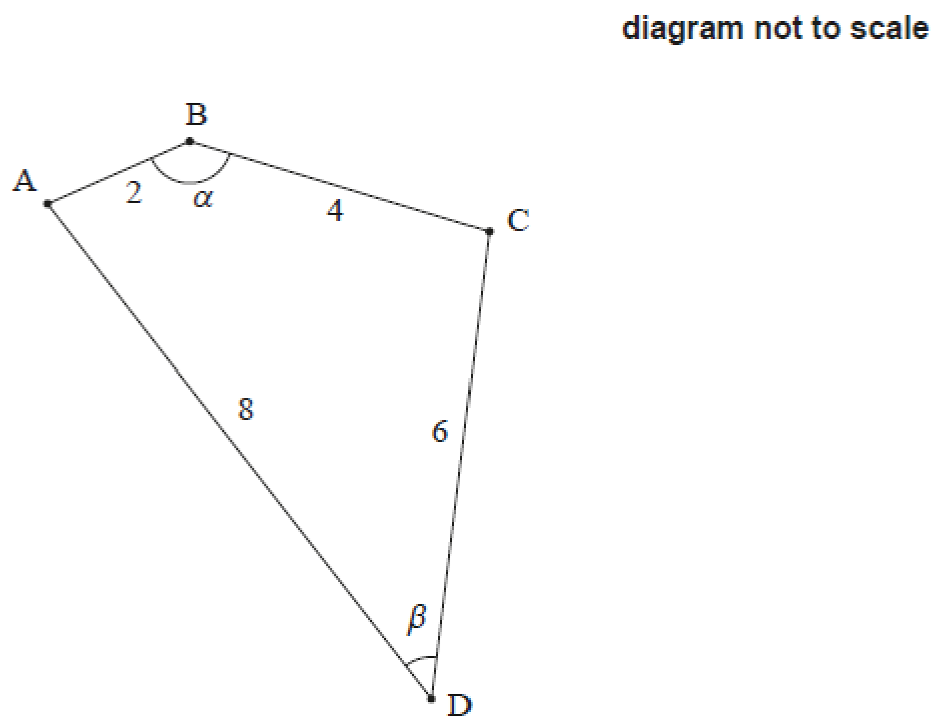
(b) Find the largest value of a for which the inverse function f^{-1} exists in the case where $k = \pi$. [1]

(c) Find the largest value of a for which the inverse function f^{-1} exists in the case where $\pi < k < 2\pi$. Give your answer in terms of k.

[2]

8. [Maximum mark: 8]

Consider a quadrilateral ABCD such that $\mathrm{AB} = 2,\ \mathrm{BC} = 4,\ \mathrm{CD} = 6$ and $\mathrm{DA} = 8$, as shown in the following diagram. Let $\alpha = \mathrm{A\widehat{B}C}$ and $\beta = \mathrm{A\widehat{D}C}$.



(a.i) Find AC in terms of α. [2]

(a.ii) Find AC in terms of β. [1]

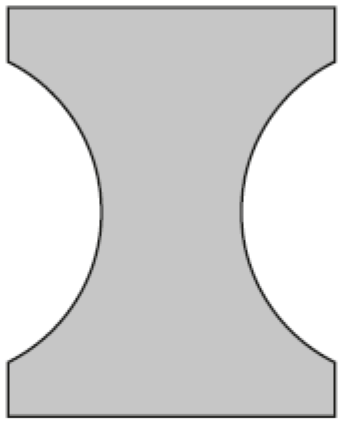
(a.iii) Hence or otherwise, find an expression for α in terms of β. [1]

(b) Find the maximum area of the quadrilateral ABCD. [4]

9. [Maximum mark: 6]

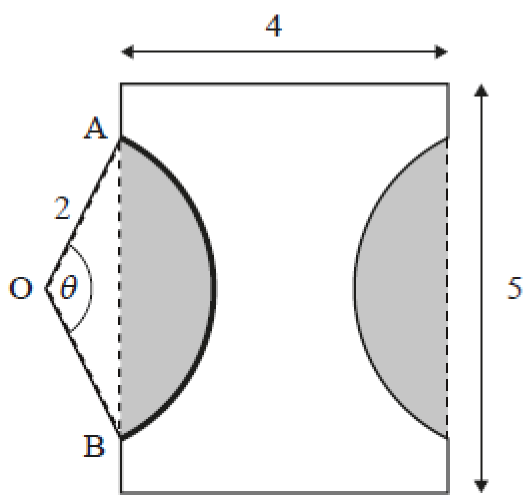
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale

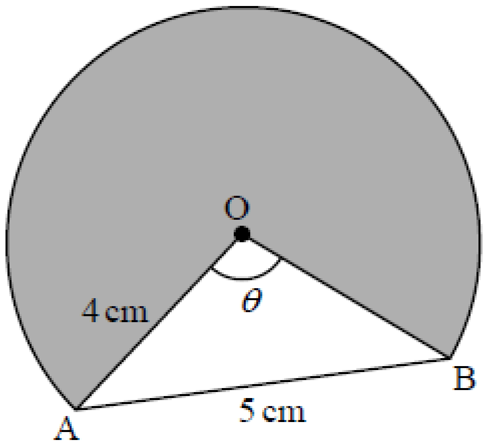


(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

10. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and $A\hat{O}B = \theta$.

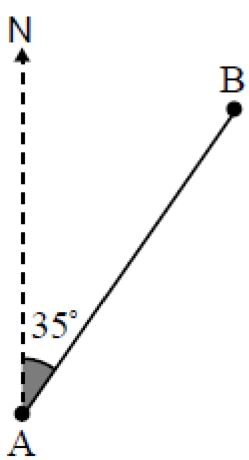
(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

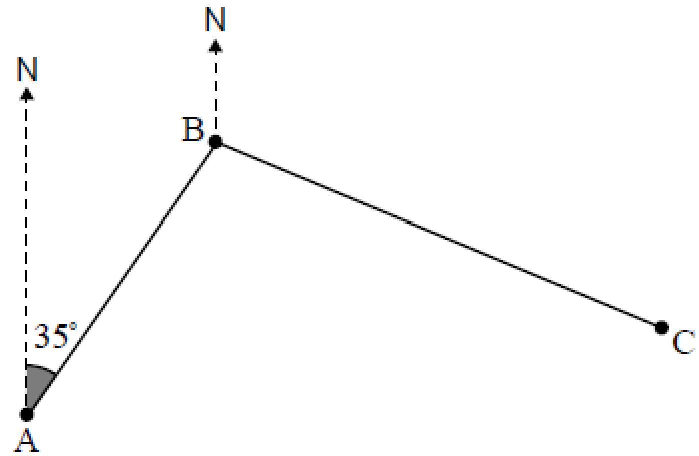
11. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



(b.i) Show that $\hat{ABC}$ is 101°. [2]

(b.ii) Find the distance from the camp to point C. [3]

(c) Find $B\hat{C}A$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C .

(d) Find the bearing that Jacob must take to point C. [3]

(e) Jacob hikes at an average speed of 3.9 km/h.

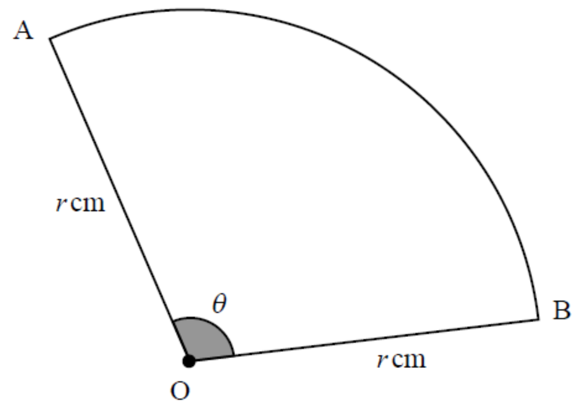
Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

12. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\ \mathrm{cm}$ and the area of the sector is $6.\ 25\ \mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

13. [Maximum mark: 16]

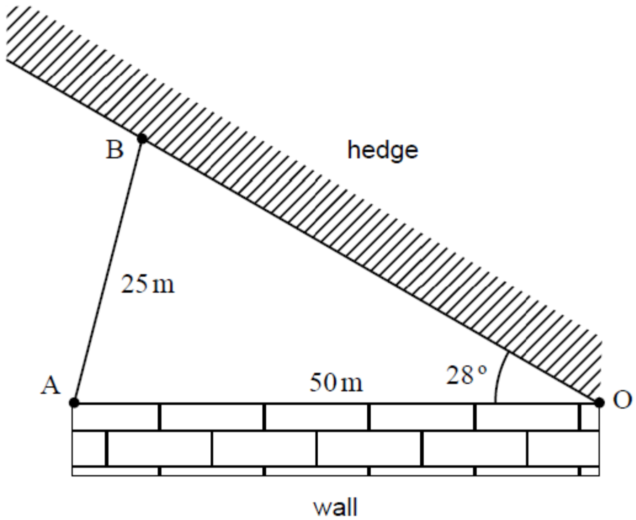
All angles in this question are given in degrees.

A farmer owns land which lies between a wall and a hedge. The wall has a length of $50\,\mathrm{m}$ and lies between points O and A. The hedge meets the wall at O and the angle between the wall and the hedge is 28°.

The farmer plans to form a triangular field for her prizewinning goats by placing a fence with a fixed length of 25 metres from point A to the hedge. The fence meets the hedge at a point B.

The information is shown on the following diagram.

diagram not to scale



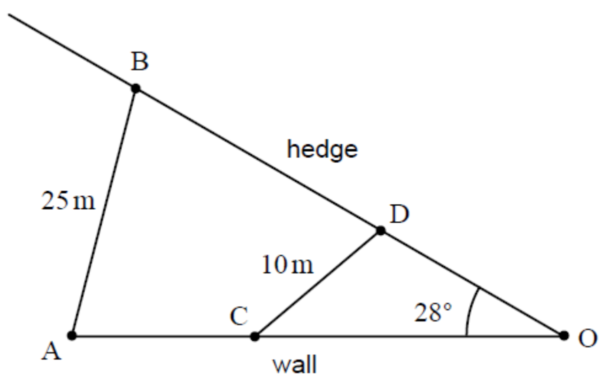
(a.i) Find the two possible sizes of $O\hat{A}B$, giving your answers in degrees. [5]

(a.ii) Hence, find the two possible areas of the triangular field. [3]

One of the goats, Brenda, fights with the other goats. The farmer plans to place a second fence with a fixed length of 10 metres between the wall and the hedge to form a small triangular field inside OAB for Brenda.

The information is shown on the following diagram.

diagram not to scale



The small triangular field OCD has an area of $60\,\mathrm{m}^2$.

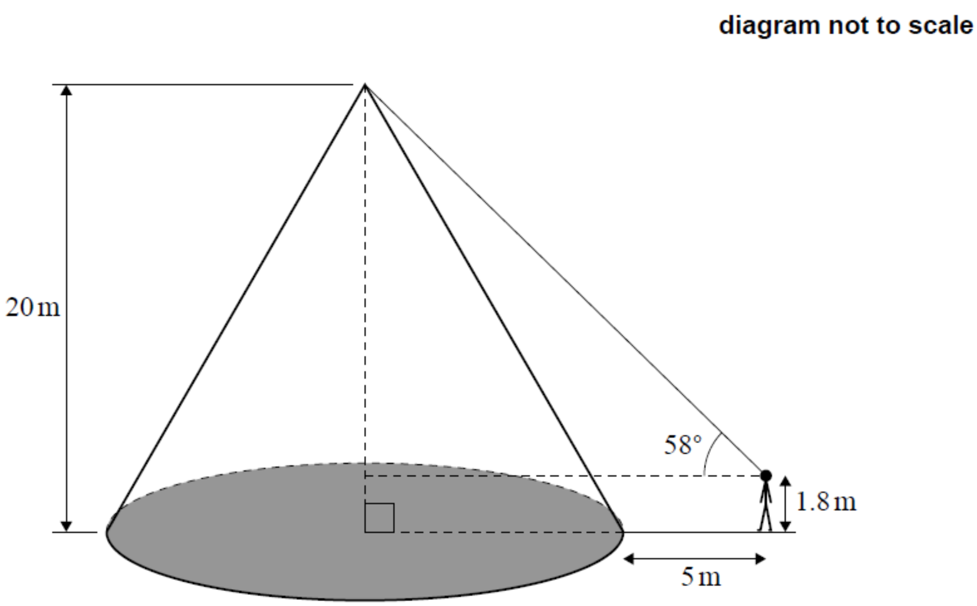
Let x be the distance OC and let y be the distance OD.

(b) Show that $x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$. [5]

(c) Hence, determine the two possible lengths of OC. [3]

14. [Maximum mark: 5]

A monument is in the shape of a right cone with a vertical height of 20 metres. Oliver stands 5 metres from the base of the monument. His eye level is 1.8 metres above the ground and the angle of elevation from Oliver's eye level to the vertex of the cone is 58°, as shown on the following diagram.



(a) Find the radius of the base of the cone. [3]

(b) Find the volume of the monument. [2]