

1.*[4 points]*

(a) Write $\sin x - \sqrt{3} \cos x$ in the form $R \sin(x - \alpha)$, where R and α are to be found with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Hence solve the equation:

$$\sin x - \sqrt{3} \cos x = \sqrt{3}$$

for $0 \leq x \leq 2\pi$.

2.

[4 points]

Solve the equation:

$$\cos 2x + \cos 4x = 0$$

for $-\pi \leq x \leq \pi$.

3.

[6 points]

(a) Show that:

$$\sin(2 \arctan x) = \frac{ax}{b + x^2}$$

where a and b are constants to be found.

(b) Hence solve the equation:

$$2 \arctan x = \arcsin\left(\frac{4}{5}\right)$$

4. [6 points]
Given that $\sec \alpha = 2$ and $\cot \beta = -3$ with $0 < \beta < \pi < \alpha < 2\pi$, find the exact value of $\sin(\alpha + 2\beta)$.