Trigonometry [134 marks]

1. [Maximum mark: 5] 22M.1.SL.TZ2.5

Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$. [5]

2. [Maximum mark: 6] 23M.1.AHL.TZ1.3

Solve $\cos 2x=\sin x$, where $-\pi \leq x \leq \pi$. [6]

3. [Maximum mark: 5] 18M.2.AHL.TZ1.H_3

Let $f\,(x)=\tan\,(x+\pi)\cos\,\left(x-\frac{\pi}{2}\right)$ where $0<x<\frac{\pi}{2}$.

Express $f\,(x)$ in terms of sin x and cos x. [5]

4. [Maximum mark: 7] 21N.1.SL.TZ0.6

(a) Show that $2x-3-\frac{6}{x-1}=\frac{2x^2-5x-3}{x-1},\ x\in\mathbb{R},\ x\neq 1$

. [2]

(b) Hence or otherwise, solve the equation

$2\sin 2\theta-3-\frac{6}{\sin 2\theta-1}=0$ for $0\leq\theta\leq\pi,\ \theta\neq\frac{\pi}{4}$. [5]

5. [Maximum mark: 7] 22M.1.SL.TZ1.4

Consider the functions $f\left(x\right)=\sqrt{3}\sin x+\cos x$ where $0\leq x\leq\pi$ and $g(x)=2x$ where $x\in\mathbb{R}$.

(a) Find $(f\circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]

6. [Maximum mark: 8] 21M.1.SL.TZ1.6

(a) Show that $\sin 2x + \cos 2x - 1 = 2 \sin x(\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$.

[6]

7. [Maximum mark: 5] 18M.1.AHL.TZ1.H_8

Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$.

Find, in terms of *b*, the solutions of $\sin 2x = -a,\ 0 \leqslant x \leqslant \pi$. [5]

8. [Maximum mark: 4] 21M.1.AHL.TZ1.6

It is given that $\operatorname{cosec} \theta = \frac{3}{2}$, where $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$. Find the exact value of $\cot \theta$. [4]

9. [Maximum mark: 7] 21M.1.AHL.TZ2.2

Solve the equation $2 \cos^2 x + 5 \sin x = 4,\ 0 \leq x \leq 2\pi$. [7]

10. [Maximum mark: 8] 19N.1.SL.TZ0.S_6

Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \leqslant x \leqslant 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$. [8]

11. [Maximum mark: 7] 19N.1.AHL.TZ0.H_4

A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$.

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$. [7]

12. [Maximum mark: 7] 18N.1.AHL.TZ0.H_3

Consider the function $g(x) = 4\cos x + 1$, $a \leqslant x \leqslant \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c.i) For the value of a found in part (b), write down the domain of g^{-1}. [1]

(c.ii) For the value of a found in part (b), find an expression for $g^{-1}(x)$. [2]

13. [Maximum mark: 19] SPM.2.AHL.TZ0.12

(a) Show that $\cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$. [1]

(b) Verify that $x = \tan\theta$ and $x = -\cot\theta$ satisfy the equation $x^2 + (2\cot 2\theta)x - 1 = 0$. [7]

(c) Hence, or otherwise, show that the exact value of $\tan\frac{\pi}{12} = 2 - \sqrt{3}$.

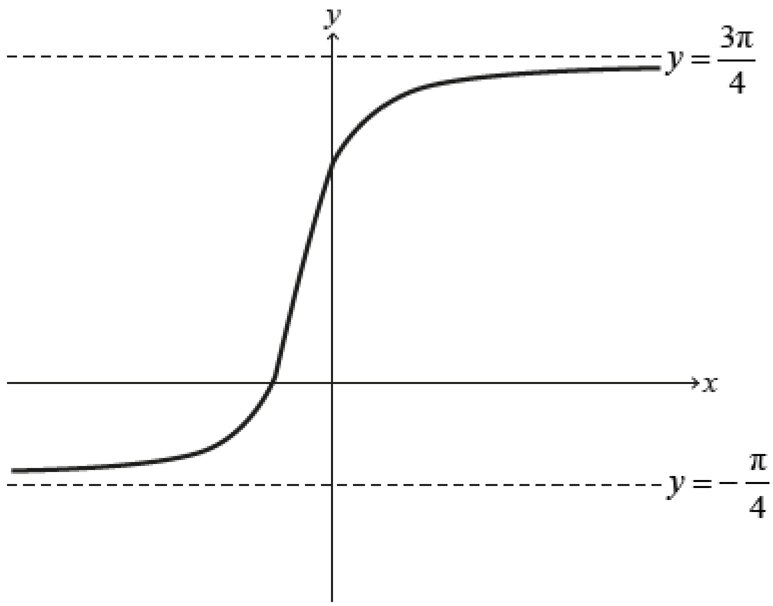
[5]

(d) Using the results from parts (b) and (c) find the exact value of $\tan\frac{\pi}{24} - \cot\frac{\pi}{24}$.

Give your answer in the form $a + b\sqrt{3}$ where $a, b \in \mathbb{Z}$. [6]

14. [Maximum mark: 19] 21M.1.AHL.TZ2.12

The following diagram shows the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, with asymptotes at $y = -\frac{\pi}{4}$ and $y = \frac{3\pi}{4}$.



(a) Describe a sequence of transformations that transforms the graph of $y = \arctan x$ to the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$. [3]

(b) Show that $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where $p,\ q > 0$ and $pq < 1$. [4]

(c) Verify that $\arctan(2x+1) = \arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}$ for $x \in \mathbb{R},\ x > 0$. [3]

(d) Using mathematical induction and the result from part (b), prove that $\sum_{r=1}^{n} \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$. [9]

15. [Maximum mark: 20] 22M.1.AHL.TZ2.11

A function f is defined by $f(x) = \frac{1}{x^2-2x-3}$, where $x \in \mathbb{R},\ x \neq -1,\ x \neq 3$.

(a) Sketch the curve $y = f(x)$, clearly indicating any asymptotes with their equations. State the coordinates of any local maximum or minimum points and any points of intersection with the coordinate axes. [6]

A function g is defined by $g(x) = \frac{1}{x^2-2x-3}$, where $x \in \mathbb{R},\ x > 3$.

The inverse of g is g^{-1}.

(b.i) Show that $g^{-1}(x) = 1 + \frac{\sqrt{4x^2+x}}{x}$. [6]

(b.ii) State the domain of g^{-1}. [1]

A function h is defined by $h(x) = \arctan\frac{x}{2}$, where $x \in \mathbb{R}$.

(c) Given that $(h \circ g)(a) = \frac{\pi}{4}$, find the value of a.

Give your answer in the form $p + \frac{q}{2}\sqrt{r}$, where $p,\ q,\ r \in \mathbb{Z}^+$. [7]