Trigonometric equations

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We will cover the following topics:

- basic trigonometric equations,
- variations of basic trigonometric equations,
- factorization of trig equations,
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- some harder examples,
- exam questions from Polish matura,
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We will start with the following equation:

$$\sin x = \frac{\sqrt{3}}{2}$$

We want to draw one period of the sine function (eg. from $-\pi$ to π) and the line $y = \frac{\sqrt{3}}{2}$.

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We can see two solutions (red points).

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We can see two solutions (red points). We should know one of those (from tables of values of standard angles), we can find the other one using symmetries of the graph Tomasz Lechowski DP1 AA HL October 16, 2024 6/140



Our solutions are $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$

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where $k \in \mathbb{Z}$, so k is an integer.

Where does the $2k\pi$ come from? We only drew one period of sine, the values repeat themselves every 2π , so adding or subtracting any multiple of 2π to x will not change the value of the function.

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So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi$$
 or $x = \frac{2\pi}{3} + 2k\pi$

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Now we want to solve:

$$\cos x = \frac{\sqrt{2}}{2}$$

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Now we want to solve:

$$\cos x = \frac{\sqrt{2}}{2}$$

We draw one period of the cosine function (again it can be from $-\pi$ to π) and the line $y = \frac{\sqrt{2}}{2}$.

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We can see two solutions (red points). We should know one of those solutions

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We can see two solutions (red points).

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We can see two solutions (red points). We should know one of those solutions and we can find the other one using symmetries of the graph, $h \in \mathbb{R}$, $h \in \mathbb{R}$, $h \in \mathbb{R}$

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One solution is $x = \frac{\pi}{4}$, the other is of course $x = -\frac{\pi}{4}$.

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Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

where $k \in \mathbb{Z}$.

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Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

$$x = \frac{\pi}{4} + 2k\pi$$
 or $x = -\frac{\pi}{4} + 2k\pi$

where $k \in \mathbb{Z}$.

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Solve:

$$\tan x = \frac{\sqrt{3}}{3}$$

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Solve:

$$\tan x = \frac{\sqrt{3}}{3}$$

We draw one period of tangent function (remember that the period of tan x is π , it's best to draw it from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$) and the line $y = \frac{\sqrt{3}}{3}$.



There's one solution (red point). We know it from the table of standard angles.

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There's one solution (red point).

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There's one solution (red point). We know it from the table of standard angles, $x = \frac{\pi}{6}$.

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We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

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We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

$$x = \frac{\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

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Solve:

$\cot x = 1$

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Solve:

$\cot x = 1$

We draw one period of cotangent function (the period is π , we'll draw it between 0 and π) and the line y = 1.



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We can see one solution (red point).

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We can see one solution (red point). It is $x = \frac{\pi}{4}$.

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Therefore the solutions to

$$\cot x = 1$$

are:

where $k\in\mathbb{Z}$

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Therefore the solutions to

$$\cot x = 1$$

are:

$$x = \frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

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Solve the following equations:



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Solve the following equations:

• Equation:

$$\sin x = \frac{1}{2}$$

 $x=rac{\pi}{6}+2k\pi$ or $x=rac{5\pi}{6}+2k\pi$ where $k\in 2$

Equation:

Solution:

 $\kappa = rac{\pi}{2} + k\pi$, where $k \in \mathbb{Z}$.

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Solve the following equations:

• Equation:

$$\sin x = \frac{1}{2}$$

Solution:

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 or $x = rac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

Equation

 $\cos x = 0$ $x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

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Solve the following equations:

• Equation:

$$\sin x = \frac{1}{2}$$

Solution:

$$x = \frac{\pi}{6} + 2k\pi$$
 or $x = \frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

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• Equation:

 $\cos x = 0$

Solution:

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 or $x = \frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

• Equation:

$$\cos x = 0$$

Solution:

$$x = \frac{\pi}{2} + k\pi$$
 where $k \in \mathbb{Z}$

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• Equation:

$$\tan x = \sqrt{3}$$

Solution:

 $x=rac{\pi}{3}+k\pi$ where $k\in\mathbb{Z}$

Equation:

 $\cot x = 0$

Solution:

 $\kappa = rac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

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• Equation:

$$\tan x = \sqrt{3}$$

Solution:

 $x=rac{\pi}{3}+k\pi$ where $k\in\mathbb{Z}$

Equation:

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Solution:

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$$\tan x = \sqrt{3}$$

Solution:

$$x = \frac{\pi}{3} + k\pi$$
 where $k \in \mathbb{Z}$

Equation:



Solution:

 $=rac{\pi}{2}+k\pi$ where $k\in\mathbb{Z}$

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• Equation:

$$\tan x = \sqrt{3}$$

Solution:

$$x = \frac{\pi}{3} + k\pi$$
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Solution:

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• Equation:

$$\tan x = \sqrt{3}$$

Solution:

$$x = \frac{\pi}{3} + k\pi$$
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Equation: •

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Solution:

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• Equation:

$$\tan x = \sqrt{3}$$

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Solution:

$$x=rac{\pi}{2}+k\pi$$
 where $k\in\mathbb{Z}$

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Solve the equation:

$$\sin x = -1$$

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Solve the equation:

 $\sin x = -1$

We draw one period of sine function and the line y = -1.

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Image: A matrix and a matrix

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We can see one solution, it's of course $x = -\frac{\pi}{2}$.

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The solutions to

$$\sin x = -1$$

are:



where $k \in \mathbb{Z}$.

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The solutions to

$$\sin x = -1$$

are:

$$x = -\frac{\pi}{2} + 2k\pi$$

where $k \in \mathbb{Z}$.

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Solve:

$$\cos x = -\frac{1}{2}$$

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Solve:

$$\cos x = -\frac{1}{2}$$

We draw one period of cosine function and the line $y = -\frac{1}{2}$.

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We can see two solutions. If we were to solve $\cos x = \frac{1}{2}$, then we would know that $x = \frac{\pi}{3}$ is one of the solutions, here we can use the symmetry to get $x = \frac{5}{2}$ as a solution and then we also get $x = -\frac{2\pi}{3}$.

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We can see two solutions. If we were to solve $\cos x = \frac{1}{2}$, then we would know that $x = \frac{\pi}{3}$ is one of the solutions, here we can use the symmetry to get $x = \frac{2\pi}{3}$ as a solution and then we also get $x = -\frac{2\pi}{3}$.

We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

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We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

$$x = -\frac{2\pi}{3} + 2k\pi$$
 or $x = \frac{2\pi}{3} + 2k\pi$

where $k \in \mathbb{Z}$.

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Solve:

$\tan x = -1$

As always we draw one period of tangent function and the line $\mathbf{y}=-1$

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Solve:

$\tan x = -1$

As always we draw one period of tangent function and the line y = -1.

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There's one solution. If we were to solve $\tan x = 1$, the solution would be x = 3

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There's one solution. If we were to solve $\tan x = 1$, the solution would be $x = \frac{\pi}{4}$, so here we of course have $x = -\frac{\pi}{4}$

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So the solutions to

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 $\tan x = -1$

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are:

where $k \in \mathbb{Z}$.

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So the solutions to

$$\tan x = -1$$

are:

$$x = -\frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

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Solve:

$$\cot x = -\sqrt{3}$$

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Solve:

$$\cot x = -\sqrt{3}$$

We draw one period of cotangent function and the line $y = -\sqrt{3}$.

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Image: A matrix and a matrix

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There's one solution. Solving $\cot x = \sqrt{3}$ would give us $x = \frac{4}{2}$, so here we have

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There's one solution. Solving $\cot x = \sqrt{3}$ would give us $x = \frac{\pi}{6}$, so here we have $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

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So the solutions to

$$\cot x = -\sqrt{3}$$

are:



where $k \in \mathbb{Z}$.

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So the solutions to

$$\cot x = -\sqrt{3}$$

are:

$$x = \frac{5\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

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Solve the following equations:



Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Equation:



Solutions:

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Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:



Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -rac{\pi}{3} + 2k\pi$$
 or $x = -rac{2\pi}{3} + 2k\pi$ wheree $k \in \mathbb{Z}$

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Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -rac{\pi}{3} + 2k\pi$$
 or $x = -rac{2\pi}{3} + 2k\pi$ wheree $k \in \mathbb{Z}$

Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

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Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -\frac{\pi}{3} + 2k\pi$$
 or $x = -\frac{2\pi}{3} + 2k\pi$ wheree $k \in \mathbb{Z}$

• Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

Solutions:

Solve the following equations:

• Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -\frac{\pi}{3} + 2k\pi$$
 or $x = -\frac{2\pi}{3} + 2k\pi$ wheree $k \in \mathbb{Z}$

• Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

Solutions:

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Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions

 $\mathbf{x} = -rac{\pi}{6} + k\pi$ where $k \in \mathbb{Z}$

Equation:

 $\cot x = -1$

Solutions:

 $x=rac{3\pi}{4}+k\pi$ where $k\in\mathbb{Z}$

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Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

 $x = -rac{\pi}{6} + k\pi$ where $k \in \mathbb{Z}$

• Equation:

 $\cot x = -1$

Solutions:

 $x=rac{3\pi}{4}+k\pi$ where $k\in\mathbb{Z}$

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Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi$$
 where $k \in \mathbb{Z}$

Equation:

 $\cot x = -1$

Solutions:

 $\mathbf{x} = rac{3\pi}{4} + k\pi$ where $k \in \mathbb{Z}$

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Equation: •

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 $\cot x = -1$

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• Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi$$
 where $k \in \mathbb{Z}$

• Equation:

 $\cot x = -1$

Solutions:

$$k \in \mathbb{Z}$$
 where $k \in \mathbb{Z}$

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• Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi$$
 where $k \in \mathbb{Z}$

• Equation:

$$\cot x = -1$$

Solutions:

$$x = rac{3\pi}{4} + k\pi$$
 where $k \in \mathbb{Z}$

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In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval.



for $0 \leq x \leq 3\pi$.

This is even simpler. We draw $y=rac{\sqrt{2}}{2}$ and $y=\sin x$, but only for $0\leq x\leq 3\pi$.

In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval. Solve

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \le x \le 3\pi$.

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In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval. Solve

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have four solutions. We should know one from the table and find the rest

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We have four solutions.

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We have four solutions. We should know one from the table and find the rest using symmetries and periodicity of the graph.

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The solutions to

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \le x \le 3\pi$ are

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The solutions to

$$\sin x = \frac{\sqrt{2}}{2}$$
for $0 \le x \le 3\pi$ are $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$ or $x = \frac{9\pi}{4}$ or $x = \frac{11\pi}{4}$

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Solve

$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi < x < \pi$.

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Image: A matrix and a matrix October 16, 2024 38 / 140

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$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi < x < \pi$.

We draw
$$y = -\frac{\sqrt{3}}{2}$$
 and $y = \cos x$, but only for $-2\pi \le x \le \pi$.

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We have 3 solutions. If we were solving $\cos x = \frac{x^2}{2}$, then we would have $x = \frac{\pi}{2}$

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We have 3 solutions.

 $=\frac{\sqrt{3}}{2}$, then we would have $x=\frac{3}{2}$

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We have 3 solutions. If we were solving $\cos x = \frac{\sqrt{3}}{2}$, then we would have $x = \frac{\pi}{6}$ as a solution, based on that and symmetries we can find the actual solutions.

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The solutions to

$$\cos x = -\frac{\sqrt{3}}{2}$$

for $-2\pi \le x \le \pi$ are

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The solutions to

$$\cos x = -\frac{\sqrt{3}}{2}$$
for $-2\pi \le x \le \pi$ are $x = -\frac{7\pi}{6}$ or $x = -\frac{5\pi}{6}$ or $x = \frac{5\pi}{6}$.

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• Solve:

 $\tan x = -1$

for $-\pi \leq x \leq \pi$.

Solve:

 $\sin x = 1$

for $-\pi \le x \le 3\pi$ Solution:

 $x = \frac{\pi}{2}$ or $x = \frac{5}{2}$

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• Solve:

 $\tan x = -1$

for $-\pi \le x \le \pi$. Solution:

Solve:

for $-\pi \le x \le 3\pi$ Solution:



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Solve:

$$\tan x = -1$$

for $-\pi \le x \le \pi$. Solution:

$$x = -\frac{\pi}{4}$$
 or $x = \frac{3\pi}{4}$

Solve:

for $-\pi \le x \le 3\pi$. Solution:

 $x = \frac{\pi}{2}$ or $x = \frac{5\pi}{2}$

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Solve:

$$\tan x = -1$$

for $-\pi \le x \le \pi$. Solution:

$$x = -\frac{\pi}{4}$$
 or $x = \frac{3\pi}{4}$

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Solve:

 $\sin x = 1$

for $-\pi \le x \le 3\pi$

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Solve:

$$\tan x = -1$$

for $-\pi \le x \le \pi$. Solution:

$$x = -\frac{\pi}{4}$$
 or $x = \frac{3\pi}{4}$

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Solve:

 $\sin x = 1$

for $-\pi \le x \le 3\pi$ Solution:

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Solve:

$$\tan x = -1$$

for $-\pi \le x \le \pi$. Solution:

$$x = -\frac{\pi}{4}$$
 or $x = \frac{3\pi}{4}$

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for $-\pi \le x \le 3\pi$ Solution:

$$x = \frac{\pi}{2}$$
 or $x = \frac{5\pi}{2}$

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The ability to solve simple trigonometric equations is the basis for more complicated equations.

In the following examples I II assume that you can solve the basic equations with ease, so make sure you practice those before moving on.

On the following slides I'll skip the step with drawing graphs, but you should still do it. It is a very useful habit. What it means is that when you get to a basic trig equation you should solve it as above - quick sketch and read of the solutions. The ability to solve simple trigonometric equations is the basis for more complicated equations. In the end we will almost always arrive at the simple ones.

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We move on to equations where some algebraic manipulation is required.

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Solve:

 $2\sin(3x) + 4 = 3$

We rewrite it in the form

and now we solve as a basic trig equation (but instead of x we have 3x), so we get:

$$3x = -rac{\pi}{6} + 2k\pi$$
 or $3x = -rac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

Now we divide by 3, to get x:

$$\mathbf{x}=-rac{\pi}{18}+rac{2k\pi}{3}$$
 or $\mathbf{x}=-rac{5\pi}{18}+rac{2k\pi}{3}$ where $k\in\mathbb{Z}$

and this is our solution.

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Solve:

$$2\sin(3x) + 4 = 3$$

We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

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and now we solve as a basic trig equation (but instead of x we have 3x), so we get:

$$3x = -\frac{\pi}{6} + 2k\pi$$
 or $3x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

Now we divide by 3, to get x:

$$x=-rac{\pi}{18}+rac{2k\pi}{3}$$
 or $x=-rac{5\pi}{18}+rac{2k\pi}{3}$ where $k\in\mathbb{Z}$

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We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

and now we solve as a basic trig equation (but instead of x we have 3x), so we get:

$$3x = -\frac{\pi}{6} + 2k\pi$$
 or $3x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

Now we divide by 3, to get

$$x = -rac{\pi}{18} + rac{2k\pi}{3}$$
 or $x = -rac{5\pi}{18} + rac{2k\pi}{3}$ where $k \in \mathbb{Z}$

and this is our solution.

Solve:

$$2\sin(3x) + 4 = 3$$

We rewrite it in the form

$$\sin(3x) = -\frac{1}{2}$$

and now we solve as a basic trig equation (but instead of x we have 3x), so we get:

$$3x = -\frac{\pi}{6} + 2k\pi$$
 or $3x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

Now we divide by 3, to get *x*:

$$x = -rac{\pi}{18} + rac{2k\pi}{3}$$
 or $x = -rac{5\pi}{18} + rac{2k\pi}{3}$ where $k \in \mathbb{Z}$

and this is our solution.

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Note that when solving

$$\sin(3x) = -\frac{1}{2}$$

We don't need to draw sin(3x) (sine squeezed by a factor of $\frac{1}{3}$). It's better to draw sin α (so the usual graph of sine), solve for α and then put 3x instead of α .

We will get back to this in a few slides.

Note that when solving

$$\sin(3x) = -\frac{1}{2}$$

We don't need to draw sin(3x) (sine squeezed by a factor of $\frac{1}{3}$). It's better to draw sin α (so the usual graph of sine), solve for α and then put 3x instead of α .

We will get back to this in a few slides.

Solve:

$$\cos(2x-\frac{\pi}{3})+1=0$$

We rewrite in the form:

$$\cos(2x-\frac{\pi}{3})=-1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

$$2x-rac{\pi}{3}=\pi+2k\pi$$
 where $k\in\mathbb{Z}$

rearrange to find x:

$$x=rac{2\pi}{3}+k\pi$$
 where $k\in\mathbb{Z}$

and that's our solution.

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Solve:

$$\cos(2x-\frac{\pi}{3})+1=0$$

We rewrite in the form:

$$\cos(2x-\frac{\pi}{3})=-1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

$$2x - \frac{\pi}{3} = \pi + 2k\pi$$
 where $k \in \mathbb{Z}$

rearrange to find x:

$$x = rac{2\pi}{3} + k\pi$$
 where $k \in \mathbb{Z}$

and that's our solution.

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Solve:

$$\cos(2x-\frac{\pi}{3})+1=0$$

We rewrite in the form:

$$\cos(2x-\frac{\pi}{3})=-1$$

and we solve as a basic equation (instead of x we have $2x - \frac{\pi}{3}$), we get:

$$2x-rac{\pi}{3}=\pi+2k\pi$$
 where $k\in\mathbb{Z}$

rearrange to find x:

$$x = rac{2\pi}{3} + k\pi$$
 where $k \in \mathbb{Z}$

and that's our solution.

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 where $k \in \mathbb{Z}$

and that's our solution.

46 / 140

Solve:

 $\tan^2(5x) - 3 = 0$

We get to:

 $\tan(5x) = -\sqrt{3}$ or $\tan(5x) = \sqrt{3}$

we solve two basic equations (instead of x we have 5x), we get:

$$5x=-rac{\pi}{3}+k\pi$$
 or $5x=rac{\pi}{3}+k\pi$ where $k\in\mathbb{Z}$

rearrange to find x:

$$x=-rac{\pi}{15}+rac{k\pi}{5}$$
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Image: A matrix and a matrix

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 or $x = rac{\pi}{15} + rac{k\pi}{5}$ where $k \in \mathbb{Z}$

and that's it.

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Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3}$$
 or $\cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$

$$rac{\lambda}{2}=-rac{\pi}{3}+k\pi$$
 or $rac{\lambda}{2}=rac{\pi}{3}+k\pi$ where $k\in\mathbb{Z}$

$$x=-rac{2\pi}{3}+2k\pi$$
 or $x=rac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

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Solve:

$$3\cot^2\left(\frac{x}{2}\right) = 1$$

Rearrange and get to:

$$\cot\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3}$$
 or $\cot\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{3}$

$$\frac{x}{2} = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$
by 2 to get x:

Tomasz Lechowski

Image: Image:

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Multiply by 2 to get x:

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Multiply by 2 to get x:

$$x = -rac{2\pi}{3} + 2k\pi$$
 or $x = rac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

and that's our solution.

Tomasz Lechowski

October 16, 2024

Solve:

$$|2\cos(3x)-1|=1$$

We rearrange and solve to get:

 $\cos(3x) = 0$ or $\cos(3x) = 1$

we solve two basic equations (instead of x we have 3x), we get:

 $3x=rac{\pi}{2}+k\pi$ or $3x=2k\pi$ where $k\in\mathbb{Z}$

divide by 3 to get x:

$$x = \frac{\pi}{6} + \frac{k\pi}{3}$$
 or $x = \frac{2k\pi}{3}$ where $k \in \mathbb{Z}$

and we have the solution.

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and we have the solution

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 $3x = \frac{\pi}{2} + k\pi \quad \text{or} \quad 3x = 2k\pi \quad \text{where } k \in \mathbb{Z}$ le by 3 to get x: $x = \frac{\pi}{6} + \frac{k\pi}{3} \quad \text{or} \quad x = \frac{2k\pi}{3} \quad \text{where } k \in \mathbb{Z}$ we have the solution.

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$$3x = \frac{\pi}{2} + k\pi$$
 or $3x = 2k\pi$ where $k \in \mathbb{Z}$

divide by 3 to get x:

$$x=rac{\pi}{6}+rac{k\pi}{3}$$
 or $x=rac{2k\pi}{3}$ where $k\in\mathbb{Z}$

and we have the solution.

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divide by 3 to get x:

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 or $x = \frac{2k\pi}{3}$ where $k \in \mathbb{Z}$

and we have the solution.

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Solve:

$$|2\sin(7x)+1|=2$$

We rearrange to get:

$$\sin(7x) = -\frac{3}{2}$$
 or $\sin(7x) = \frac{1}{2}$

the first equation has no solutions, we solve the second one (instead of x we have 7x), we get:

$$7x=rac{\pi}{6}+2k\pi$$
 or $7x=rac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$

divide by 7 to get x:

$$x = rac{\pi}{42} + rac{2k\pi}{7}$$
 or $x = rac{5\pi}{42} + rac{2k\pi}{7}$ where $k \in \mathbb{Z}$

and that's our solution.

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divide by 7 to get x:

$$x = rac{\pi}{42} + rac{2k\pi}{7}$$
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divide by 7 to get x:
$$x = \frac{\pi}{42} + \frac{2k\pi}{7} \quad \text{or} \quad x = \frac{5\pi}{42} + \frac{2k\pi}{7} \quad \text{where } k \in \mathbb{Z}$$

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Tomasz Lechowski

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and that's our solution. Tomasz Lechowski

• Equation:

$$2\sin^2(5x)-1=0$$

Solution



Equation:

Solution:

 $x=-\pi+6k\pi$ or $x=\pi+6k\pi$ where $k\in\mathbb{Z}$

Tomasz Lechowski

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• Equation:

$$2\sin^2(5x) - 1 = 0$$

Solution:

$$x = rac{\pi}{20} + rac{k\pi}{5}$$
 or $x = rac{3\pi}{20} + rac{k\pi}{5}$ where $k \in \mathbb{Z}$

Equation:

$$|2\cos\left(\frac{x}{3}\right) - 3| = 2$$

Solution

 $x = -\pi + 6k\pi$ or $x = \pi + 6k\pi$ where $k \in \mathbb{Z}$

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Image: A matrix and a matrix

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Equation:

$$3\tan^2(2x-\frac{\pi}{2})-1=0$$

Solution:



Equation:

 $|2\cot(4x) - 1| = 1$

Solution:

 $x=rac{\pi}{16}+rac{k\pi}{4}$ or $x=rac{\pi}{8}+rac{k\pi}{4}$ where $k\in$

Tomasz Lechowski

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Equation:

$$3\tan^2(2x-\frac{\pi}{2})-1=0$$

Solution:



Equation:

 $|2\cot(4x)-1|=1$

Solution:

where $k \in \mathbb{Z}$

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• Equation:

$$3 \tan^2(2x-\frac{\pi}{2}) - 1 = 0$$

Solution:

$$x = \frac{\pi}{6} + \frac{k\pi}{2}$$
 or $x = \frac{\pi}{3} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$

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Again, we will usually have a specified interval for x:

 $2\cos 4x - 1 = 0$

for $0 \leq x \leq \pi$.

Here we can use two methods (I recommend the latter). The first is to forget about the interval for a moment and solve as above We get:

$$4x = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad 4x = -\frac{\pi}{3} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$
$$x = \frac{\pi}{12} + \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{12} + \frac{k\pi}{2} \quad \text{where } k \in \mathbb{Z}$$

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Tomasz Lechowski

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for $-2\pi \leq x \leq 6\pi$.

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We go back to x. Since $\alpha = \frac{\pi}{2}$, so we have $x = 2\alpha$, this gives the following solutions:

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Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \le x \le \frac{\pi}{2}$.



Solve

 $\tan^2(2x) = 3$

with $-\pi \leq x \leq \frac{\pi}{2}$. Solution:



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Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \le x \le \frac{\pi}{2}$. Solution:

$$= \frac{\pi}{12}$$
 or $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{12}$

Solve

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with $-\pi \leq x \leq \frac{\pi}{2}$. Solution:



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 or $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{12}$

Solve

 $\tan^2(2x) = 3$

with $-\pi \leq x \leq \frac{\pi}{2}$. Solution:

$$x \in \left\{ -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3} \right\}$$

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Solve:

$$\cos^2(3x) = \frac{1}{2}$$

for $0 \le x \le \frac{\pi}{2}$. Solution:

$$x = \frac{\pi}{12}$$
 or $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{12}$

Solve

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with $-\pi \leq x \leq \frac{\pi}{2}$.

$$\mathfrak{c} \in \left\{-rac{5\pi}{6}, -rac{2\pi}{3}, -rac{\pi}{3}, -rac{\pi}{6}, rac{\pi}{6}, rac{\pi}{3}
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$$\tan^2(2x)=3$$

with $-\pi \le x \le \frac{\pi}{2}$. Solution:

$$x \in \left\{-\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}\right\}$$

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Now we move on to equations which can be easily factored resulting in two or more basic equations.

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Solve:

$2\sin^2 x + \sin x - 1 = 0$

We have a disguised quadratic (we could substitute *s* = sin x and solve), let's try factoring. We rewrite the LHS in a factored form:

 $(2\sin x - 1)(\sin x + 1) = 0$

This gives:

$$sin(x) = \frac{1}{2}$$
 or $sin(x) = -1$

We solve these basic equations to get:

$$x=rac{\pi}{6}+2k\pi$$
 or $x=rac{5\pi}{6}+2k\pi$ or $x=-rac{\pi}{2}+2k\pi$ where $k\in\mathbb{Z}$

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Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

We have a disguised quadratic (we could substitute $s = \sin x$ and solve), let's try factoring. We rewrite the LHS in a factored form:

$$(2\sin x - 1)(\sin x + 1) = 0$$

This gives:

$$\sin(x) = \frac{1}{2}$$
 or $\sin(x) = -1$

We solve these basic equations to get:

 $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$ or $x = -\frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

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Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

We factorize:

 $(2\cos x + 1)(\cos x - 2) = 0$

We get:

$$\cos(x) = -\frac{1}{2}$$
 lub $\cos(x) = 2$

There's no solutions to the second equation, solving the first one gives:

$$x=-rac{2\pi}{3}+2k\pi$$
 or $x=rac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

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Solve:

$$2\cos^2 x - 3\cos x - 2 = 0$$

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Tomasz Lechowski

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We factorize:

$$(2\cos x+1)(\cos x-2)=0$$

We get:

$$\cos(x) = -\frac{1}{2}$$
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There's no solutions to the second equation, solving the first one gives:

$$x = -rac{2\pi}{3} + 2k\pi$$
 or $x = rac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

Tomasz Lechowski

October 16, 2024

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Solve:

$2\sin x \cos x - 2\sin x + \cos x - 1 = 0$

We can first factor out $2 \sin x$ from the first two terms, this gives: $2 \sin x (\cos x - 1) + \cos x - 1 = 0$

So we have:

 $(2\sin x + 1)(\cos x - 1) = 0$

So:

 $\sin(x) = -\frac{1}{2} \qquad \text{lub} \qquad \cos(x) = 1$

Now we solve and get:

 $\mathbf{x}=-rac{\pi}{6}+2k\pi$ or $\mathbf{x}=-rac{5\pi}{6}+2k\pi$ or $\mathbf{x}=2k\pi$ where $k\in\mathbb{Z}$

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We can first factor out $2 \sin x$ from the first two terms, this gives:

$$2\sin x(\cos x-1)+\cos x-1=0$$

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So we have:

$$(2\sin x+1)(\cos x-1)=0$$

So:

$$sin(x) = -\frac{1}{2}$$
 lub $cos(x) = 1$

Now we solve and get:

$$x = -\frac{\pi}{6} + 2k\pi$$
 or $x = -\frac{5\pi}{6} + 2k\pi$ or $x = 2k\pi$ where $k \in \mathbb{Z}$

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Solve:

$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set *t* = tan² x, but let's try factoring again:

 $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

We can continue factoring (using difference of squares) or we can just write that:

$$\tan(x) = \pm \frac{\sqrt{3}}{3}$$
 lub $\tan(x) = \pm \sqrt{3}$

We solve and get:

$$x = rac{\pi}{6} + rac{k\pi}{2}$$
 or $x = rac{\pi}{3} + rac{k\pi}{2}$ where $k \in \mathbb{Z}$

Think about this solution. Make sure you understand where it came from.

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$$3\tan^4 x - 10\tan^2 x + 3 = 0$$

We can set $t = \tan^2 x$, but let's try factoring again:

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Think about this solution. Make sure you understand where it came from.

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$$\tan(x) = \pm \frac{\sqrt{3}}{3}$$
 lub $\tan(x) = \pm \sqrt{3}$

We solve and get:

$$x = \frac{\pi}{6} + \frac{k\pi}{2}$$
 or $x = \frac{\pi}{3} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$

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Solve:

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Think about this solution. Make sure you understand where it came from.

Tomasz Lechowski	DP1 AA HL	October 16, 2024	63 / 140
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Solve:

$$2\cos^2 x - \cos x - 3 = 0$$

Solution:

 $x=\pi+2k\pi$ where $k\in\mathbb{Z}$

Solve:

 $\sin x \cos x + \sin x - \cos x - 1 = 0$

Solution:

 $x=rac{\pi}{2}+2k\pi$ or $x=\pi+2k\pi$ where $k\in\mathbb{Z}$

Tomasz Lechowski

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Solution:

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Solution

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Solution:

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 or $x=\pi+2k\pi$ where $k\in\mathbb{Z}$

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• Solve:

$$\cot^3 x - \cot^2 x - 3\cot x + 3 = 0$$

Solution

$$x=rac{\pi}{4}+k\pi$$
 or $x=rac{\pi}{6}+k\pi$ or $x=rac{5\pi}{6}+k\pi$ where $k\in\mathbb{Z}$

Solve:

$$\sin^3 x - 4\sin^2 x - \sin x + 4 = 0$$

Solution:

$$x=rac{\pi}{2}+k\pi$$
 where $k\in\mathbb{Z}$

Tomasz Lechowski

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Solve:

$$\cot^3 x - \cot^2 x - 3 \cot x + 3 = 0$$

Solution:

$$x = \frac{\pi}{4} + k\pi$$
 or $x = \frac{\pi}{6} + k\pi$ or $x = \frac{5\pi}{6} + k\pi$ where $k \in \mathbb{Z}$

Solve:

$$\sin^3 x - 4\sin^2 x - \sin x + 4 = 0$$

Solution

$$\mathbf{x} = rac{\pi}{2} + k\pi$$
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• Solve:

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Tomasz Lechowski

October 16, 2024

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We increase the difficulty slightly.

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We increase the difficulty slightly. We add the Pythagorean identity to our arsenal.

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 $\sin^2 x + \cos^2 x = 1$

We can use it to solve simple problems like:

Simple problem

Given an angle α , such that $\cos \alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin \alpha$ and $\cot \alpha$.

We have sin $x = -\frac{2\sqrt{2}}{3}$ and cot $x = -\frac{\sqrt{2}}{4}$. Refer to chapter 8E in Core HL if you forgot about these types of problems.

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 and $\cot x = -\frac{\sqrt{2}}{4}$.
The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

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Remember that the Pythagorean identity works for any angle x, so we have

 $\sin^2 31^\circ + \cos^2 31^\circ = 1$

 $\sin^2rac{\pi}{7}+\cos^2rac{\pi}{7}=1$

but also:

 $\sin^2(3lpha) + \cos^2(3lpha) = 1$

$$\sin^2\left(\frac{x}{2} - \pi\right) + \cos^2\left(\frac{x}{2} - \pi\right) = 1$$

Solving trig equations using Pythagorean identity boils down to simplifying the equation so that it can be solved using previous methods.

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Note that there are two very simple consequences of Pythagorean identity, namely:

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

They of course can be derived by dividing the Pythagorean identity by cos² x and sin² x respectively.

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They of course can be derived by dividing the Pythagorean identity by $\cos^2 x$ and $\sin^2 x$ respectively.

Solve:

 $5\sin x - 2\cos^2 x = 1$

We use Pythagorean identity to replace $-2\cos^2 x$ with $2\sin^2 x - 2$ and we get:

 $2\sin^2 x + 5\sin x - 3 = 0$

Factorize:

 $(2\sin x - 1)(\sin x + 3) = 0$

We now solve and get:

$$x=rac{\pi}{6}+2k\pi$$
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 $2\sin x = 2 + \cos^2 x$

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 $x=rac{\pi}{2}+2k\pi$ where $k\in\mathbb{Z}$

Equation:

 $2\cos^2 2x + 7\sin 2x + 2 = 0$

Solution:



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We increase the difficulty.

 $(\sin 3x - 1)(2\cos 2x - 1) = 0$

then there's no problem. We have 3x and 2x, but we easily get two basic equations. The solutions are:

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It may however happen that it's not so simple and then the goal would be to make sure that we have the same angle everywhere.

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Formulae that you have to remember:

 $\sin 2x = 2\sin x \cos x$

$$\cos 2x = \cos^2 x - \sin^2 x =$$
$$= 2\cos^2 x - 1 =$$
$$= 1 - 2\sin^2 x$$

In case of *cosine* we in fact have 3 formulae and we use the one which suits us.

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

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$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Remember that these formulae work regardless of the angle, so in particular we have:

 $\sin 10^\circ = 2 \sin 5^\circ \cos 5^\circ$

 $\sin 8lpha = 2\sin 4lpha \cos 4lpha$

 $\cos(10x) = 1 - 2\sin^2(5x)$

$$\cos(\mathbf{x}) = \cos^2\left(\frac{\mathbf{x}}{2}\right) - \sin^2\left(\frac{\mathbf{x}}{2}\right)$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{2\tan(\frac{\theta}{4})}{1 - \tan^2(\frac{\theta}{4})}$$

The angle on the left hand side has to be twice the angle on the right hand side.

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Remember that these formulae work regardless of the angle, so in particular we have:

 $\sin 10^\circ = 2 \sin 5^\circ \cos 5^\circ$

 $\sin 8\alpha = 2\sin 4\alpha \cos 4\alpha$

 $\cos(10x) = 1 - 2\sin^2(5x)$

$$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{2\tan\left(\frac{\theta}{4}\right)}{1 - \tan^2\left(\frac{\theta}{4}\right)}$$

The angle on the left hand side has to be twice the angle on the right hand side.

75 / 140

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Tomasz Lechowski

October 16, 2024

Solve:

 $\sin 2x + \sin x = 0$

We use double angle formula for sine (sin $2x = 2 \sin x \cos x$) and get:

 $2 \sin x \cos x + \sin x = 0$

We factor out sin x and we get:

 $\sin x(2\cos x+1)=0$

solve the above to get:

 $\mathbf{x}=k\pi$ or $\mathbf{x}=-rac{2\pi}{3}+2k\pi$ or $\mathbf{x}=rac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

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$\cos(6x) - 3\cos(3x) + 1 = 0$

We use double angle formula for cosine $(\cos 6x = 2\cos^2(3x) - 1)$, we get:

 $2\cos^2(3x) - 3\cos(3x) = 0$

Factor out cos(3x):

 $\cos(3x)(2\cos(3x)-3)=0$

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77 / 140

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 $x = \frac{\pi}{6} + \frac{k\pi}{3}$ where $k \in \mathbb{Z}$

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Solve:

 $\cos 4x + 4\sin 2x + 5 = 0$

We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get: $-2\sin^2 2x + 4\sin 2x + 6 = 0$

Divide by -2 and factorize:

 $(\sin 2x + 1)(\sin 2x - 3) = 0$

solve and get (the second equation has no solutions):

$$2x = rac{3\pi}{2} + 2k\pi$$
 where $k \in \mathbb{Z}$

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• Equation:

$$\sin x - 2\cos\frac{x}{2} = 0$$

Solution:

 $x=\pi+2k\pi$ where $k\in\mathbb{Z}$

Equation:

 $\cos 4x + 2 \sin 2x \cos 2x + 1 = 0$

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$$x = rac{\pi}{4} + rac{k\pi}{2}$$
 or $x = -rac{\pi}{8} + rac{k\pi}{2}$ where $k \in \mathbb{Z}$

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We have the following formulae for sine and cosine of sum and difference of angles:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

They can be used to calculate for example $\sin(\frac{t\pi}{12})$ or $\cos 15^\circ$:

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They can be used to calculate for example $sin(\frac{7\pi}{12})$ or $cos 15^{\circ}$:

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) =$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos 15^{\circ} = \cos(45^{\circ} - 30^{\circ}) = \cos 45^{\circ} \cos 30^{\circ} + \sin 45^{\circ} \sin 30^{\circ} =$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

We get the same result. This, of course, is no accident, we have $\frac{72}{12} = 105^\circ$, so $\sin 105^\circ = \sin(180 - 75^\circ) = \sin 75^\circ = \cos(90^\circ - 75^\circ) = \cos 15^\circ$.

When solving equations we will in most cases use the formulae in the opposite direction.

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When solving equations we will in most cases use the formulae in the opposite direction.
Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

We have a sum so it's appropriate to change it to cosine of a difference or sine of a sum. We will do the later. We want to change 1 into cosine and $\sqrt{3}$ into a sine. By drawing an appropriate triangle we can see that the hypotenuse is 2 (so we need to divide both sides by 2) and the required angle is $\alpha = \frac{\pi}{2}$:



So we get:

Image: Image:

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$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

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$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

So we get:

$$\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = \frac{\sqrt{2}}{2}$$

Now we can apply the formula for the sine of the sum of angles to get:

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

Note that we could have tried to use the formula for cosine of a difference of the angles. In which case we would need to change 1 into sine and $\sqrt{3}$ into cosine. The hypotenuse is still 2, but the angle is $\alpha = \frac{\pi}{6}$, so we would get:



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Applying the formula for the cosine of a difference we get:

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$$\sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \frac{\sqrt{2}}{2}$$

Applying the formula for the cosine of a difference we get:

$$\cos\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

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Going back, we have:

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

This is simple now, we have:

 $x + \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi$ lub $x + \frac{\pi}{3} = \frac{3\pi}{4} + 2k\pi$ gdzie $k \in \mathbb{Z}$

$$x = -rac{\pi}{12} + 2k\pi$$
 lub $x = rac{5\pi}{12} + 2k\pi$ gdzie $k \in \mathbb{Z}$

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If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

This gives:

$$x-rac{\pi}{6}=-rac{\pi}{4}+2k\pi$$
 lub $x-rac{\pi}{6}=rac{\pi}{4}+2k\pi$ gdzie $k\in\mathbb{Z}$

and the final answer is of course the same.

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and the final answer is of course the same.

Solve:

 $\sin x - \cos x = \sqrt{2}$

We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1$$

$$\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x = 1$$

Now apply the formula for sine of a difference.

$$\sin\left(x-\frac{\pi}{4}
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$$\sin\left(x-\frac{\pi}{4}\right)=1$$

$$\sin\left(x-\frac{\pi}{4}\right)=1$$

this gives:

 $x-rac{\pi}{4}=rac{\pi}{2}+2k\pi$ where $k\in\mathbb{Z}$

So finally we get:

 $x=rac{3\pi}{4}+2k\pi$ where $k\in\mathbb{Z}$

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Solve:

 $\sqrt{3}\sin x + \cos x = 1$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

 $\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$

$$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \frac{1}{2}$$

Applying the formula we get:

$$\sin\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$$

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so:

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$$x=2k\pi$$
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Tomasz Lechowski

October 16, 2024

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Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have 3x makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{\epsilon}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

SO:

 $\cosrac{\pi}{4}\sin 3x + \sinrac{\pi}{4}\cos 3x = -rac{\sqrt{3}}{2}$

Using formula for sine of a sum we get

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$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

SO:

$$\cos\frac{\pi}{4}\sin 3x + \sin\frac{\pi}{4}\cos 3x = -\frac{\sqrt{3}}{2}$$

Using formula for sine of a sum we get:

$$\sin\left(3x+\frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(3x+\frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

we get

 $3x + \frac{\pi}{4} = -\frac{\pi}{3} + 2k\pi$ or $3x + \frac{\pi}{4} = -\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

So in the end we get:

$$x = -\frac{7\pi}{36} + \frac{2k\pi}{3}$$
 or $x = -\frac{11\pi}{36} + \frac{2k\pi}{3}$ where $k \in \mathbb{Z}$

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Image: Image:

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Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

 $x = rac{\pi}{4} + k\pi$ or $x = rac{7\pi}{12} + k\pi$ where $k \in \mathbb{Z}$

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Formula for sine/cosine of sum/difference of angles exercise

• Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

$$x=rac{\pi}{4}+k\pi$$
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Formula for *sine/cosine* of sum/difference of angles - exercise

Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

$$x=rac{\pi}{4}+k\pi$$
 or $x=rac{7\pi}{12}+k\pi$ where $k\in\mathbb{Z}$

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Now we move to the final set of examples, where we apply formulae for sums and differences of sines and cosines.

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Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by heart (in fact it's best to learn the ones in the formula booklet by heart as well).

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$
$$\sin \alpha - \sin \beta = 2 \sin \left(\frac{\alpha - \beta}{2}\right) \cos \left(\frac{\alpha + \beta}{2}\right)$$
$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2}\right) \cos \left(\frac{\alpha - \beta}{2}\right)$$
$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2}\right) \sin \left(\frac{\alpha - \beta}{2}\right)$$

Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by heart (in fact it's best to learn the ones in the formula booklet by heart as well).

$$\begin{aligned} \sin \alpha + \sin \beta &= 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) \\ \sin \alpha - \sin \beta &= 2 \sin \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right) \\ \cos \alpha + \cos \beta &= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) \\ \cos \alpha - \cos \beta &= -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \end{aligned}$$

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Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2}=0$$

so we get:

$$rac{3x}{2} = k\pi$$
 or $rac{x}{2} = rac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

and finally:

$$x=rac{2k\pi}{3}$$
 or $x=\pi+2k\pi$ where $k\in\mathbb{Z}$

We've solve the above equation earlier using $\sin 2x = 2 \sin x \cos x$. Compare the answers. At first glance you may think that we got different solutions, but if you study it carefully you will see that they are indeed the same

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Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

$$rac{3x}{2}=k\pi$$
 or $rac{x}{2}=rac{\pi}{2}+k\pi$ where $k\in\mathbb{Z}$

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Solve:

 $\cos x + \cos 2x + \cos 3x = 0$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x\cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression: $2\cos 2x\cos(-x) + \cos 2x = 0$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

 $\cos 2x(2\cos x+1)=0$

This gives:

$$2x = \frac{\pi}{2} + k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

the end:

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Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$.

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 or $x = -\frac{2\pi}{3} + 2k\pi$ or $x = \frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

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In the end:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

Solve:

 $\sin 2x - \cos 3x = 0$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Now apply the formula for difference of sines:

$$2\sin\left(\frac{2x - (\frac{\pi}{2} - 3x)}{2}\right)\cos\left(\frac{2x + (\frac{\pi}{2} - 3x)}{2}\right) = 0$$

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Simplify to get



This gives:

$$rac{5 imes}{2}-rac{\pi}{4}=k\pi$$
 or $-rac{ imes}{2}+rac{\pi}{4}=rac{\pi}{2}+k\pi$ where $k\in\mathbb{Z}$

And finally:

$$x=rac{\pi}{10}+rac{2k\pi}{5}$$
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• Equation:





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• Equation:

$$\sin x = \sin 3x$$

Solution:

$$x = k\pi$$
 or $x = \frac{\pi}{4} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$

Equation:

 $\sin x + \sin 3x = \sin 2x$

Solution:

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Tomasz Lechowski
Advanced examples

On the next slides we will look at some advanced examples, where we need to make some important observations.

Make sure you think about these example before looking at the solutions. There may be multiple ways to solve those.

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On the next slides we will look at some advanced examples, where we need to make some important observations.

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Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$, and the bracket is just 1 (using Pythagorean identity). So we get:

 $1 - 2\sin^2 x \cos^2 x = \cos 2x$

Now we have two options. We can try to make all angles equal 2x or make them all equal x. Let's analyse both options.

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The first observation is that the left hand side can be written as $(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$,

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We try to make all angles equal to 2x. Recall that $\sin 2x = 2 \sin x \cos x$, so $\sin^2 2x = 4 \sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

 $2 - 4\sin^2 x \cos^2 x = 2\cos 2x$

so we get:

$$2 - \sin^2 2x = 2\cos 2x$$

Now we can use Pythagorean identity to change $-\sin^2 2x$ into $\cos^2 2x - 1$, and we get (moving all terms to the left hand side):

 $\cos^2 2x - 2\cos 2x + 1 = 0$

This is equivalent to:

$$(\cos 2x - 1)^2 = 0$$

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 or $\mathbf{x}=rac{\pi}{6}+2k\pi$ or $\mathbf{x}=rac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$

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 $\sin^3 x + \cos^3 x = 1$

We will change 1 into $sin^2 x + cos^2 x$ and move all terms to one side

 $\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$

We factor out sin² x and cos² x:

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Now an important observation. $\sin^2 x \ge 0$, but $\sin x - 1 \le 0$, because *sine* cannot be greater than 1. Similarly $\cos^2 x \ge 0$ and $\cos x - 1 \le 0$. So $\sin^2 x (\sin x - 1) \le 0$ oraz $\cos^2 x (\cos x - 1) \le 0$

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$$x=2k\pi$$
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 $\cos x - \cos 3x = \sin x - \sin 3x$

Seems obvious that we want to apply the formula for sum of sines and cosines:

 $-2\sin 2x\sin(-x) = 2\sin(-x)\cos 2x$

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Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x\sin(-x)=2\sin(-x)\cos 2x$$

Sine is an odd function, so sin(-x) = -sin x, using this and moving all terms to one side:

$$2\sin 2x\sin x + 2\sin x\cos 2x = 0$$

Factoring out $2 \sin x$:

$$2\sin x(\sin 2x + \cos 2x) = 0$$

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$2\sin x(\sin 2x + \cos 2x) = 0$

So sin x = 0 or sin $2x = -\cos 2x$. The second equation can be turned into tan 2x = -1 (by dividing both sides by $\cos x$). We solve and get:

$$x=k\pi$$
 or $2x=-rac{\pi}{4}+k\pi$ where $k\in\mathbb{Z}$

So finally we have:

$$x = k\pi$$
 or $x = -\frac{\pi}{8} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$.

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So $\sin x = 0$ or $\sin 2x = -\cos 2x$.

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Solve:

 $\sin^2 x + \sin^2 2x = \sin^2 3x$

Move all terms to one side:

 $\sin^2 x - \sin^2 3x + \sin^2 2x = 0$

We can use difference of squares (with the hope that we can get sin 2x to factor out):

 $(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$

We use the formula for sum and difference of sines:

 $2\sin(-x)\cos 2x \cdot 2\sin 2x\cos x + \sin^2 2x = 0$

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Tomasz Lechowski

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We have that sin(-x) = -sin x and we get:

 $-4 \sin x \cos x \cos 2x \sin 2x + \sin^2 2x = 0$

Now we have the expression $\sin x \cos x$ which should remind us of the formula $\sin 2x = 2 \sin x \cos x$, we use it to get:

 $-2\cos 2x\sin^2 2x + \sin^2 2x = 0$

Now it's a breeze, we factor out $\sin^2 2x$:

 $\sin^2 2x(1-2\cos 2x)=0$

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$$2x = k\pi$$
 or $2x = -\frac{\pi}{3} + 2k\pi$ or $2x = \frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

So the final answer is:

$$x=rac{k\pi}{2}$$
 or $x=-rac{\pi}{6}+k\pi$ or $x=rac{\pi}{6}+k\pi$ where $k\in\mathbb{Z}$

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Solve:

 $\cot 8x \cot 10x = -1$

We will start with the domain (usually the domain of the equation is specified in IB questions, but it's useful to do it anyway)

 $8x
eq k\pi$ and $10x
eq k\pi$

SO



Now we will use the fact that $\cot x = \frac{\cos x}{\sin x}$:

 $\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -\frac{1}{2}$

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 $\frac{\cos 8x \cos 10x}{\sin 8x \sin 10x} = -1$

Multiply by the denominator (which we know is non-zero) and move to one side to get:

 $\cos 8x \cos 10x + \sin 8x \sin 10x = 0$

This looks like a formula for a cosine of a difference. We get:

 $\cos 2x = 0$

So:

$$x=rac{\pi}{4}+rac{k\pi}{2}$$
 where $k\in\mathbb{Z}$.

But beware, all of these solutions are outside of our domain, so in the end our equation has no solutions.

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$$x \in \emptyset$$

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The next slides include problems that appeared on a Polish Matura (advanced level).

Image: A matrix and a matrix

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May 2015,

Zadanie 4. (0-1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

- A. nie ma rozwiązań rzeczywistych.
- B. ma dokładnie jedno rozwiązanie rzeczywiste.
- C. ma dokładnie dwa rozwiązania rzeczywiste.
- D. ma więcej niż dwa rozwiązania rzeczywiste.

This is a very important question, because it shows that it's always important to think about the equation before solving it. sin x is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course (find it) $\sqrt{13}$), so there'll be no solutions - answer A.

May 2015, simple multiple choice question to begin with:

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May 2017,

Zadanie 10. (0-4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

 $2\cos^2 x + 3\cos x + 1 = 0$

Factorize:

 $(2\cos x + 1)(\cos x + 1) = 0$

Now we sketch the graph of cosine for $0 \le x \le 2\pi$. We get:

$$x = \frac{2\pi}{3}$$
 or $x = \pi$ or $x = \frac{4\pi}{3}$

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May 2017,

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$$x = \frac{2\pi}{3}$$
 or $x = \pi$ or $x = \frac{4\pi}{3}$

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May 2017,

Zadanie 10. (0-4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

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Tomasz Lechowski

May 2018,

Zadanie 11. (0-4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2 \sin 3x \cos 3x$:

 $2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$

Factor out cos 3x and move all terms to one side:

 $\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$

Now we can factor out $(2 \sin 3x + 1)$:

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119/140
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Since $\alpha = 3x$, then $x = \frac{\alpha}{3}$, so:

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May 2019,

Zadanie 2. (0–1) Liczba cos² 105° – sin² 105° jest równa

A.
$$-\frac{\sqrt{3}}{2}$$
 B. $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get:

 $\cos^2 105^\circ - \sin^2 105^\circ = \cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$ Answer A.

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121 / 140

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May 2019,

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May 2019,

Zadanie 14. (0–4)
Rozwiąż równanie
$$(\cos x) \left[\sin \left(x - \frac{\pi}{3} \right) + \sin \left(x + \frac{\pi}{3} \right) \right] = \frac{1}{2} \sin x$$
.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos \frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple. Of course we have $\cos \frac{\pi}{3} = \frac{1}{2}$. We move all terms to one side and we get:

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This gives:

$$x=k\pi$$
 or $x=-rac{\pi}{3}+2k\pi$ or $x=rac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

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Tomasz Lechowski

October 16, 2024

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Now we move on to IB exam questions.

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^{\circ} \le x < 360^{\circ}$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1-\tan^2 x} = 0$$

Now it makes sense to multiply both sides by $1 - \tan^2 x$ to get:

 $\tan x - \tan^3 x + 2 \tan x = 0$

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$$\tan x + \frac{2\tan x}{1 - \tan^2 x} = 0$$

Now it makes sense to multiply both sides by $1 - \tan^2 x$ to get:

 $\tan x - \tan^3 x + 2 \tan x = 0$

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IB exam - problem 1

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Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides. We get:

 $2 \sin x \cos x - 2 \cos^2 x + 1 = 1 + \sin x - \cos x$

Moving all terms to one side we get:

 $2\sin x \cos x - 2\cos^2 x - \sin x + \cos x = 0$

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 $(\sin x - \cos x) - (\sin x - \cos x) = 0$

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So we have sin $x = \cos x$, which gives $\tan x = 1$ or from the second bracket $\cos x = \frac{1}{2}$. These are easy to solve. We draw $\tan x$ and $\cos x$ in the interval $-\pi \le x \le \pi$ and get that:

$$x \in \left\{-\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{3}\right\}$$

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[Maximum mark: 8]

Consider the equation
$$\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}, \ 0 < x < \frac{\pi}{2}$$
. Given that $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$
and $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that
$$x = \frac{\pi}{12}$$
 is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0 < x < \frac{\pi}{2}$. [5]

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We will start with the left hand side:

$$LHS = \frac{\sqrt{3} - 1}{\sin\frac{\pi}{12}} + \frac{\sqrt{3} + 1}{\cos\frac{\pi}{12}} =$$

$$= \frac{4(\sqrt{3} - 1)}{\sqrt{6} - \sqrt{2}} + \frac{4(\sqrt{3} + 1)}{\sqrt{6} + \sqrt{2}} =$$

$$= \frac{4(\sqrt{3} - 1)}{\sqrt{2}(\sqrt{3} - 1)} + \frac{4(\sqrt{3} + 1)}{\sqrt{2}(\sqrt{3} + 1)} =$$

$$= \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} =$$

$$= \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} = RHS$$

so $x = \frac{\pi}{12}$ is a solution.

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$(\sqrt{3} - 1)\cos x + (\sqrt{3} + 1)\sin x = 4\sqrt{2}\sin x\cos x$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3} - 1$, the adjacent side is $\sqrt{3} + 1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$. We divide both sides by $2\sqrt{2}$:

$$\frac{\sqrt{3} - 1}{2\sqrt{2}}\cos x + \frac{\sqrt{3} + 1}{2\sqrt{2}}\sin x = 2\sin x \cos x$$

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get $(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$

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Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3} - 1$, the adjacent side is $\sqrt{3} + 1$. The hypotenuse is $2\sqrt{2}$. The angle then becomes $\frac{\pi}{12}$. We divide both sides by $2\sqrt{2}$:



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We get:

$$\sin\frac{\pi}{12}\cos x + \cos\frac{\pi}{12}\sin x = \sin 2x$$

Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

Now there are two ways to proceed. We can move all terms to one side and use the formula for difference of sines:

$$\sin\left(\frac{\pi}{12} + x\right) - \sin 2x = 0$$

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$$\sin\left(\frac{\pi}{12} + x\right) - \sin 2x = 0$$
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$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right) = 0$$

Solving the first part (sine) in the required interval gives:

$$\frac{\frac{\pi}{12} - x}{2} = 0$$

which gives $x = \frac{\pi}{12}$ an answer we already had. Solving the second part (cosine) gives:

$$\frac{\frac{1}{12}+3x}{2}=\frac{\pi}{2}$$

So $x = \frac{11\pi}{36}$ and this is our second solution

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Let's go back to:

$$\sin\!\left(\frac{\pi}{12} + x\right) = \sin 2x$$

and let's discuss another approach.

We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

and this gives $x = \frac{\pi}{12}$. But two sines are also equal if one argument is π minus the other (sin $\alpha = \sin(\pi - \alpha)$), so we could also have:

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Note that in general if we have:

 $\sin \alpha = \sin \beta$

Then:

 $\alpha = \beta$ or $\alpha = \pi - \beta$ or $\alpha = 2\pi + \beta$ or $\alpha = 3\pi - \beta$ or ...

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lpha=eta or $lpha=2\pi-eta$ or $lpha=2\pi+eta$ or $lpha=4\pi-eta$ or ...

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 If we have:

$$\cos \alpha = \cos \beta$$

Then

$$\alpha=\beta \quad \text{or} \quad \alpha=2\pi-\beta \quad \text{or} \quad \alpha=2\pi+\beta \quad \text{or} \quad \alpha=4\pi-\beta \quad \text{or} \quad \dots$$

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IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ), 0^\circ \le x \le 180^\circ$.

(b) Show that
$$\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$$
. [3]

Note that this is part of a longer question which involved topics we haven't covered yet.

for part (a) it makes sense to apply the formula for the sine and cosine of sums.

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[5]

IB exam - problem 4

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We get:

 $2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$

Now this becomes:

$$\sin x + \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$

This gives:

which is equivalent to:

In the given interval we have only one solution, namely $x=150^\circ$

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For part (b) we can calculate both sine and cosine separately using angles 60° and $45^\circ,$ so that we have:

 $LHS = \sin 105^{\circ} + \cos 105^{\circ} =$ = $\sin(60^{\circ} + 45^{\circ}) + \cos(60^{\circ} + 45^{\circ}) =$ = $\sin 60^{\circ} \cos 45^{\circ} + \sin 45^{\circ} \cos 60^{\circ} + \cos 60^{\circ} \cos 45^{\circ} - \sin 60^{\circ} \sin 45^{\circ} =$ = $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS$

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= $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS$

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Alternatively we can change $\cos 105^\circ$ into $-\sin 15^\circ$ and apply the formula for difference of sines:

$$S = \sin 105^{\circ} + \cos 105^{\circ} =$$

$$= \sin 105^{\circ} - \sin 15^{\circ} =$$

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That's it. That's all the basics. Note that all IB questions will require you to solve trigonometric equations in a specific interval, but it's still useful to be aware of the general solution. Make sure you understand all examples discussed in the presentation.

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