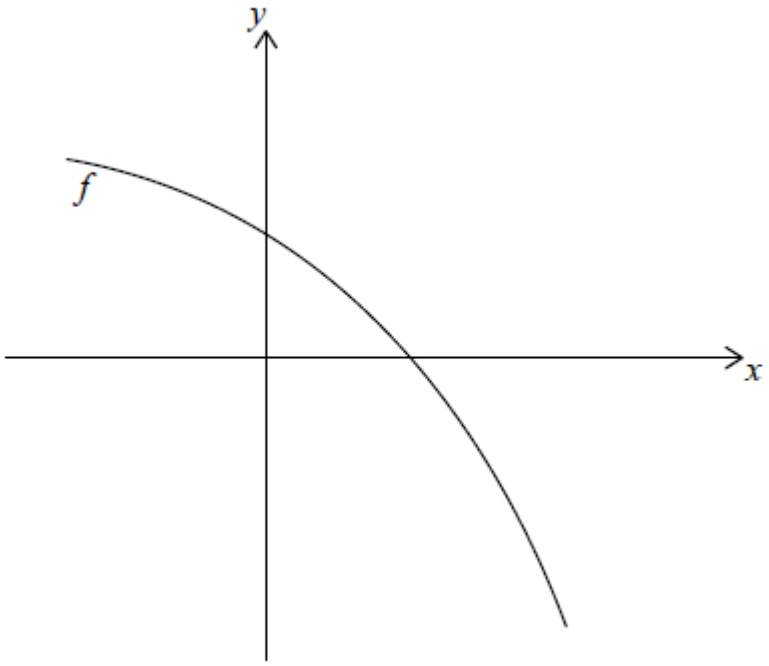
Volume of solid of revolution [60 marks]

1. [Maximum mark: 5] 19M.2.SL.TZ2.S_2

Let $f(x) = 4 - 2e^x$. The following diagram shows part of the graph of f.



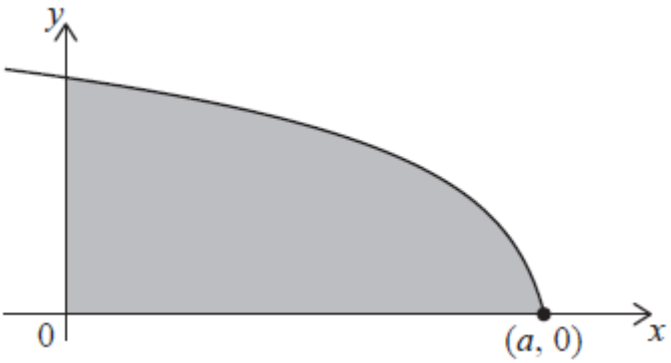
(a) Find the x-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the x-axis and the y-axis is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

2. [Maximum mark: 7] 20N.1.SL.TZ0.S_3

Let $f(x) = \sqrt{12 - 2x},\ x \leq a$. The following diagram shows part of the graph of f.

The shaded region is enclosed by the graph of f, the x-axis and the y-axis.



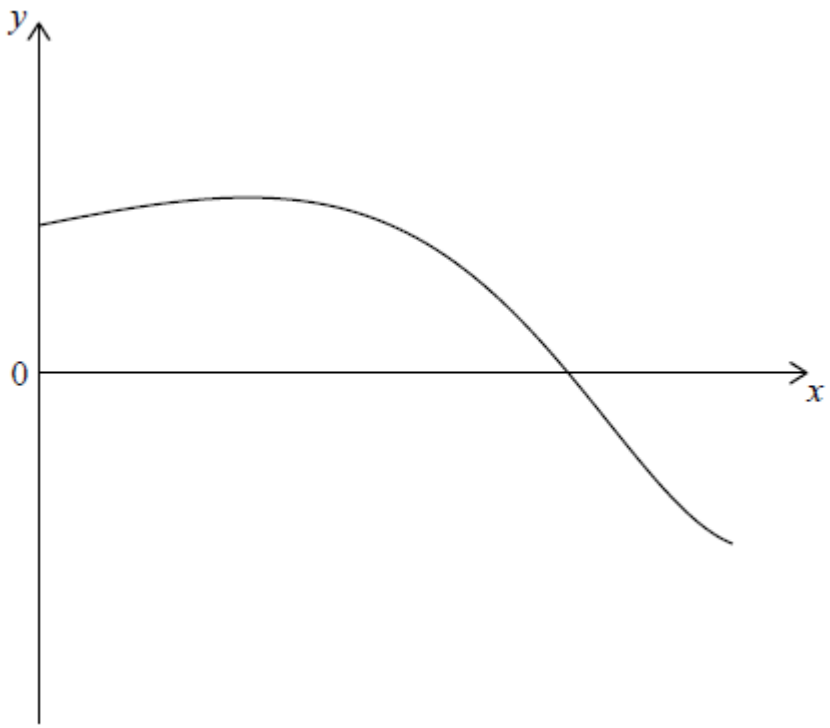
The graph of f intersects the x-axis at the point $(a,\ 0)$.

(a) Find the value of a. [2]

(b) Find the volume of the solid formed when the shaded region is revolved 360° about the x-axis. [5]

3. [Maximum mark: 5] 18M.2.SL.TZ2.S_3

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of f.



(a) Find the *x*-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the *y*-axis and the *x*-axis is rotated 360° about the *x*-axis.

Find the volume of the solid formed. [3]

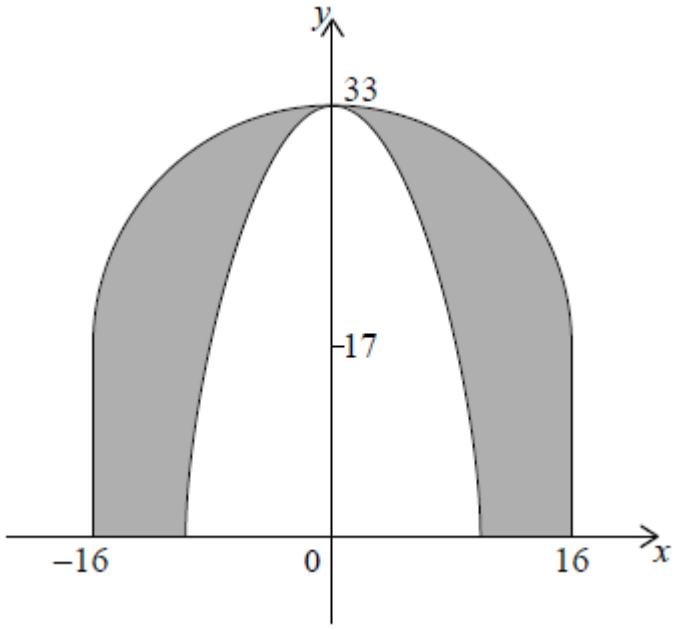
4. [Maximum mark: 8] SPM.1.AHL.TZ0.14

The graph of $y = -x^3$ is transformed onto the graph of $y = 33 - 0.08x^3$ by a translation of a units vertically and a stretch parallel to the x-axis of scale factor b.

(a.i) Write down the value of a. [1]

(a.ii) Find the value of b. [2]

(b) The outer dome of a large cathedral has the shape of a hemisphere of diameter 32 m, supported by vertical walls of height 17 m. It is also supported by an inner dome which can be modelled by rotating the curve $y = 33 - 0.08x^3$ through 360° about the y-axis between $y = 0$ and $y = 33$, as indicated in the diagram.



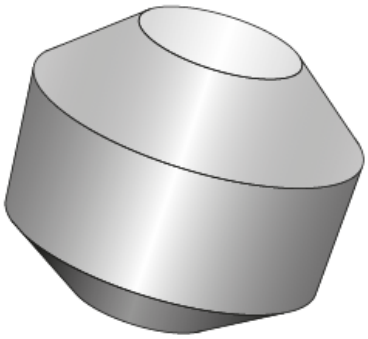
[5]

Find the volume of the space between the two domes.

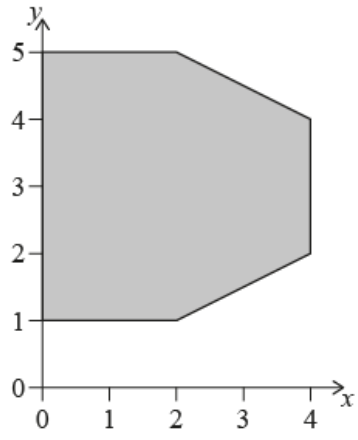
5. [Maximum mark: 6] 23M.1.AHL.TZ1.15

The solid shown is formed by rotating the hexagon with vertices $(2,\ 1), (0,\ 1), (0,\ 5), (2,\ 5), (4,\ 4)$ and $(4,\ 2)$ about the y-axis.

Solid



Hexagon

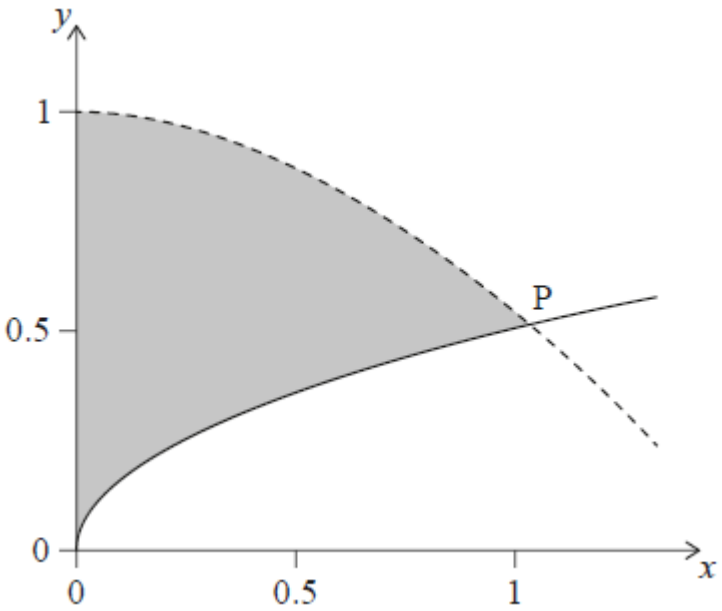


Find the volume of this solid. [6]

6. [Maximum mark: 9] 23M.1.AHL.TZ2.16

The following diagram shows parts of the curves of $y = \cos x$ and $y = \frac{\sqrt{x}}{2}$.

P is the point of intersection of the two curves.



(a) Use your graphic display calculator to find the coordinates of P. [2]

The shaded region is rotated 360° **about the** y**-axis** to form a volume of revolution V.

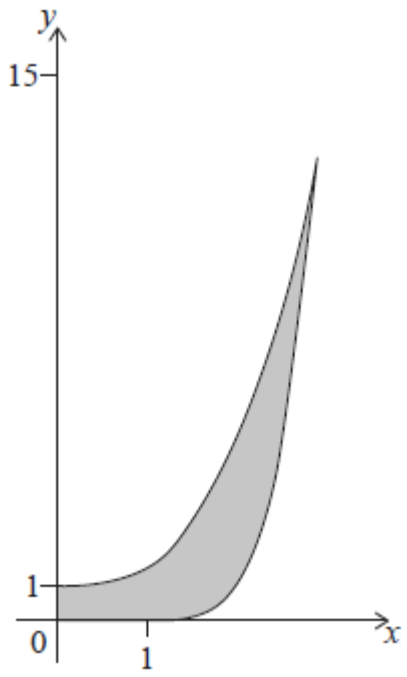
(b) Express V as the sum of two definite integrals. [5]

(c) Hence find the value of V. [2]

7. [Maximum mark: 13] 22N.2.AHL.TZ0.5

Adesh is designing a glass. The glass has an inner surface and an outer surface.
Part of the cross section of his design is shown in the following graph, where the

shaded region represents the glass. The two surfaces meet at the top of the glass. 1 unit represents $1\,\text{cm}$.



The inner surface is modelled by $f(x) = \frac{1}{2}x^3 + 1$ for $0 \leq x \leq p$.

The outer surface is modelled by $g(x)\begin{cases} 0 & \text{for } 0 \leq x < 1 \\ (x-1)^4 & \text{for } 1 \leq x \leq p \end{cases}$.

(a) Find the value of p. [2]

The glass design is finished by rotating the shaded region in the diagram through 360° about the y-axis.

(b) Find the volume of liquid that can be contained inside the finished glass. [5]

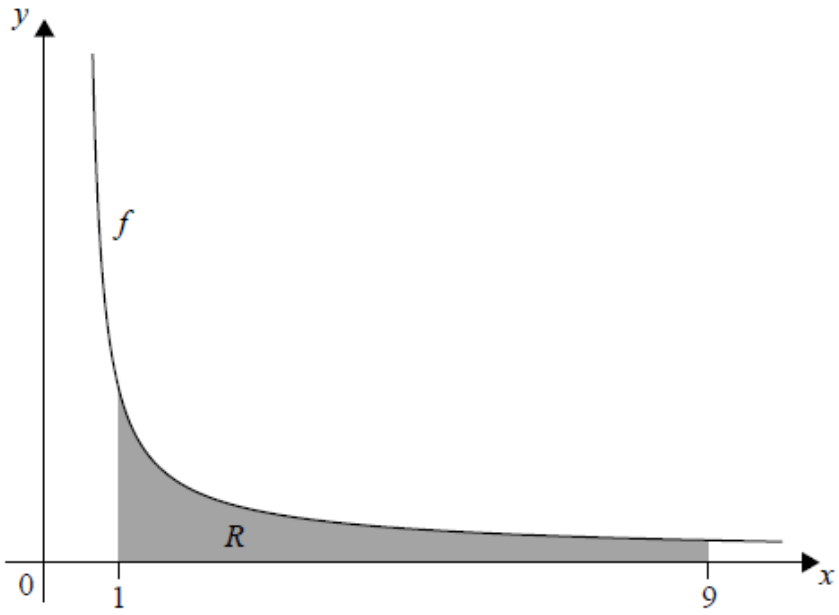
(c) Find the volume of the region between the two surfaces of the finished glass. [6]

8. [Maximum mark: 7] 18M.1.SL.TZ1.S_5

Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

(a) Find $\int (f(x))^2 \mathrm{d}x$. [3]

(b) Part of the graph of f is shown in the following diagram.



The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 9$. Find the volume of the solid formed when R is revolved 360° about the x-axis. [4]